

# ROBUST SPARSE MULTICHANNEL ACTIVE NOISE CONTROL

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## ABSTRACT

Multichannel active noise control (MC-ANC) aims to cancel low-frequency noise in an enclosure. If noise sources are distributed sparsely in space, adding an  $\ell_1$ -norm constraint to the standard MC-ANC helps to reduce the complexity of the system and accelerate the convergence rate. However, the convergence performance of  $\ell_1$ -norm constrained MC-ANC ( $c\ell_1$ -MC-ANC) degrades significantly in reverberant environments. In this paper, we analyze the necessity of using sparsity-inducing algorithms with distinct zero-attracting strengths over loudspeakers, and then derive three algorithms of this kind in the complex domain. Simulation results show that, compared to  $c\ell_1$ -MC-ANC, the proposed algorithms exhibit faster convergence or higher noise reduction at steady state in both free field and reverberant environments.

**Index Terms**— Sparsity, ANC, convergence performance, robust

## 1. INTRODUCTION

Active noise control (ANC) is based on the principle of superposition, that is, a secondary noise with equal amplitude and opposite phase is introduced to counteract the primary noise [1, 2]. Multichannel active noise control (MC-ANC) is adopted to deal with noise in an enclosure, which requires multiple secondary sources and error sensors, as well as possible reference sensors depending on whether it is a feedforward [3–7] or feedback [8–10] structure. This paper focuses on the feedback MC-ANC where reference signals are not easily available.

In many applications of MC-ANC, using the secondary noise sources that are close to the primary noise sources is sufficient for noise control. This motivates the development of sparse MC-ANC [11, 12] for a common scenario that primary noise sources are sparsely distributed in space. In [11],

$\ell_1$ -norm constrained MC-ANC ( $c\ell_1$ -MC-ANC) is proposed to inactive the majority of the secondary sources and reduce the complexity of the system. Moreover, it exhibits faster convergence compared to the standard MC-ANC. However, it was only evaluated in the free field. When the sparsity condition is not strictly satisfied, for example in a mildly or even moderately reverberant environment, we find that its convergence performance significantly deteriorates, as shown later in Section 4. In  $c\ell_1$ -MC-ANC, all the secondary sources are attracted to zero with the same strength, which can accelerate the convergence speed of near-zero sources, but introduces bias for large-valued sources. In the case of free field, near-zero sources are in the majority, thus bias is limited. While under reverberation conditions, the effect of reverberation can be represented by using a set of images [13], hence the sparsity of the secondary sources is reduced and more sources tend to have large values. In this case, shrinking all sources uniformly will introduce significant bias and make  $c\ell_1$ -MC-ANC suffer from high steady state residual energy.

Motivated by research on sparse system identification [14–16], this paper proposes three kinds of sparsity constrained MC-ANCs to improve the robustness in reverberant environments. Essentially, we employ a reweighted zero attractor for each secondary source, that is, only loudspeakers with small outputs are shrunk to zero. Simulation results show that the proposed algorithms exhibit faster convergence or higher noise reduction at steady state in both free field and reverberant environments.

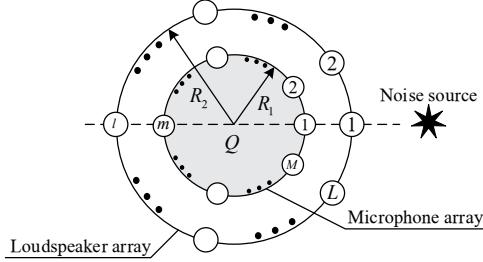
The remainder of this paper is organized as follows. In Section 2, we formulate the problem of feedback MC-ANC. We describe our proposed algorithms in Section 3. The simulation setup and evaluation results are presented in Section 4. We conclude this paper in Section 5.

## 2. BACKGROUND

Let the desired quiet region  $Q$  be a circular zone with radius  $R_1$  and a noise source is located outside of this region as shown in Fig. 1. A uniform circular array, composed of  $M$

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**Fig. 1.** Multichannel feedback ANC.

microphones, is placed along the boundary of the region to pick up residual signals. A uniform concentric circular array with radius  $R_2$  ( $R_2 > R_1$ ), consisting of  $L$  loudspeakers, is placed outside of the region  $Q$  to generate secondary signals.

In the frequency domain, residual signals recorded by error microphones can be written as

$$\epsilon(f) = p(f) + \mathbf{H}d(f), \quad (1)$$

where error signals  $\epsilon$ , primary signals  $p$  are vectors of length  $M$ , the driving signal of loudspeakers  $d$  is a vector of length  $L$ ,  $\mathbf{H}$  is an  $M \times L$  matrix to represent the acoustic transfer functions (ATFs), and  $f$  is frequency.

In order to minimize the total energy of the residual signals, the cost function  $J(n)$  in the standard MC-ANC is defined as  $J(n) = \frac{1}{2}\epsilon^H(n)\epsilon(n)$ , where superscript  $(\cdot)^H$  denotes complex conjugate transpose and  $n$  is the number of iteration. Note that for convenience, frequency  $f$  is not shown.

In the  $c\ell_1$ -MC-ANC [11], a new cost function  $J_{\ell_1}(n)$  is defined by incorporating a complex  $\ell_1$ -norm penalty, that is,

$$J_{\ell_1}(n) = \frac{1}{2}\epsilon^H(n)\epsilon(n) + \lambda\|d(n)\|_1, \quad (2)$$

where  $\lambda \geq 0$  is a regularization parameter to balance the residual energy and the sparsity of driving signals, and  $\|\cdot\|_1$  denotes the  $\ell_1$ -norm of a complex-valued vector.

### 3. PROPOSED ALGORITHMS

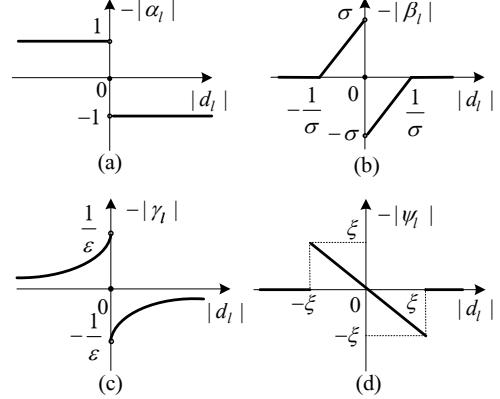
#### 3.1. Problem formulation

Instead of using the  $\ell_1$ -norm, properly designing a sparsity-inducing term can be beneficial in many settings [14–18]. We then write a general form of (2) as

$$J(n) = \frac{1}{2}\epsilon^H(n)\epsilon(n) + \lambda\Omega(d(n)) \quad (3)$$

where  $\Omega(d(n))$  can be any sparse regularization term. By minimizing (3) using the gradient descent method, the driving signals are updated as

$$d(n+1) = d(n) - 2\mu\nabla_d J(n), \quad (4)$$



**Fig. 2.** Zero-attractors of (a)  $c\ell_1$ -MC-ANC (b)  $Q\text{-}c\ell_0$ -MC-ANC (c)  $R\text{-}c\ell_1$ -MC-ANC (d)  $C\text{-}c\ell_2$ -MC-ANC.

where  $\mu > 0$  is the step size, and  $\nabla_d J(\cdot)$  denotes the gradient of  $J(\cdot)$  with respect to the complex vector  $d$ .

As a special case, the gradient of  $J_{\ell_1}(n)$  is derived using Wirtinger calculus [19] as

$$\nabla_d J_{\ell_1}(n) = \frac{1}{2}[\mathbf{H}^H\epsilon(n) + \lambda\alpha(n)] \quad (5)$$

where  $\alpha(n)$ , called *zero-attractor*, is an  $L \times 1$  vector with the  $l$ -th entry given by

$$\alpha_l(n) = \begin{cases} \frac{d_l(n)}{|d_l(n)|}, & |d_l(n)| \neq 0 \\ 0, & |d_l(n)| = 0 \end{cases}, \quad (6)$$

where  $|\cdot|$  denotes the modulus of its complex argument,  $\alpha_l(n)$  and  $d_l(n)$  denote the  $l$ -th entries of  $\alpha(n)$  and  $d(n)$ , respectively.

The relation between  $\alpha_l$  and  $|d_l|$  is shown in Fig. 2(a). It is observed that the zero-attractors of the  $c\ell_1$ -MC-ANC shrink all entries of  $d$  uniformly, which may result in a significant bias if the sparsity of the secondary sources is reduced.

#### 3.2. Proposed sparsity constrained algorithms

To enhance robustness against reverberation, here we introduce three new regularization terms  $\Omega(d(n))$ , which only shrink small-valued sources and avoid introducing significant bias. As will be shown in the simulation part, each of these regularization terms has its own favorable properties, and can be used accordingly in different environments.

##### 3.2.1. Quasi complex $\ell_0$ -norm MC-ANC

The quasi  $\ell_0$ -norm constrained LMS is proposed in [16], but is restricted to the real-valued case. In this paper, we extend it to the MC-ANC with complex-valued operations. The quasi complex  $\ell_0$ -norm constrained MC-ANC (Q- $c\ell_0$ -MC-ANC) is

derived via

$$J_{\ell_0}(n) = \frac{1}{2} \boldsymbol{\epsilon}^H(n) \boldsymbol{\epsilon}(n) + \lambda \sum_{l=1}^L (1 - e^{-\sigma|d_l(n)|}), \quad (7)$$

where  $\sigma > 0$  controls the shrinkage range and strength. The gradient of  $J_{\ell_0}(n)$  is

$$\nabla_d J_{\ell_0}(n) = \frac{1}{2} [\mathbf{H}^H \boldsymbol{\epsilon}(n) + \lambda \boldsymbol{\beta}(n)], \quad (8)$$

where  $\boldsymbol{\beta}(n)$  is an  $L \times 1$  vector with the  $l$ th entry given by

$$\beta_l(n) = \begin{cases} \sigma e^{-\sigma|d_l(n)|} \frac{d_l(n)}{|d_l(n)|}, & |d_l(n)| \neq 0 \\ 0, & |d_l(n)| = 0 \end{cases}, \quad (9)$$

with  $\beta_l(n)$  being the  $l$ th entry of  $\boldsymbol{\beta}(n)$ . Further, we adopt a first-order Taylor expansion to approximate the exponential term in (9) as in [16] to reduce the computational complexity, which leads to

$$e^{-\sigma|d_l(n)|} \approx \begin{cases} 1 - \sigma|d_l(n)|, & |d_l(n)| < 1/\sigma \\ 0, & \text{otherwise} \end{cases}. \quad (10)$$

Visualizing the zero attractors of (9) in Fig. 2(b), it is clear that the Q- $\ell_0$ -MC-ANC does not shrink large  $d_l(n)$ .

### 3.2.2. Reweighted complex $\ell_1$ -norm MC-ANC

Motivated by previous works in system identification [14, 15], we propose a reweighted complex  $\ell_1$ -norm constrained MC-ANC (R- $\ell_1$ -MC-ANC), with the cost function defined by

$$J_R(n) = \frac{1}{2} \boldsymbol{\epsilon}^H(n) \boldsymbol{\epsilon}(n) + \lambda \sum_{l=1}^L \log[1 + |d_l(n)|/\varepsilon], \quad (11)$$

where the positive parameter  $\varepsilon$  controls the shrinkage range. The log-sum penalty  $\sum_{l=1}^L \log[1 + |d_l(n)|/\varepsilon]$  is considered, as it behaves more similarly to the  $\ell_0$ -norm than to  $\|\mathbf{d}(n)\|_1$  [20]. The derivative of  $J_R(n)$  is

$$\nabla_d J_R(n) = \frac{1}{2} [\mathbf{H}^H \boldsymbol{\epsilon}(n) + \lambda \boldsymbol{\gamma}(n)], \quad (12)$$

with the  $l$ th entry of  $\boldsymbol{\gamma}(n)$  given by

$$\gamma_l(n) = \begin{cases} \frac{1}{|d_l(n)|+\varepsilon} \cdot \frac{d_l(n)}{|d_l(n)|}, & |d_l(n)| \neq 0 \\ 0, & |d_l(n)| = 0 \end{cases}. \quad (13)$$

The property of  $\gamma_l(n)$  is shown in Fig. 2(c), which shrinks less to larger  $d_l(n)$ .

### 3.2.3. Clipped complex $\ell_2$ -norm MC-ANC

It is obvious that both Q- $\ell_0$ -MC-ANC and R- $\ell_1$ -MC-ANC are dedicated to avoid introducing bias for  $d_l$  with large values. However, for  $d_l$  with small values, the shrinkage strength

**Table 1.** Hyperparameter Settings.

	free field					reverberant field				
	$\mu$	$\lambda$	$\sigma$	$\varepsilon$	$\xi$	$\mu$	$\lambda$	$\sigma$	$\varepsilon$	$\xi$
MC-ANC	5	-	-	-	-	4	-	-	-	-
c $\ell_1$ -MC-ANC	5	0.01	-	-	-	4	0.02	-	-	-
Q-c $\ell_0$ -MC-ANC	5	0.02	0.5	-	-	4	0.05	0.25	-	-
R-c $\ell_1$ -MC-ANC	5	0.02	-	2.2	-	4	0.05	-	3	-
C-c $\ell_2$ -MC-ANC	5	0.02	-	-	2	4	0.05	-	-	1

is preferably proportional to the value of  $|d_l|$ . Based on this, we propose a clipped complex  $\ell_2$ -norm MC-ANC (C-c $\ell_2$ -MC-ANC), that is,

$$J_{c\ell_2}(n) = \frac{1}{2} \boldsymbol{\epsilon}^H(n) \boldsymbol{\epsilon}(n) + \frac{\lambda}{2} \sum_{l=1}^L \min\{d_l^2(n), \xi^2\}, \quad (14)$$

where  $\xi > 0$  is a predefined threshold used to control the shrinkage range, and  $\min\{\cdot, \cdot\}$  denotes the minimum operator. The gradient of  $J_{c\ell_2}(n)$  is

$$\nabla_d J_{c\ell_2}(n) = \frac{1}{2} [\mathbf{H}^H \boldsymbol{\epsilon}(n) + \lambda \boldsymbol{\psi}(n)], \quad (15)$$

with the  $l$ th entry of  $\boldsymbol{\psi}(n)$  derived as

$$\psi_l(n) = \begin{cases} d_l(n), & |d_l(n)| < \xi \\ 0, & \text{otherwise} \end{cases}. \quad (16)$$

The property of  $\psi_l(n)$  is illustrated in Fig. 2(d).

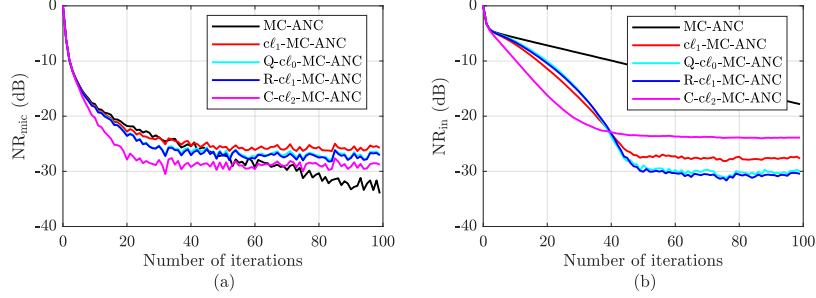
## 4. SIMULATION

### 4.1. Simulation setup

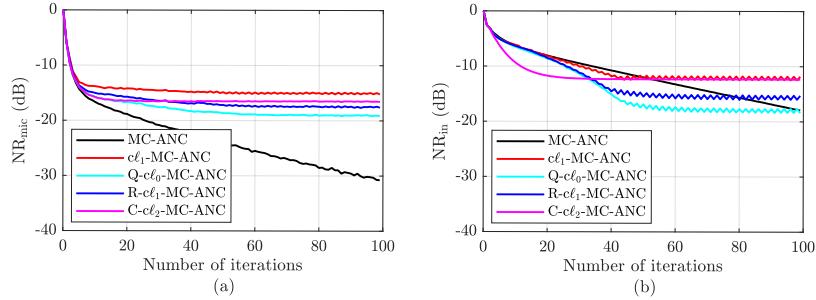
Assume that the desired quiet zone  $Q$  is a circular region with a radius of 1 m. We place 11 microphones equally along the boundary of the region  $Q$  to pick up residual signals, and 11 loudspeakers equally on a concentric circle with a radius of 2 m to generate secondary signals. A primary noise source is located 2.5 m away from the circle center at the same height, and its magnitude and frequency are 10 and 200 Hz, respectively. As in [11], we assume that both the noise source and loudspeakers as cylindrical wave sources, and white Gaussian noise with SNR of 40 dB is added to each microphone recordings to model internal thermal noise or measurement errors. The speed of sound is  $c = 343$  m/s.

To mimic a moderate reverberation environment, we generate a simulated room with size 6 m  $\times$  5 m (length  $\times$  width) using the image source method [20]. The center of the room coincides with the center of the loudspeaker/microphone array. The ceiling and floor are perfectly absorbing and all side walls have a reflection coefficient of 0.5.

All parameters used in the algorithms are listed in Table 1. We evaluate the performance according to the noise reduction (NR) at the position of error microphones [ $\text{NR}_{\text{mic}}(n)$ ] and



**Fig. 3.** Analysis of the convergence speed and noise reduction ability of the compared methods in a free field environment.  
(a)  $\text{NR}_{\text{mic}}(n)$ , (b)  $\text{NR}_{\text{in}}(n)$ .



**Fig. 4.** Analysis of the convergence speed and noise reduction ability of the compared methods in a reverberant environment.  
(a)  $\text{NR}_{\text{mic}}(n)$ , (b)  $\text{NR}_{\text{in}}(n)$ .

noise reduction inside the control region [ $\text{NR}_{\text{in}}(n)$ ] as in [11]. For the latter, 1296 sampling points are uniformly chosen in the control region to evaluate the system performance.

## 4.2. Results

Figure 3 illustrates the convergence performance of all algorithms in the free field. It is observed that, compared to MC-ANC, all sparsity-constrained ANC algorithms increase the convergence rate, especially for noise reduction inside the region as shown in Fig. 3(b). For the sparsity-constrained ANC algorithms, the proposed Q-cℓ<sub>0</sub>-MC-ANC and R-cℓ<sub>1</sub>-MC-ANC have similar convergence speed compared to cℓ<sub>1</sub>-MC-ANC, but have higher noise reduction at steady state. Among all these algorithms, C-cℓ<sub>2</sub>-MC-ANC has the fastest convergence rate but least noise reduction at steady state, which is preferable for rapidly-changing environments.

We then evaluate the performance of all above algorithms in the case of a reverberant environment, and the results are shown in Fig. 4. In Fig. 4(a) for  $\text{NR}_{\text{mic}}(n)$ , MC-ANC has the best performance and all sparsity-promoting algorithms prematurely reach steady state. Inside the control region, Fig. 4(b) shows that, the cℓ<sub>1</sub>-MC-ANC has a particularly high misadjustment at steady state due to the bias introduced by ℓ<sub>1</sub>-norm, and shows almost no advantage over MC-ANC. All proposed sparsity-constrained algorithms have larger noise reduction at steady state than cℓ<sub>1</sub>-MC-ANC and exhibit faster

convergence than MC-ANC, which is a favorable property for an adaptive system. Among the proposed algorithms, Q-cℓ<sub>0</sub>-MC-ANC and R-cℓ<sub>1</sub>-MC-ANC have higher noise reduction at steady state, while C-cℓ<sub>2</sub>-MC-ANC has faster convergence. This suggests that the selection of Q-cℓ<sub>0</sub>-MC-ANC, R-cℓ<sub>1</sub>-MC-ANC, or C-cℓ<sub>2</sub>-MC-ANC depends on the degree of nonstationarity of the environment. The C-cℓ<sub>2</sub>-MC-ANC is preferable for a rapidly-changing environment and Q-cℓ<sub>0</sub>-MC-ANC is a better choice for a relatively stable environment.

## 5. CONCLUSION

In this paper, we proposed three kinds of sparse MC-ANC algorithms, called Q-cℓ<sub>0</sub>-MC-ANC, R-cℓ<sub>1</sub>-MC-ANC, and C-cℓ<sub>2</sub>-MC-ANC, to improve the robustness of MC-ANC in reverberant environments when noise sources are sparsely distributed in space. Compared to the existing cℓ<sub>1</sub>-MC-ANC algorithm, the proposed algorithms selectively shrink those secondary sources with small outputs. Simulation results demonstrated the effectiveness of the proposed algorithms in both free field and reverberant environments, in terms of both noise reduction ability and convergence speed. Our future work includes finding a systematic way to choose model parameters and investigate the variable-parameter method to further improve the robustness.

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