Fluid Simulation for Computer Animation

Why Simulate Fluids?

- Feature film special effects
- Computer games
- Medicine (e.g. blood flow in heart)
- Because it's fun

Fluid Simulation

- Called *Computational Fluid Dynamics* (CFD)
- Many approaches from math and engineering
- Graphics favors *finite differences*
- Jos Stam introduced fast and stable methods to graphics [Stam 1999]

Navier-Stokes Equations

$$\nabla \cdot \mathbf{u} = 0$$ Incompressibility

$$\underbrace{\mathbf{u}_t}_{\text{Change in Velocity}} = \underbrace{k\nabla^2 \mathbf{u}}_{\text{Diffusion}} - \underbrace{(\mathbf{u} \cdot \nabla)\mathbf{u}}_{\text{Advection}} - \underbrace{\nabla p}_{\text{Pressure}} + \underbrace{\mathbf{f}}_{\text{Body Forces}}$$

Navier-Stokes Equations

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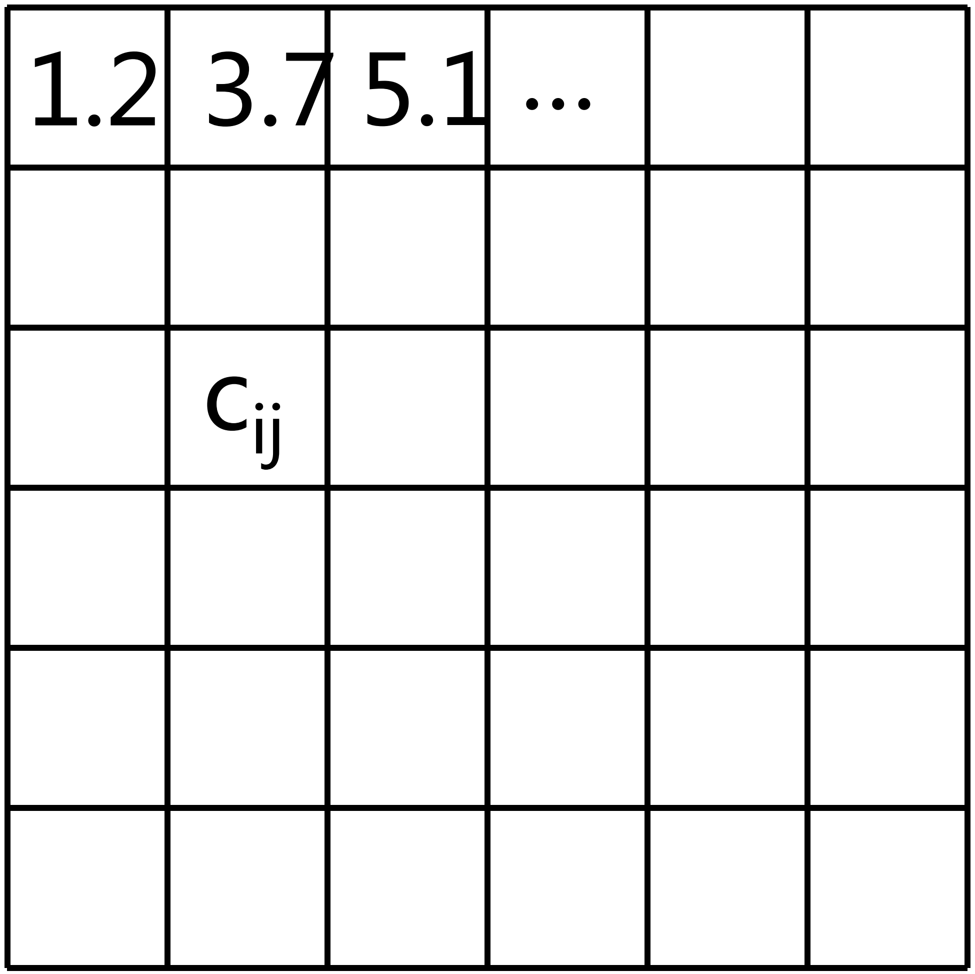
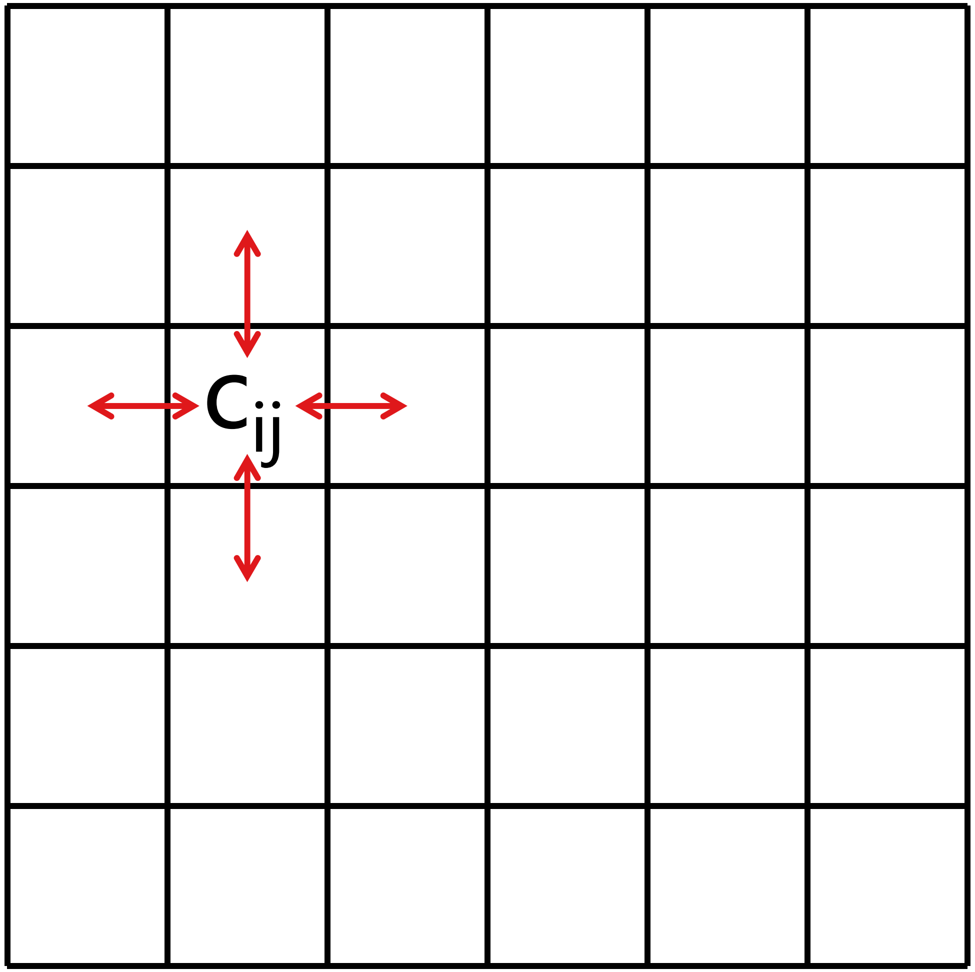
Finite Differences Grids

- All values live on regular grids
- Need *scalar* and *vector* fields
- Scalar fields: amount of smoke or dye
- Vector fields: fluid velocity
- Subtract adjacent quantities to approximate derivatives

Scalar Field (Smoke, Dye)



Diffusion



Diffusion

$$c_t = k\nabla^2 c$$

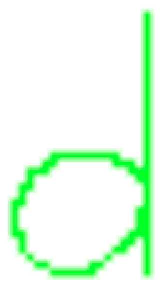
change in value

value relative to neighbors

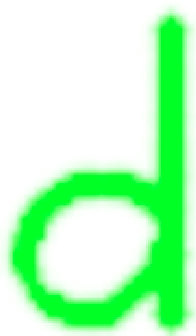
	1	
1	-4	1
	1	

$$c_{ij}^{new} = c_{ij} + k\,\Delta t\,(c_{i-1j} + c_{i+1j} + c_{ij-1} + c_{ij+1}$$

Diffusion = Blurring



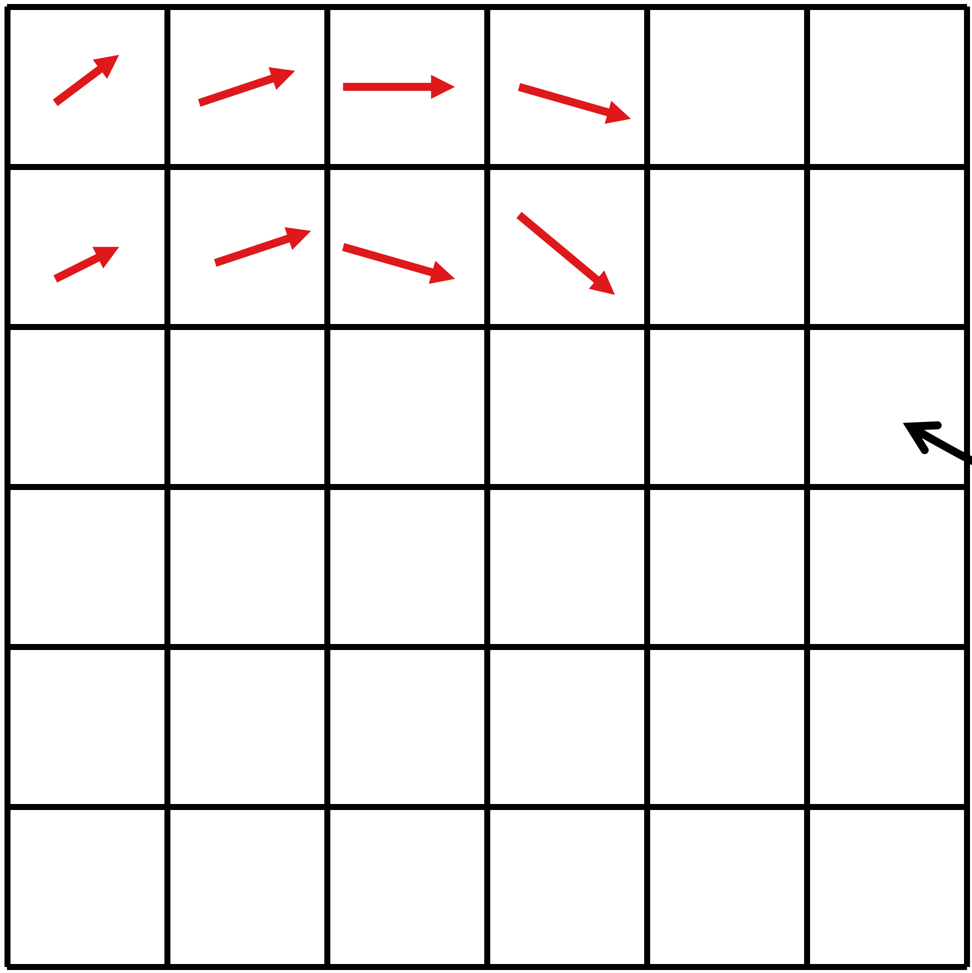
Original



Some Diffusion

More Diffusion

Vector Fields (Fluid Velocity)



$\mathbf{u}_{ij} = (u^x, u^y)$

Vector Field Diffusion

$$\mathbf{u}_t = \mathrm{k}\nabla^2\mathbf{u}$$

viscosity

Two separate diffusions:

$$\mathrm{u}^{\mathrm{x}}{}_t = \mathrm{k}\nabla^2\mathrm{u}^{\mathrm{x}}$$

$$\mathrm{u}^{\mathrm{y}}{}_t = \mathrm{k}\nabla^2\mathrm{u}^{\mathrm{y}}$$

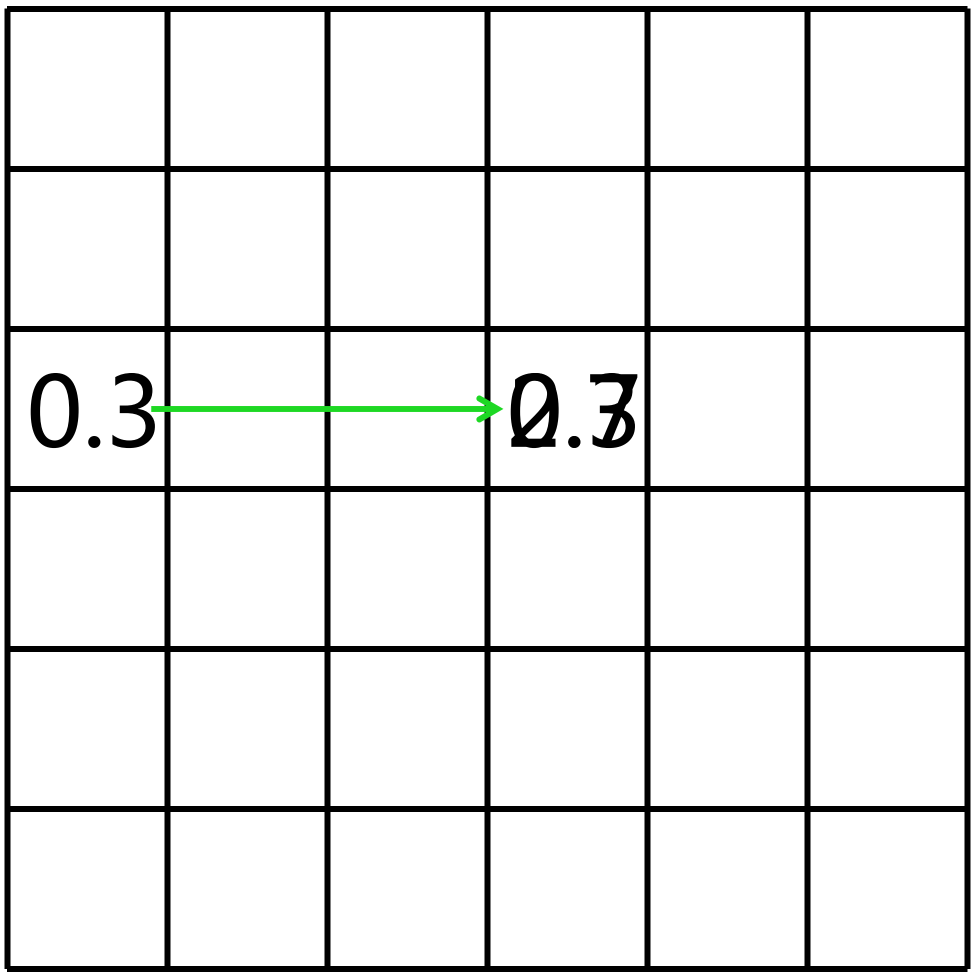
… blur the *x*-velocity and the *y*-velocit

Navier-Stokes Equations

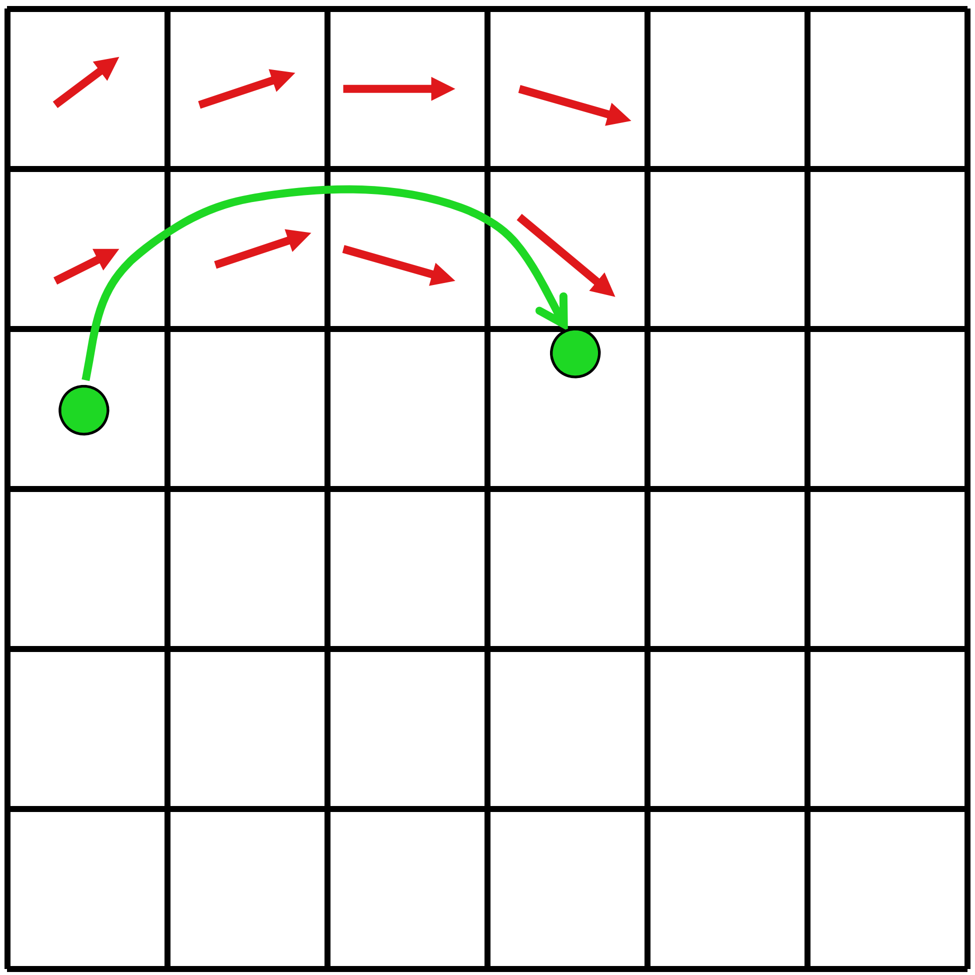
$\nabla \cdot \mathbf{u} = 0$ Incompressibility

$$\underbrace{\mathbf{u}_t}_{\text{Change in Velocity}} = \underbrace{k\nabla^2 \mathbf{u}}_{\text{Diffusion}} \underbrace{-(\mathbf{u}\cdot\nabla)\mathbf{u}}_{\text{Advection}} \underbrace{- \nabla p}_{\text{Pressure}} + \underbrace{\mathbf{f}}_{\text{Body Forces}}$$

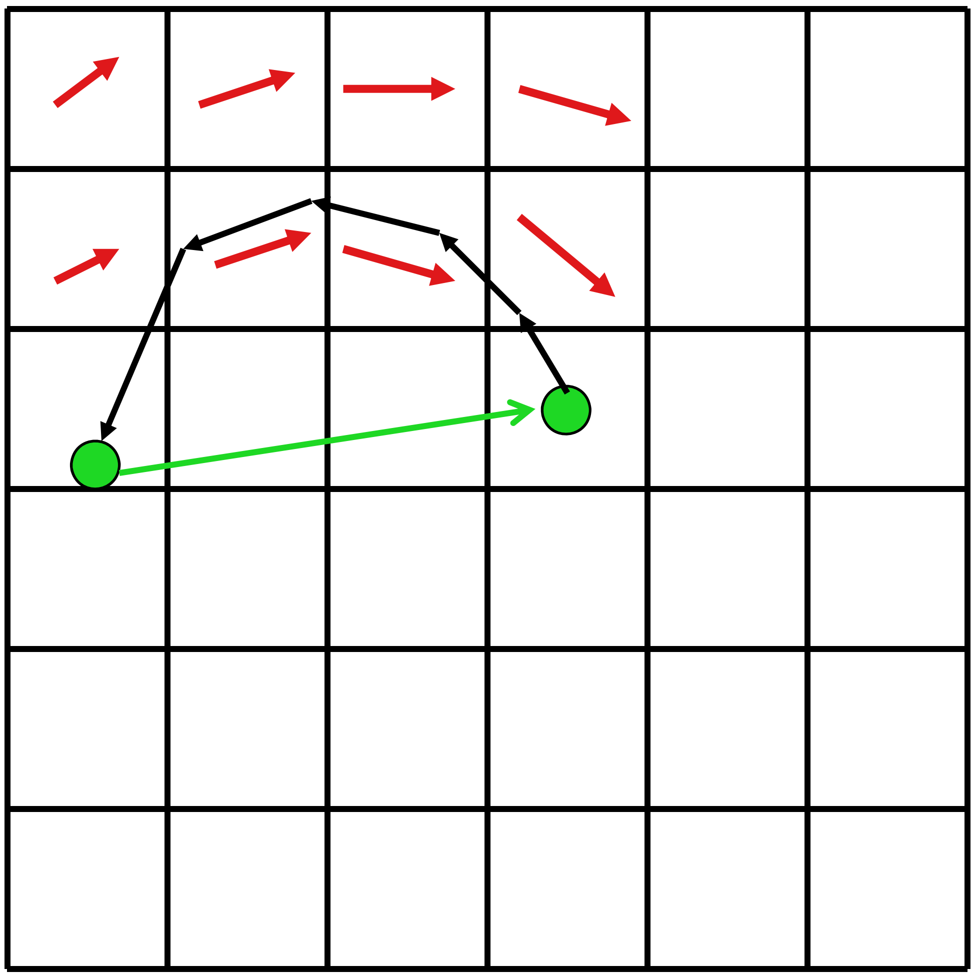
Advection: stuffs get pushed by the flow



Forward Advection



Backward Advection



Scalar Field Advection

$$c_t = -(\mathbf{u} \cdot \nabla) c$$

change in
value

advectio
n

current values

Vector Field Advection

$$\mathbf{u}_t = -(\mathbf{u} \cdot \nabla)\mathbf{u}$$

Two separate advections:

$$\mathrm{u}^{\mathrm{x}}{}_t = -(\mathbf{u} \cdot \nabla)\mathrm{u}^{\mathrm{x}}$$

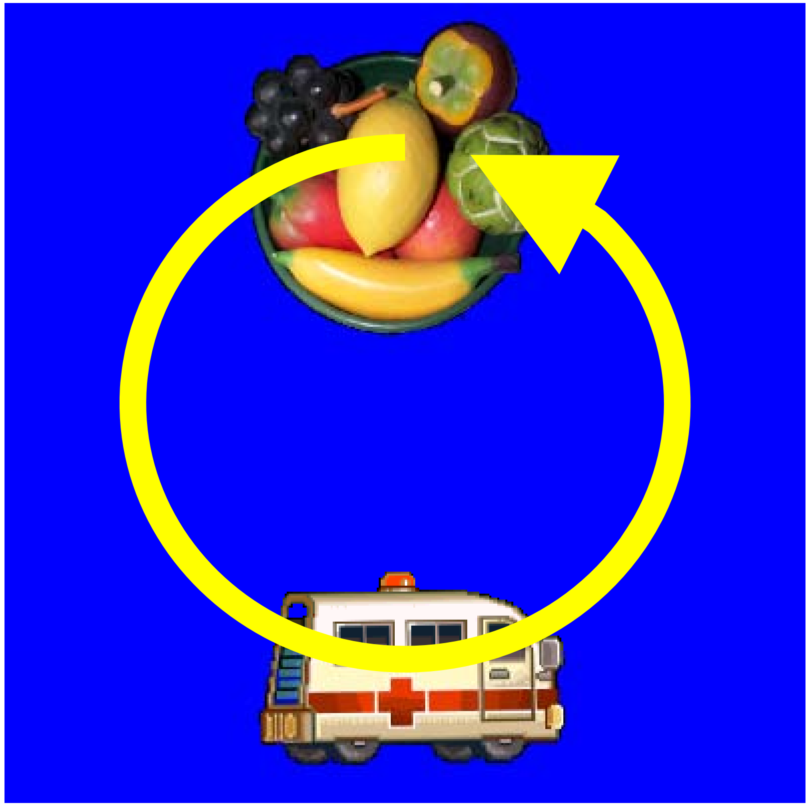
$$\mathrm{u}^{\mathrm{y}}{}_t = -(\mathbf{u} \cdot \nabla)\mathrm{u}^{\mathrm{y}}$$

… push around *x*-velocity and *y*-velocity

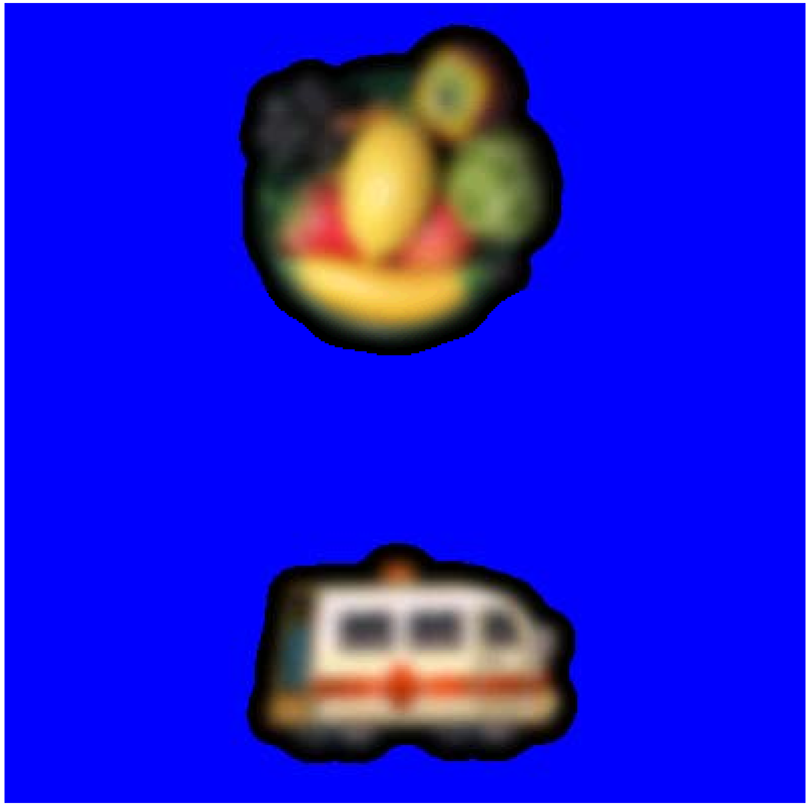
Advection

- Easy to code
- Method stable even at large time steps
- Problem: numerical inaccuracy diffuses flow

Diffusion/dissipation in first order advection



Original Image



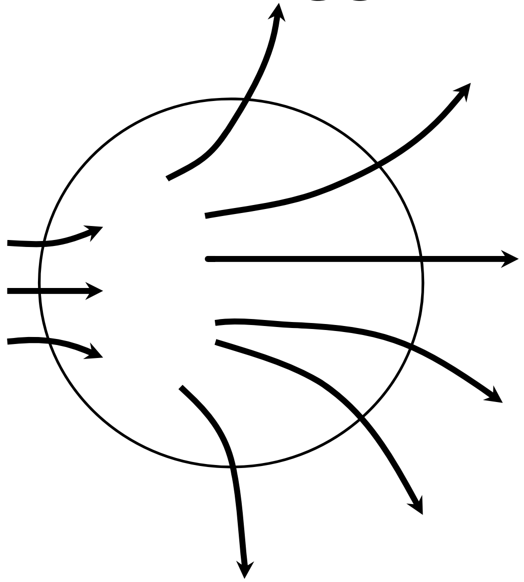
After 360 degree rotation using first order advection

Navier-Stokes Equations

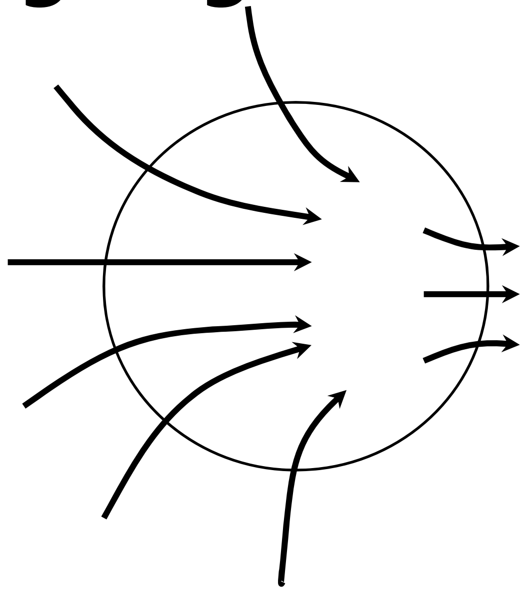
$$\nabla\cdot\mathbf{u}=0 \quad \text{Incompressibility}$$

$$\underbrace{\mathbf{u}_t}_{\text{Change in Velocity}} = \underbrace{k\nabla^2\mathbf{u}}_{\text{Diffusion}} - \underbrace{(\mathbf{u}\cdot\nabla)\mathbf{u}}_{\text{Advection}} - \underbrace{\nabla p}_{\text{Pressure}} + \underbrace{\mathbf{f}}_{\text{Body Forces}}$$

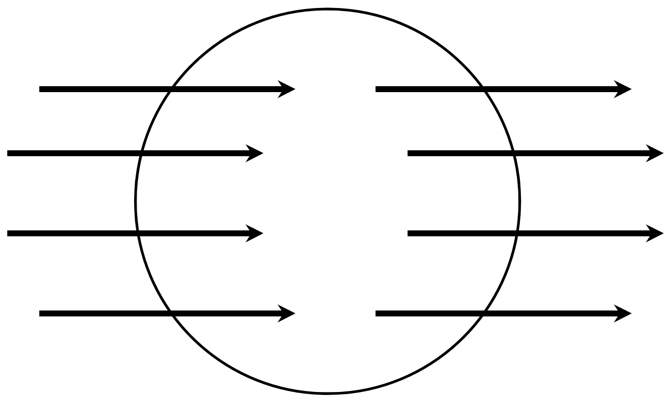
Divergence: Difference between incoming and outgoing flow



High divergence



Low divergence



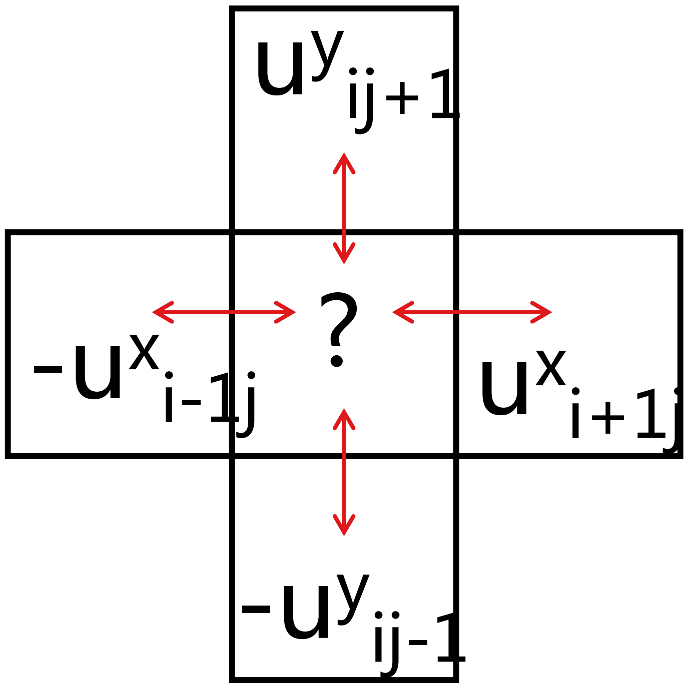
Zero divergence

Enforcing Incompressibility

- First do velocity diffusion and advection
- Find "closest" vector field that is divergence-free
- Need to calculate divergence
- Need to find and use pressure

Measuring Divergence

$$\nabla\cdot\mathbf{u}=?$$



$$\nabla\cdot\mathbf{u}_{ij}=(u^x_{i+1j} - u^x_{i-1j}) + (u^y_{ij+1} - u^y_{ij-1})$$

Pressure Term

$$\mathbf{u}^{new} = \mathbf{u} - \nabla p$$

Take divergence of both sides…

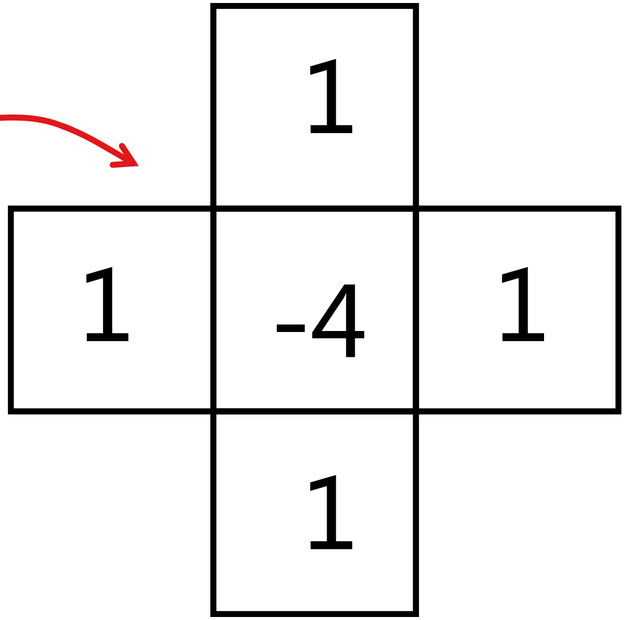
$$\underbrace{\nabla \cdot \mathbf{u}^{new}}_{\text{zero}} = \nabla \cdot \mathbf{u} - \nabla \cdot \nabla p$$

$$\nabla \cdot \mathbf{u} = \nabla^2 p$$

Pressure Term

$$\underbrace{\nabla\cdot\mathbf{u}}_{\text{known}} = \underbrace{\nabla^2 p}_{\text{unknown}}$$

$$p^{new} = p + \varepsilon(\nabla\cdot\mathbf{u} - \nabla^2 p)$$



Let $d_{ij} = \nabla\cdot\mathbf{u}_{ij}$

$$p^{new}_{ij} = p_{ij} + \varepsilon(d_{ij} - (p_{i-1j} + p_{i+1j} + p_{ij-1} + p_{ij+1} - 4$$

Pressure Term

$$\mathbf{u}^{new} = \mathbf{u} - \nabla p$$

…and velocity is now divergence-free

Found “nearest” divergence-free ve field to original.

Fluid Simulator

1) Diffuse velocity
2) Advect velocity
3) Add body forces (e.g. gravity)
4) Pressure projection
5) Diffuse dye/smoke
6) Advect dye/smoke

“Fluid simulation”

SIGGRAPH 2006/2007 Course Note

https://www.cs.ubc.ca/~rbridson/

"Real-Time Fluid Dynamics for Game
JosStam, March 2003
(CDROM link is to source code)

www.dgp.toronto.edu/people/stam/reality/Research/

Research Areas

1) Thin Features
2) Surface tension
3) Coupling
4) Fluid-like behaviors (sand, snow, et