CSE 541

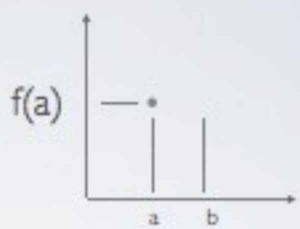
ELEMENTARY NUMERICAL METHODS

Taylor's Series

Functions - approximations

$f(b) = ?$

$f(b) \approx f(a)$



what's the error?

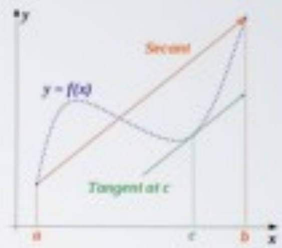
$$f'(x) < k$$

a<x<b

$$E < k(b-a)$$
$$E = f'(\xi)(b-a)$$

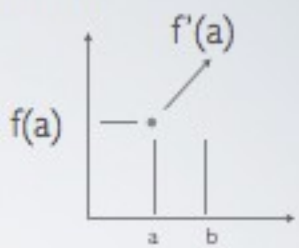
Mean Value Theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Functions - approximations

$$f(b) \approx f(a) + f'(a)(b - a)$$



what's the error?

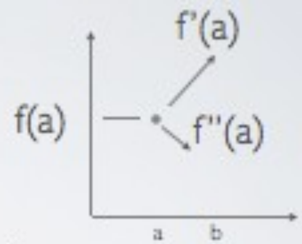
$$f''(x) < k$$

a<x<b

$$E < \frac{1}{2}k(b-a)^2$$

$$E = \frac{1}{2}f''(\xi)(b-a)^2$$

Functions - approximations



$$f(b) \approx f(a) + f'(a)(b-a) + \frac{1}{2} f''(a)(b-a)^2$$

what's the error?

Taylor's theorem

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(x)}{2!}(x-c)^2 + \frac{f'''(x)}{3!}(x-c)^3 + \dots$$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k + E_{n+1}$$

$$E_{n+1} = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-c)^{n+1}$$

Taylor's theorem

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k + E_{n+1}$$

$$E_{n+1} = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-c)^{n+1}$$

$$f(x+h) = \sum_{k=0}^{n} \frac{f^{(k)}(x)}{k!}h^k + E_{n+1}$$

$$E_{n+1} = \frac{f^{(n+1)}(\xi)}{(n+1)!}h^{n+1}$$

Alternating Series

$$sin(x) = x - \frac{s^3}{3!} + \frac{s^5}{5!} - \frac{s^7}{7!} + ... = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

If:

alternating in sign

terms are monotonically decreasing to zero

Error (remainder) term

$$E_{n+1} < |\frac{f^{(n+1)}(c)}{(n+1)!}(x-c)^{n+1}|$$