Computer Animation Algorithms and Techniques

Collisions & Contact

Collision handling
detection & response

detection

Particle-plane collision detection

Polyhedron-polyhedron collision detection

- **overlap of Bounding volumes**
- **Vertex inside polyhedron test**
  - **Concave case**
  - **Convex case**
- **Edge-face intersection test**

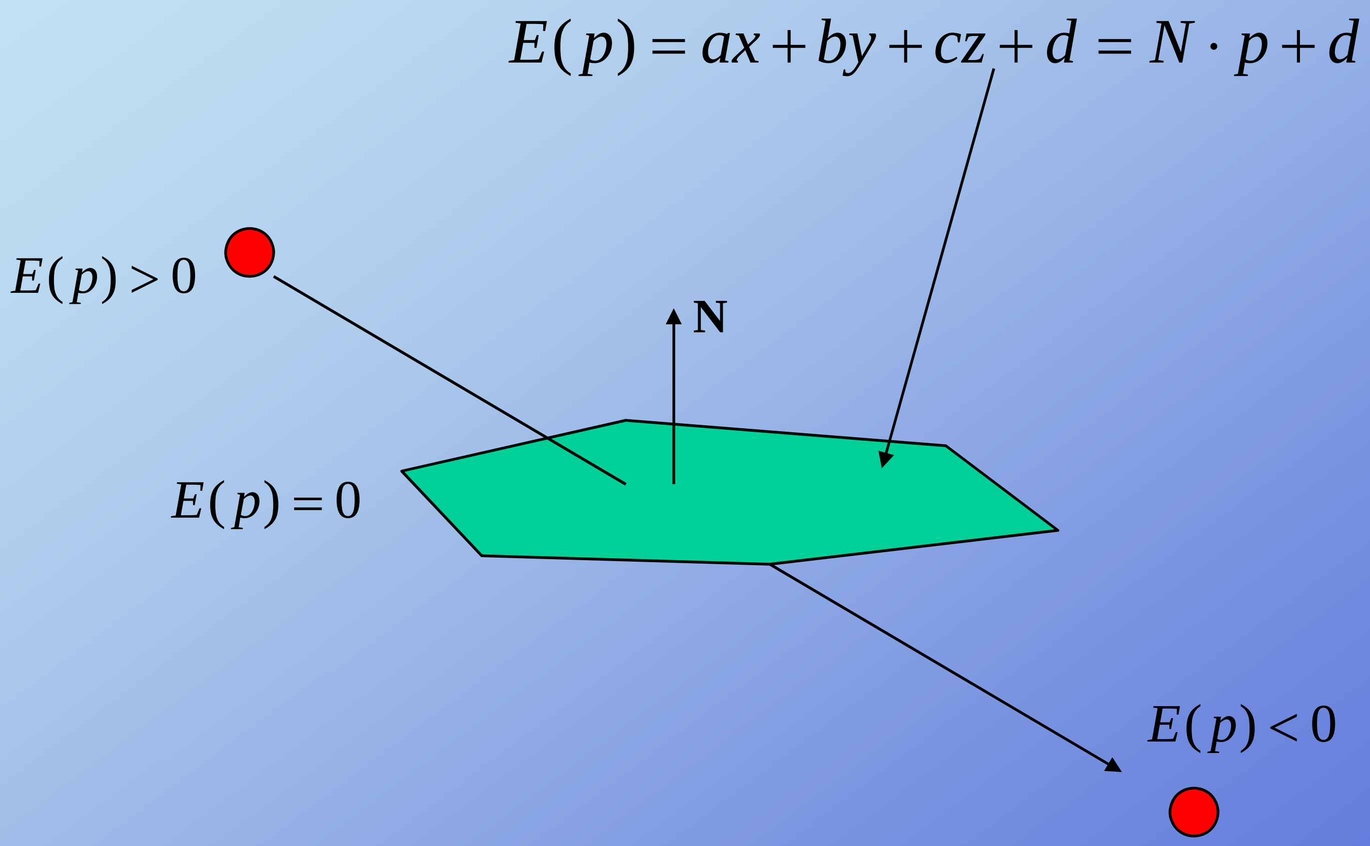
response

kinematic response

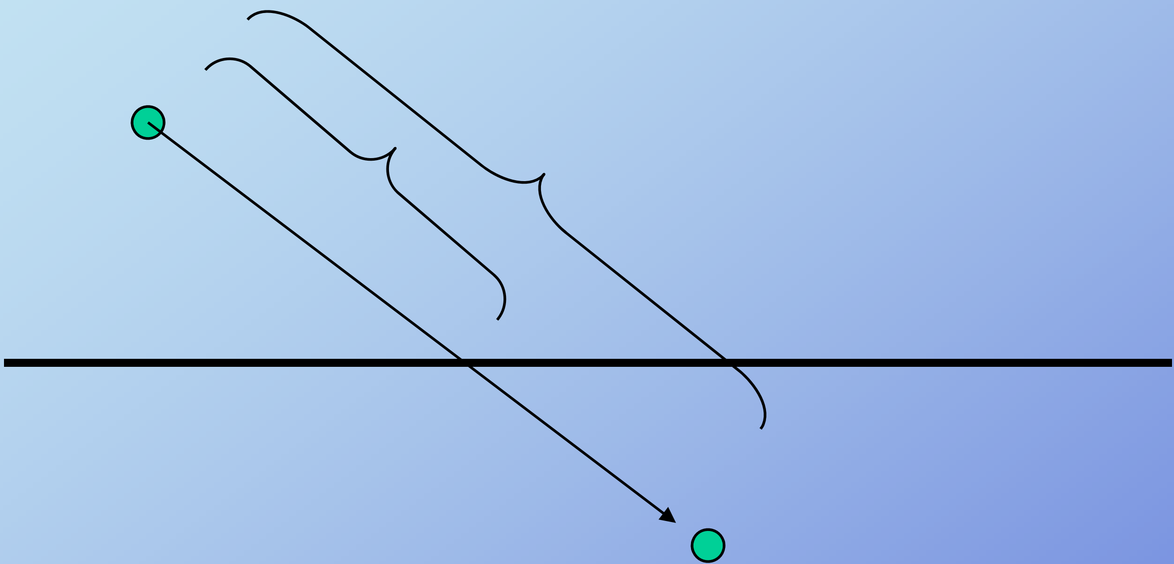
Penalty method

Impulse force of collision

Collision detection: point-plane



Collision detection: time of impact



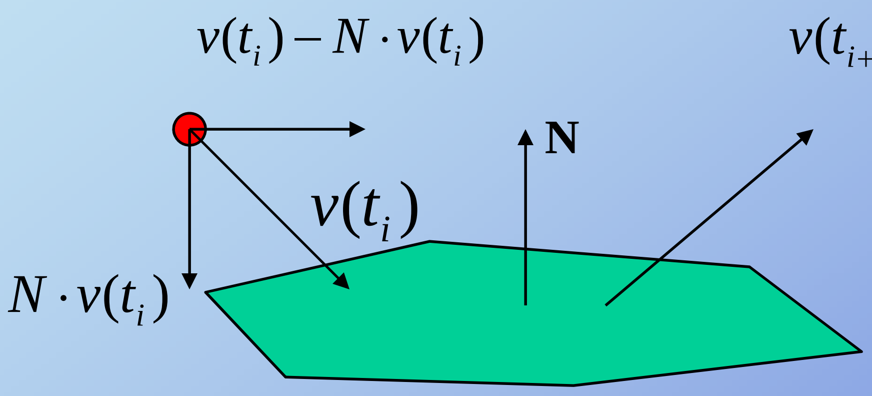
<u>2 options</u>

Consider collision at next time step

Compute fractional time at which collision actually occurred

Tradeoff: accuracy v. complexity

Collision response: kinematic

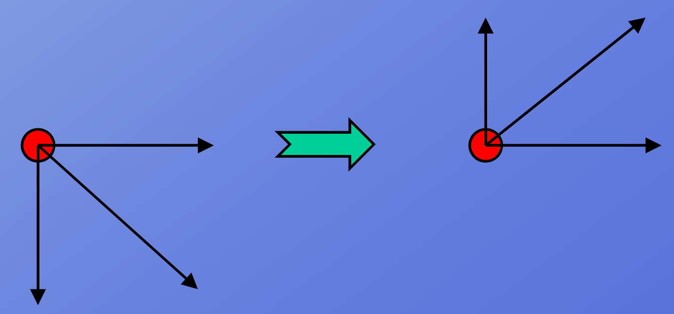


$$
\begin{aligned}
v(t_{i+1}) &= v(t_i) - N \bullet v(t_i) - k(N \bullet v(t_i)) \\
&= v(t_i) - (1+k)N \bullet v(t_i)
\end{aligned}
$$

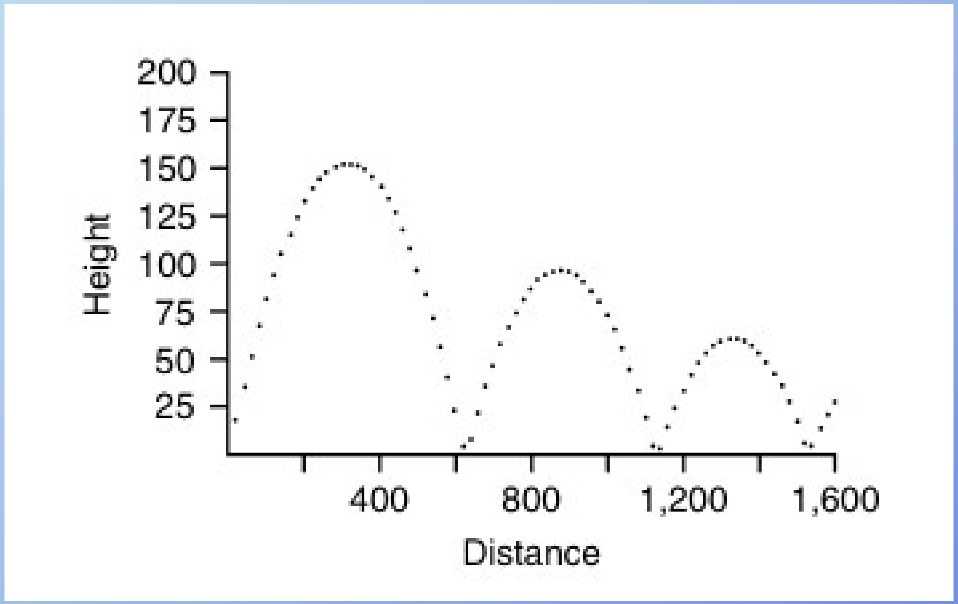
k – damping factor
=1 indicates no energy loss

Negate component of velocity in direction of normal

No forces involved!

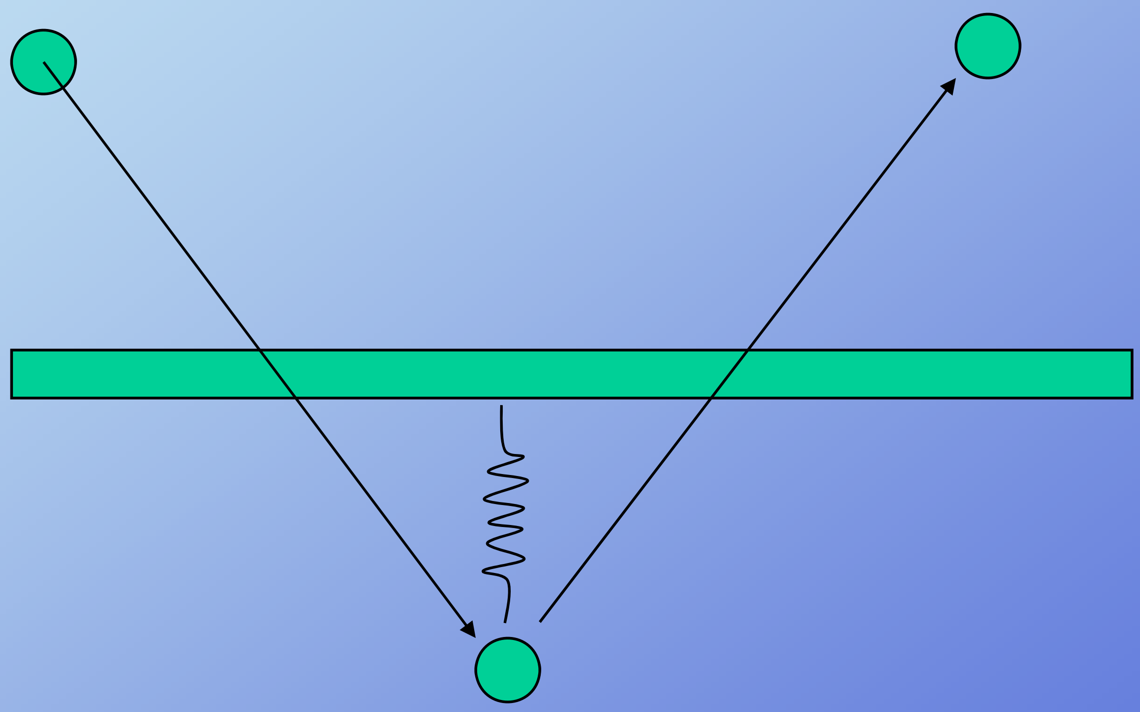


Collision response: damped

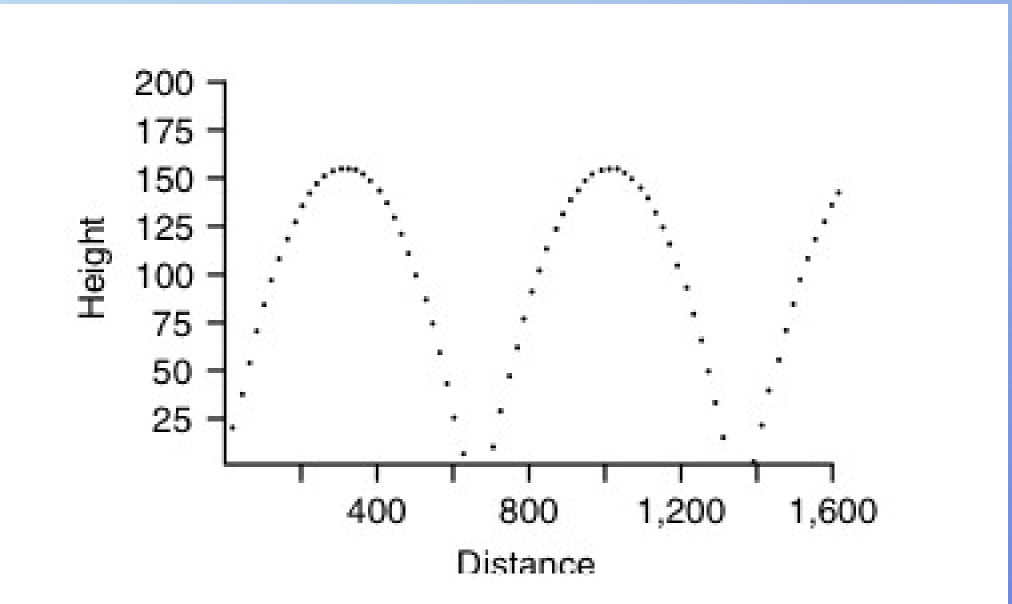


Damping factor = 0.8

Collision response – penalty method



Collision response: penalty



Collision detection: polyhedra

Order tests according to computational complexity and power of detection

1. test bounding volumes for overlap

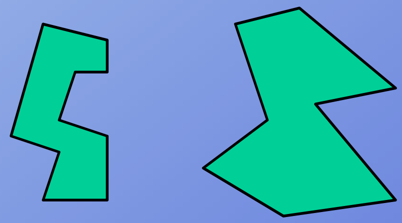
2. test for vertex of one object inside of other object

3. test for edge of one object intersecting face of other object

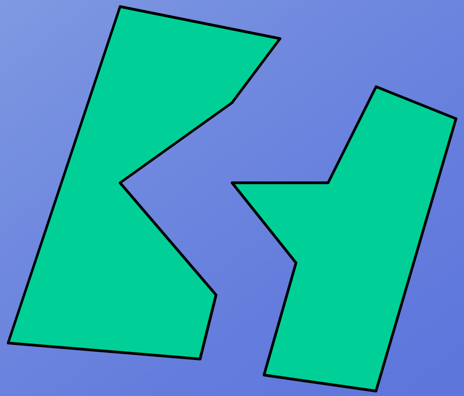
Collision detection: bounding volumes

Don't do vertex/edge intersection testing if there's no chance of an intersection between the polyhedra

Want a simple test to remove easy cases



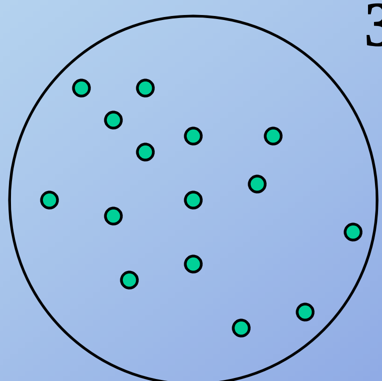
Tradeoff complexity of test with power to reject non-intersecting polyhedra (goodness of fit of bounding volume)

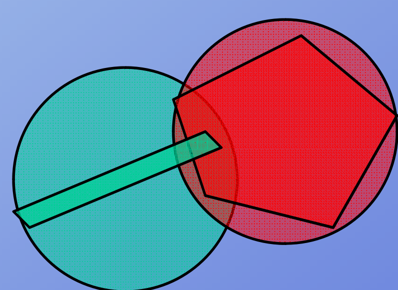


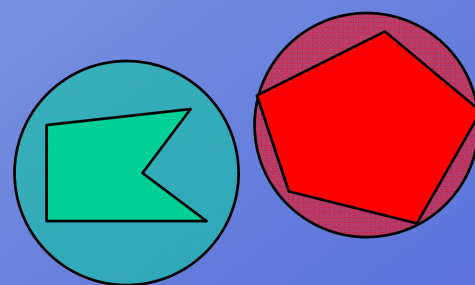
Bounding Spheres

Compute bounding sphere of vertices
Compute in object space and transform with object

1. Find min/max pair of points in each dimension
2. use maximally separated pair – use to create initial bounding sphere (midpoint is center)
3. for each vertex adjust sphere to include point



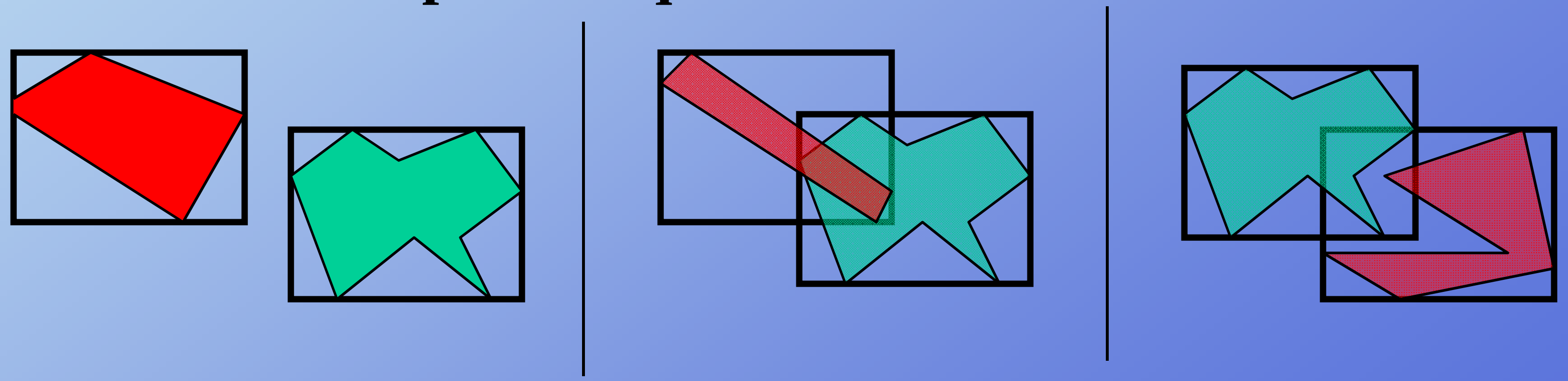




Bounding Boxes

Axis-aligned (AABB): use min/max in each dimension

Oriented (OBB): e.g., use AABB in object space and transform with object. Vertex is inside of OBB iff on inside of 6 planar equations

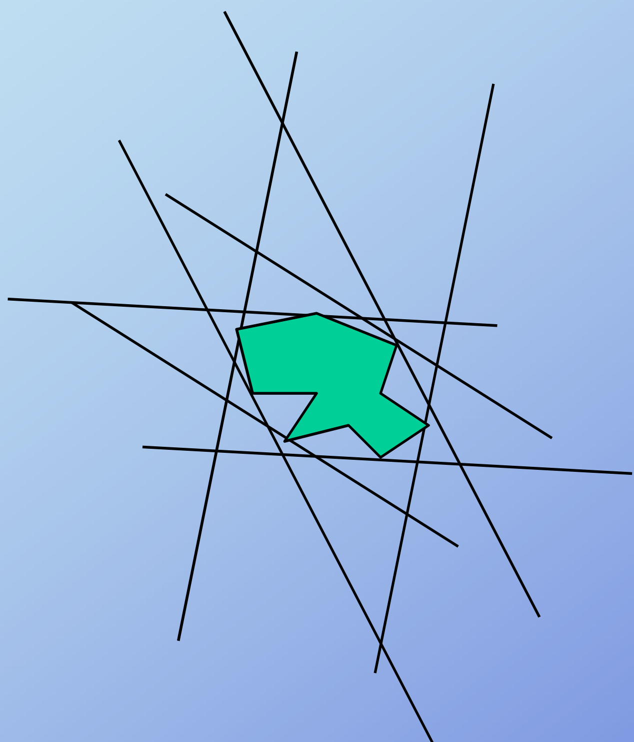


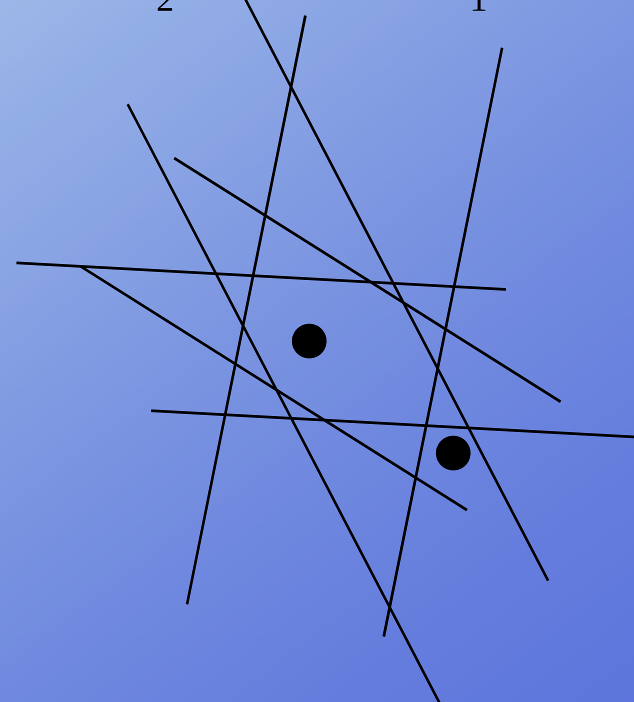
Bounding Slabs

For better fit bounding polyhedron: use arbitrary (user-specified) collection of bounding plane-pairs

Is a vertex between each pair?

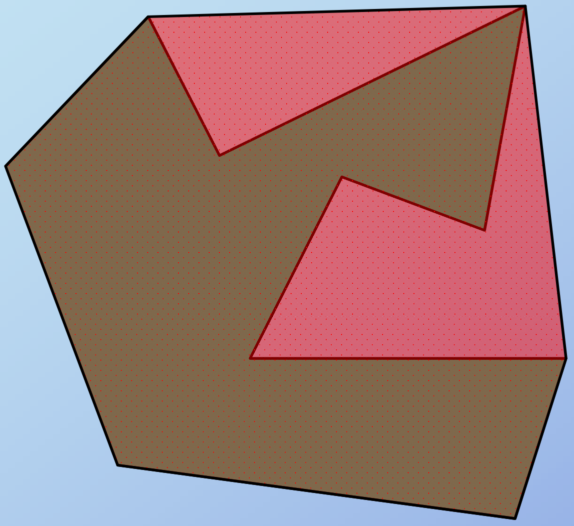
$$d_2 < N \bullet P < d_1$$





Convex Hull

Best fit convex polyhedron to concave polyhedron but takes some (one-time) computation



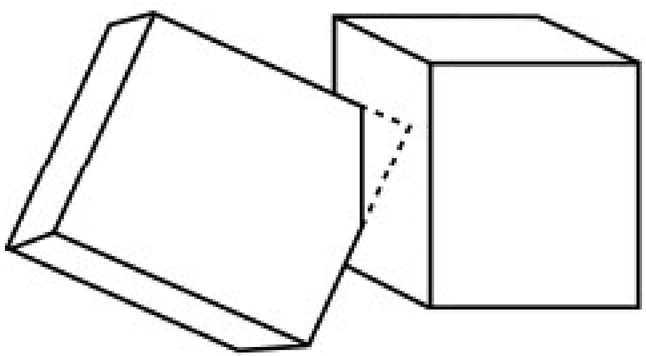
1. Find highest vertex, V1
2. Find remaining vertex that minimizes angle with horizontal plane through point. Call edge L
3. Form plane with this edge and horizontal line perpendicular to L at V1
4. Find remaining vertex that for triangle that minimizes angle with this plane. Add this triangle to convex hull, mark edges as *unmatched*
5. For each unmatched edge, find remaining vertex that minimizes angle with the plane of the edge's triangle

Collision detection: polyhedra

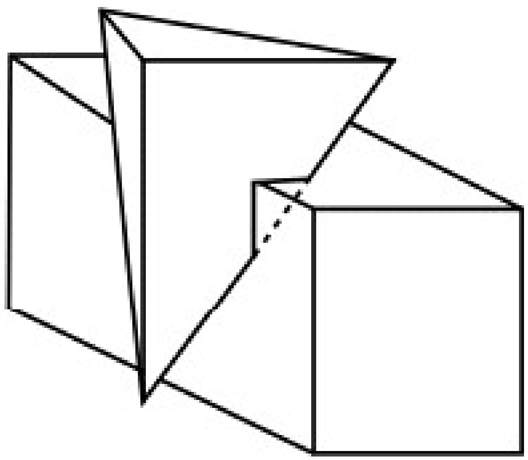
1. test bounding volumes for overlap

2. test for vertex of one object inside of other object

3. test for edge of one object intersecting face of other object



Vertex inside a polyhedron

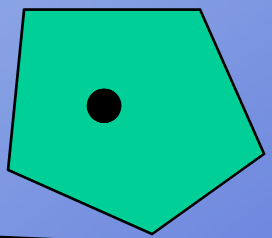


Object penetration without a vertex of one object contained in the other

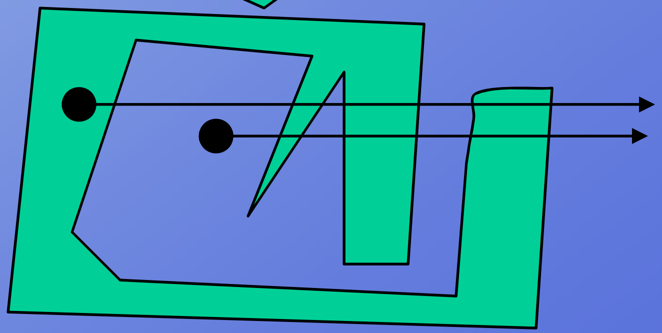
Collision detection: polyhedra

Intersection = NO
For each vertex, V, of object A
    if (V is inside of B) intersection = YES
For each vertex, V, of object B
    if (V is inside of A) intersection = YES

A vertex is inside a convex polyhedron if it's on the 'inside' side of all faces

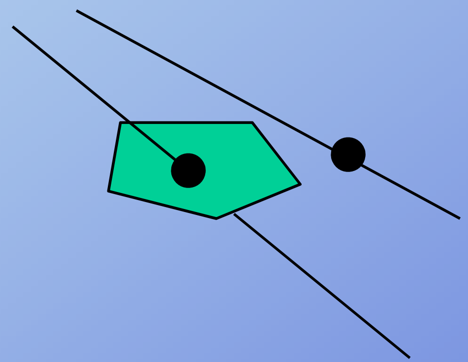


A vertex is inside a cancave polyhedron if a semi-infinite ray from the vertex intersects an odd number of faces



Collision detection: polyhedra

Edge intersection face test
Finds ALL polyhedral intersections
But is most expensive test



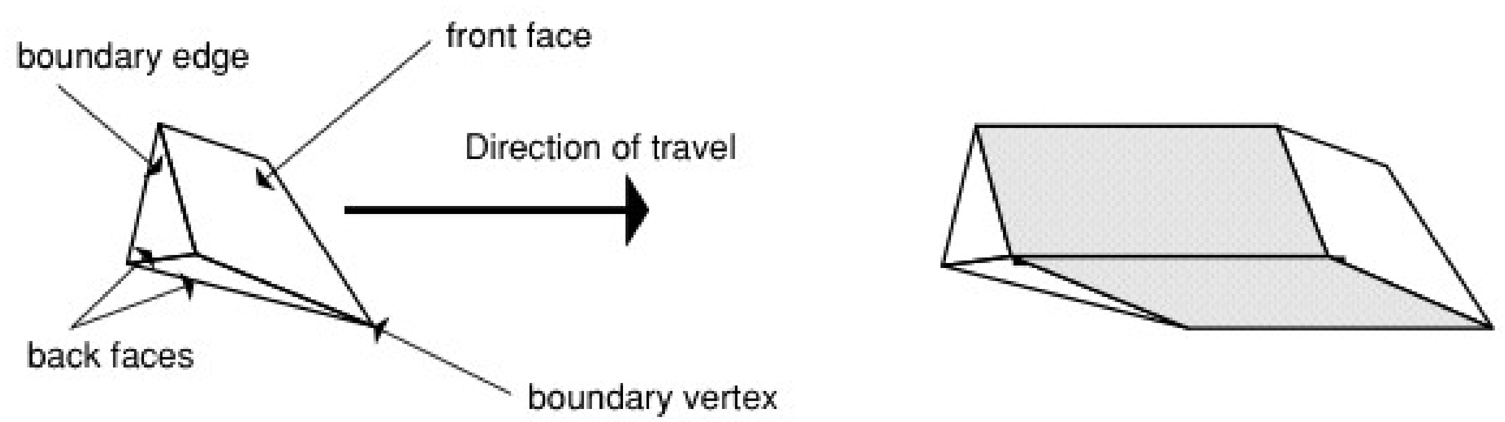
If vertices of edges are on opposite side of plane of face

Calculate intersection of edge with plane

Test vertex for inside face (2D test in plane of face)

Collision detection: swept volume

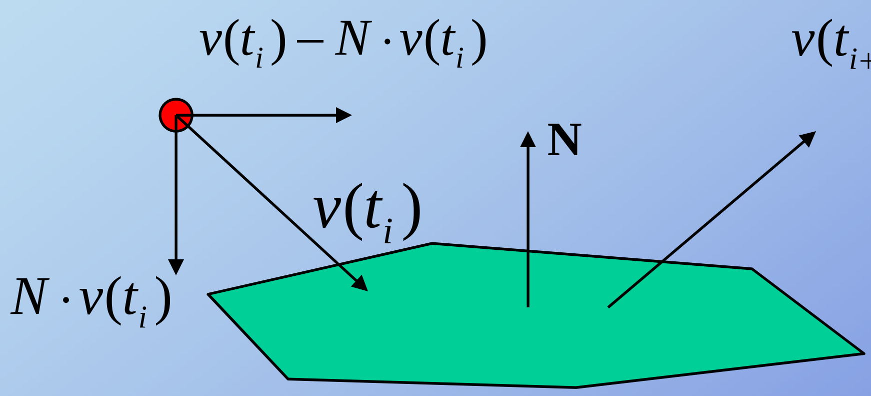
Time & relative direction of travel sweeps out a volume
Only tractable in simple cases (e.g. linear translation)



a) Original object b) Swept volume

If part of an object is in the volume, it was intersected by object

Collision reaction
Coefficient of restitution



$$v(t_{i+1}) = v(t_i) - N \bullet v(t_i) - k(N \bullet v(t_i))$$
$$= v(t_i) - (1+k)N \bullet v(t_i)$$

k – coefficient of restitution

But now want to add angular velocity contribution to separation velocity

Rigid body simulation

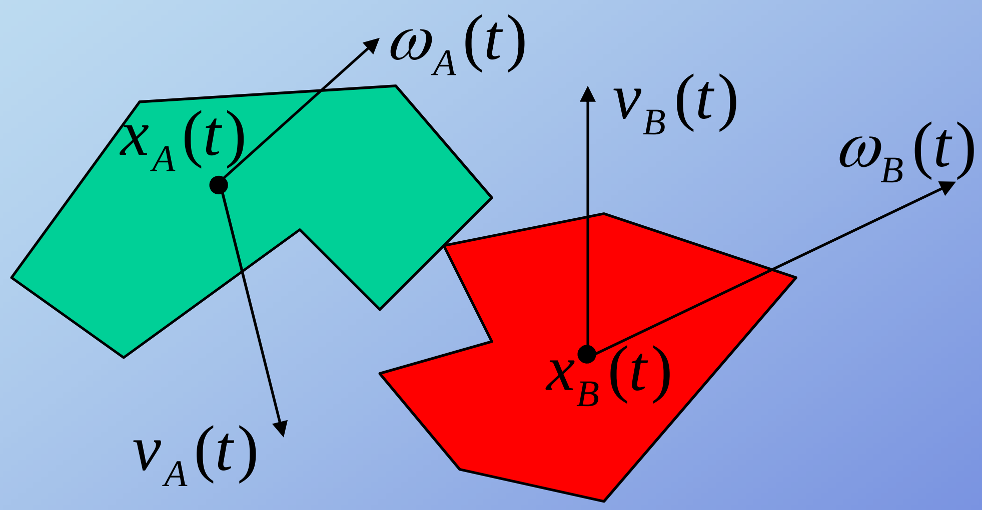
<u>Object Properties</u>
Mass
Position
linear & angular velocity
linear & angular momentum

<u>Calculate forces</u>
Wind
Gravity
Viscosity
Collisions

<u>Calculate accelerations</u>
Linear & angular using mass and inertia tensor

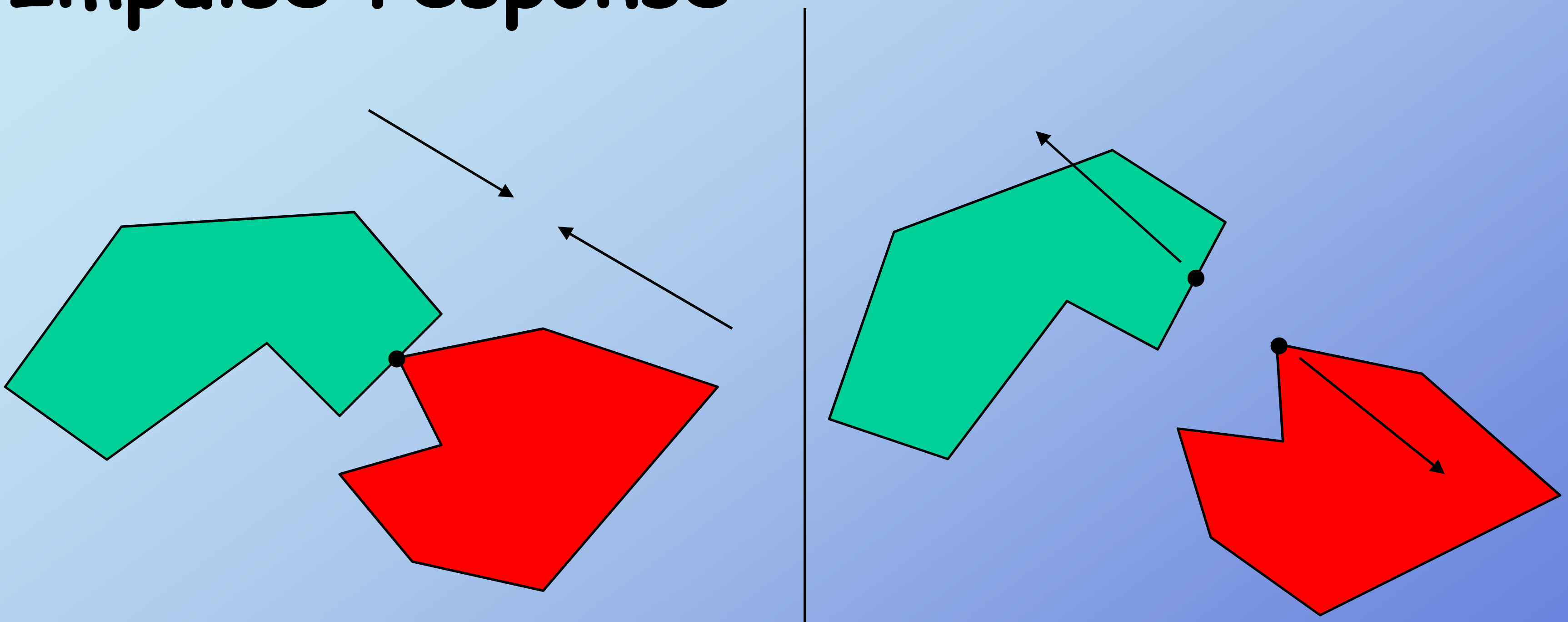
<u>Calculate change in attributes</u>
Position
linear & angular velocity
linear & angular momentum

Impulse response



How to compute the collision response of two rotating rigid objects?

Impulse response



<u>Given</u>

Separation velocity is to be negative of colliding velocity

<u>Compute</u>

Impulse force that produces sum of linear and angular velocities that produce desired separation velocity

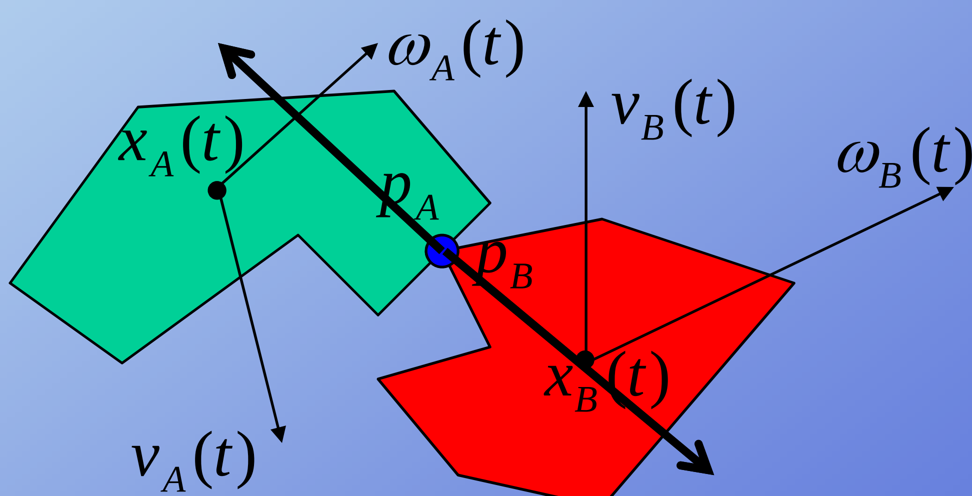
Rigid body simulation

Impulse force

$$j = f\Delta t$$

Separation velocity

$$v_{rel}^{+} = -\varepsilon v_{rel}^{-}$$



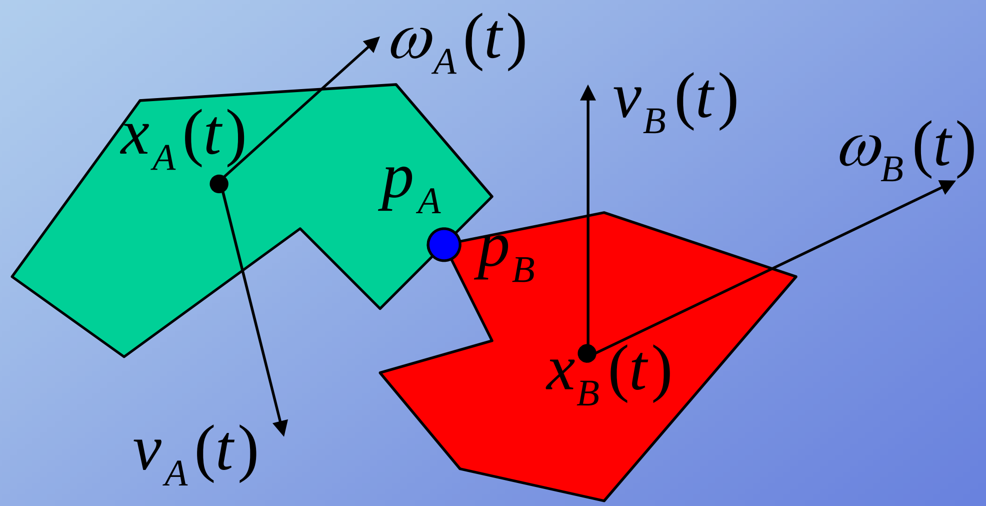
Update linear and angular velocities as a result of impulse force

$$v_A^{\;+} = v_A^{\;-} + \frac{jn}{M_A}$$

$$v_B^{\;+} = v_B^{\;-} + \frac{jn}{M_B}$$

$$\omega_A^{\;+} = \omega_A^{\;-} + I_A^{\;-1}(t)(r_A \times jn)$$

$$\omega_B^{\;+} = \omega_B^{\;-} + I_B^{\;-1}(t)(r_B \times jn)$$



Velocities of points of contact

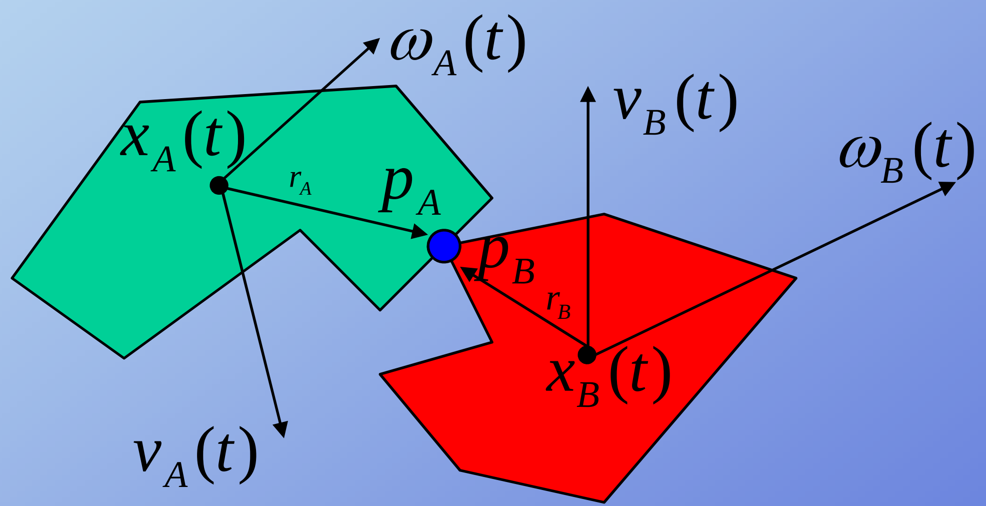
$$r_A = p_A - x_A(t)$$

$$r_B = p_B - x_B(t)$$

$$v_{rel} = (\dot{p}_A(t) - \dot{p}_B(t)) \cdot n$$

$$\dot{p}_A(t) = v_A(t) + \omega_A(t) \times r_A$$

$$\dot{p}_B(t) = v_B(t) + \omega_B(t) \times r_B$$



Rigid body simulation

$$v_{rel}^{+} = n \cdot (\dot{p}_A^{+}(t) - \dot{p}_B^{+}(t))$$

$$v_{rel}^{+} = n \cdot (v_A^{+}(t) + \omega_A^{+}(t) \times r_A - v_B^{+}(t) - \omega_B^{+}(t) \times r_B)$$

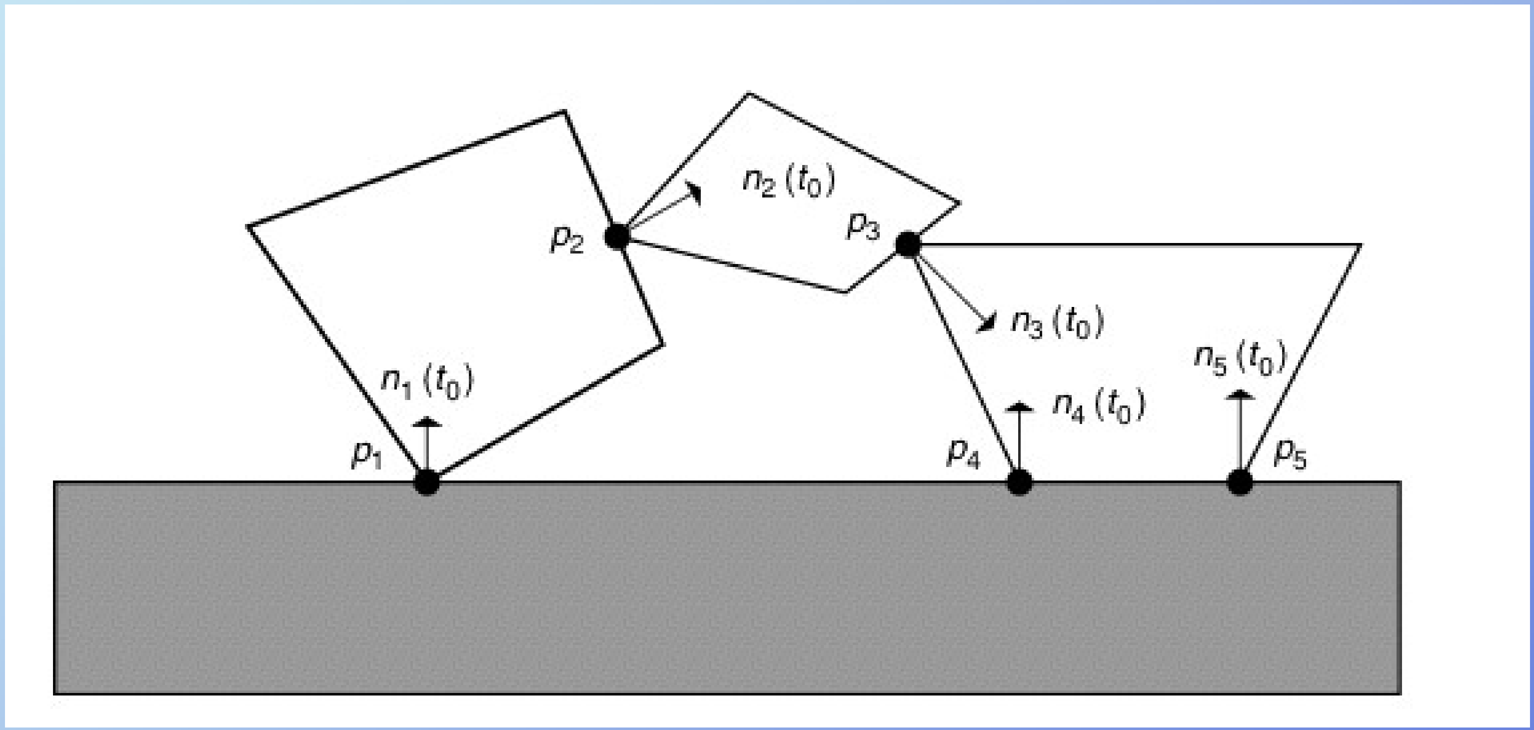
$$\varepsilon v_{rel}^{-} = n \cdot (v_A^{-} + \frac{jn}{M_A} + (\omega_A^{-} + I_A^{-1}(r_A \times jn)) \times r_A - v_B^{-} + \frac{jn}{M_B} - (\omega_B^{-} - I_B^{-1}(r_B \times jn)) \times r_B)$$

$$j = \frac{-((1+\varepsilon)v_{rel}^{-})}{\frac{1}{M_A} + \frac{1}{M_B} + n \cdot (I_A^{-1}(r_A \times n) \times r_A + I_B^{-1}(r_B \times n) \times r_B)}$$

v_{rel}^{-}

j applied to object A; -j applied to B

Resting contact



Complex situations: need to solve for forces that prevent penetration, push objects apart, if the objects are separating, then the contact force is zero