Computer Animation Algorithms and Techniques

Computational Fluid Dynamics

Fluids

<u>types</u>
viscous v. non-viscous
compressible v. incompressible

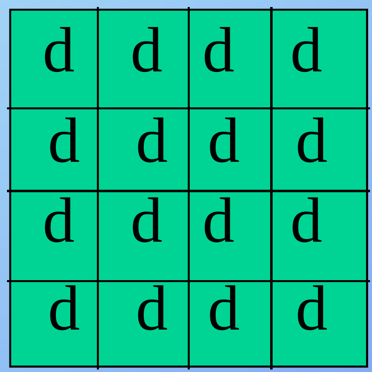
<u>conservation of</u>
mass
momentum
energy

<u>characteristics</u>
laminar flow
turbulent
steady state flow

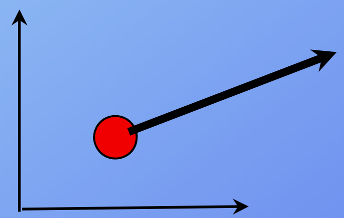
<u>properties</u>
velocity
density
pressure
heat
gravity

Models

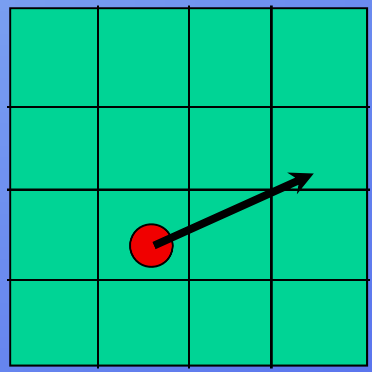
grid-based (Eulerian)



particle-based (Lagrangian)



Hybrid



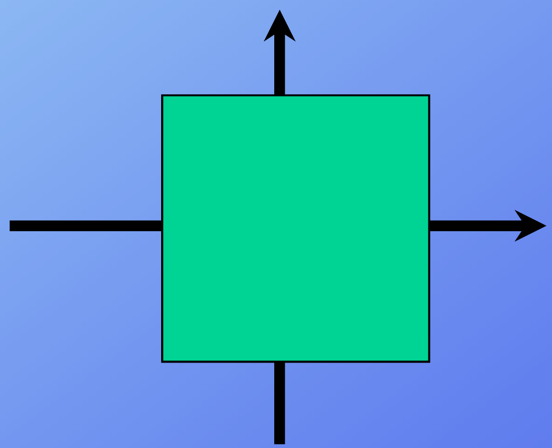
Notation

Gradient

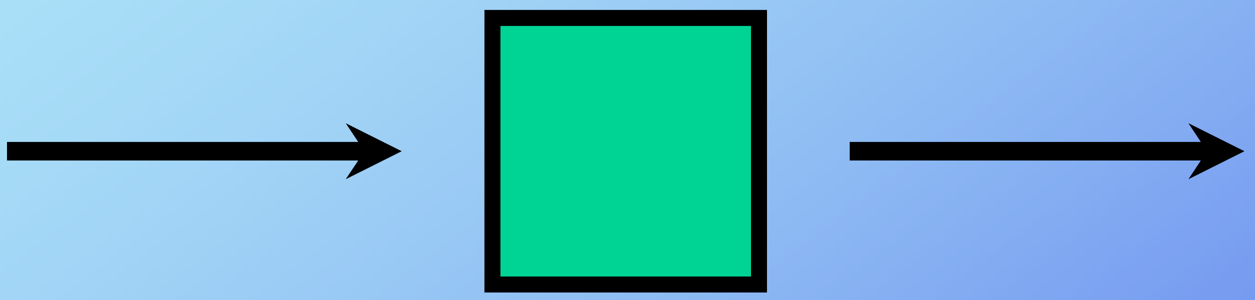
$$\nabla F = \left(\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y}, \frac{\delta F}{\delta z}\right)$$

Divergence

$$\nabla \cdot F = \frac{\delta F}{\delta x} + \frac{\delta F}{\delta y} + \frac{\delta F}{\delta z}$$

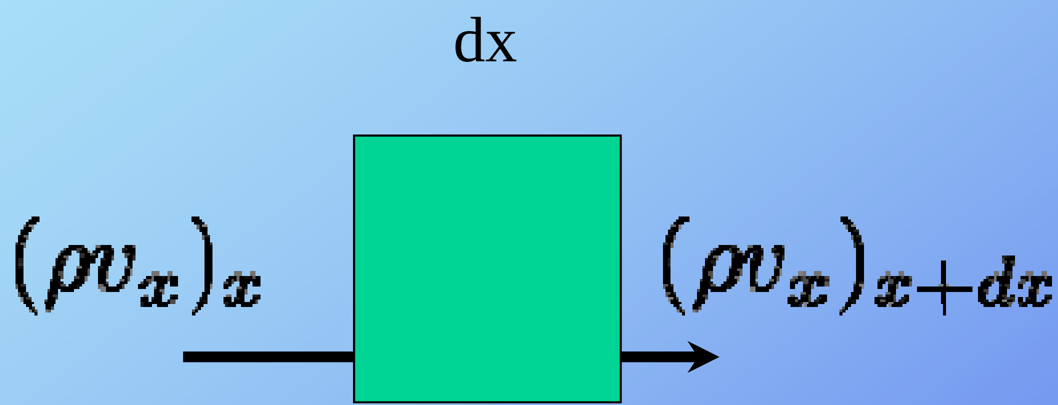


Divergence Theorem



The integral of the flow field's divergence over the volume of the cell is the same as the intgral of the flow field over the cell's boundary

Conservation of Mass



$$-\frac{d\rho}{dt} = \frac{\delta(\rho v_x)}{\delta x} + \frac{\delta(\rho v_y)}{\delta y} + \frac{\delta(\rho v_z)}{\delta z}$$

Conservation of Mass

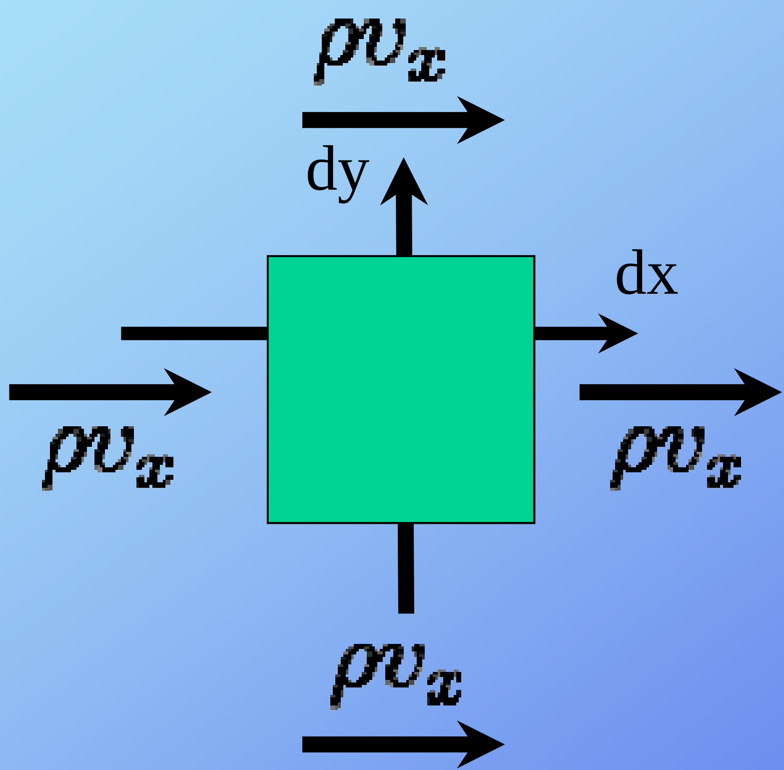
$$-\frac{d\rho}{dt} = \frac{\delta(\rho v_x)}{\delta x} + \frac{\delta(\rho v_y)}{\delta y} + \frac{\delta(\rho v_z)}{\delta z}$$

$$-\frac{d\rho}{dt} = \nabla \cdot (\rho v)$$

If incompressible: $0 = \nabla \cdot (\rho v)$

Conservation of Momentum

in x



$$-\frac{d\rho}{dx} = \frac{\delta(\rho v_x^2)}{\delta x} + \frac{\delta(\rho v_y v_x)}{\delta y} + \frac{\delta(\rho v_z v_x)}{\delta z} + \frac{d(\rho v_x)}{dt}$$

Conservation of Momentum

$$-\frac{d\rho}{dx} = \frac{\delta(\rho v_x^2)}{\delta x} + \frac{\delta(\rho v_y v_x)}{\delta y} + \frac{\delta(\rho v_z v_x)}{\delta z} + \frac{d(\rho v_x)}{dt}$$

$$-\frac{d\rho}{dy} = \frac{\delta(\rho v_x v_y)}{\delta x} + \frac{\delta(\rho v_y^2)}{\delta y} + \frac{\delta(\rho v_z v_y)}{\delta z} + \frac{d(\rho v_y)}{dt}$$

$$-\frac{d\rho}{dz} = \frac{\delta(\rho v_x v_z)}{\delta x} + \frac{\delta(\rho v_y v_z)}{\delta y} + \frac{\delta(\rho v_z^2)}{\delta z} + \frac{d(\rho v_z)}{dt}$$

Euler Equations - 2D

Velocity: u,v
Coordinates: x,y
Density: ρ
Pressure: p

$$\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} = 0$$

$$\frac{\delta(\rho u^2)}{\delta x} + \frac{\delta(\rho uv)}{\delta y} = -\frac{\delta p}{\delta x}$$

$$\frac{\delta(\rho uv)}{\delta x} + \frac{\delta(\rho v^2)}{\delta y} = -\frac{\delta p}{\delta y}$$

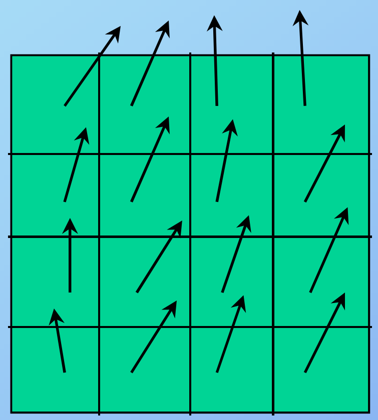
Euler Equations - incompressible form

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$$

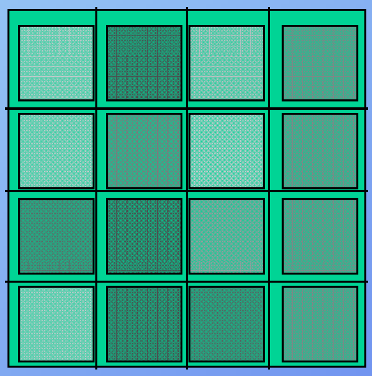
$$u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} = -\frac{1}{\rho}\frac{\delta p}{\delta x}$$

$$u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} = -\frac{1}{\rho}\frac{\delta p}{\delta y}$$

Stable Fluids



Velocity grid



Density grid

Move densities around using velocity grid and dissipate densities

Move velocities around using velocity grid and dissipate velocities

Stable Fluids

Source

$$\frac{d\rho}{dt} = s + \kappa \nabla^2 \rho - (u \cdot \nabla)\rho$$

Force

Dissipation

Advection

$$\frac{du}{dt} = f + \mu \nabla^2 u - (u \cdot \nabla)u$$