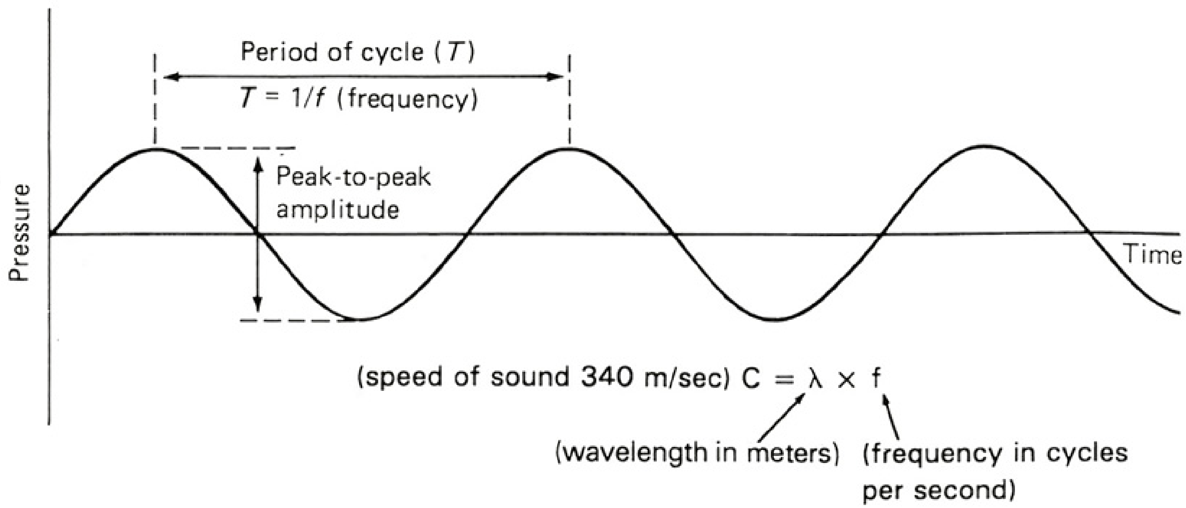
Basics of sound

- **Mathematics of the pure tone**

$$x(t) = A\sin(2\pi t / T + \phi)$$

or $$x(t) = A\sin(2\pi f t + \phi)$$

  - A: amplitude
  - ϕ: phase
  - T: period
  - f: frequency



Phase lead – phase lag

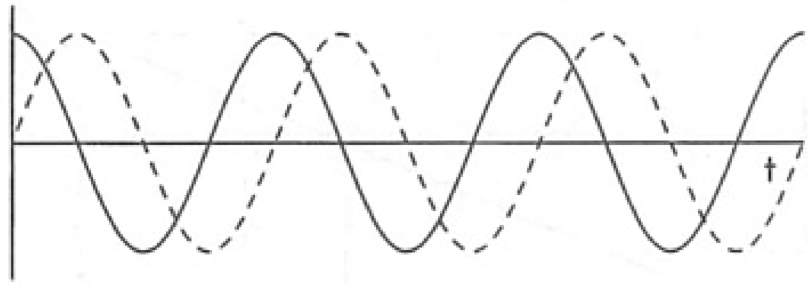


FIGURE 1.3. The wave shown by the solid line is said to lead the wave shown by the dashed line because every waveform feature——peak, positive-going zero crossing etc.——occurs at an earlier time for the solid line. Alternatively the dashed-line wave can be said to lag the solid-line wave. Both waves have the same frequency and amplitude, but their starting phases are different.

Power, intensity, and decibels

- **Treat signal *x*(*t*) as voltage**
- **By Ohm's law, the current *i*(*t*) = *x*(*t*)/*R***
- **Then the instantaneous power is**

$$P(t) = x(t)i(t) = x^2(t) / R$$

- **Energy is the integrated power over a certain time period (e.g. kilowatt vs. kilowatthour)**

Power, intensity, and decibels (cont.)

- **Treat signal *x*(*t*) as sound pressure**
- **Then the instantaneous intensity is**

$$I(t) = x^2(t)/(\rho c)$$

  - $I(t)$ is measured in watts/m^2 ($x(t)$: pressure, Newtons/m^2 or pascals)
  - ρ: the density of the medium
  - c: speed of sound

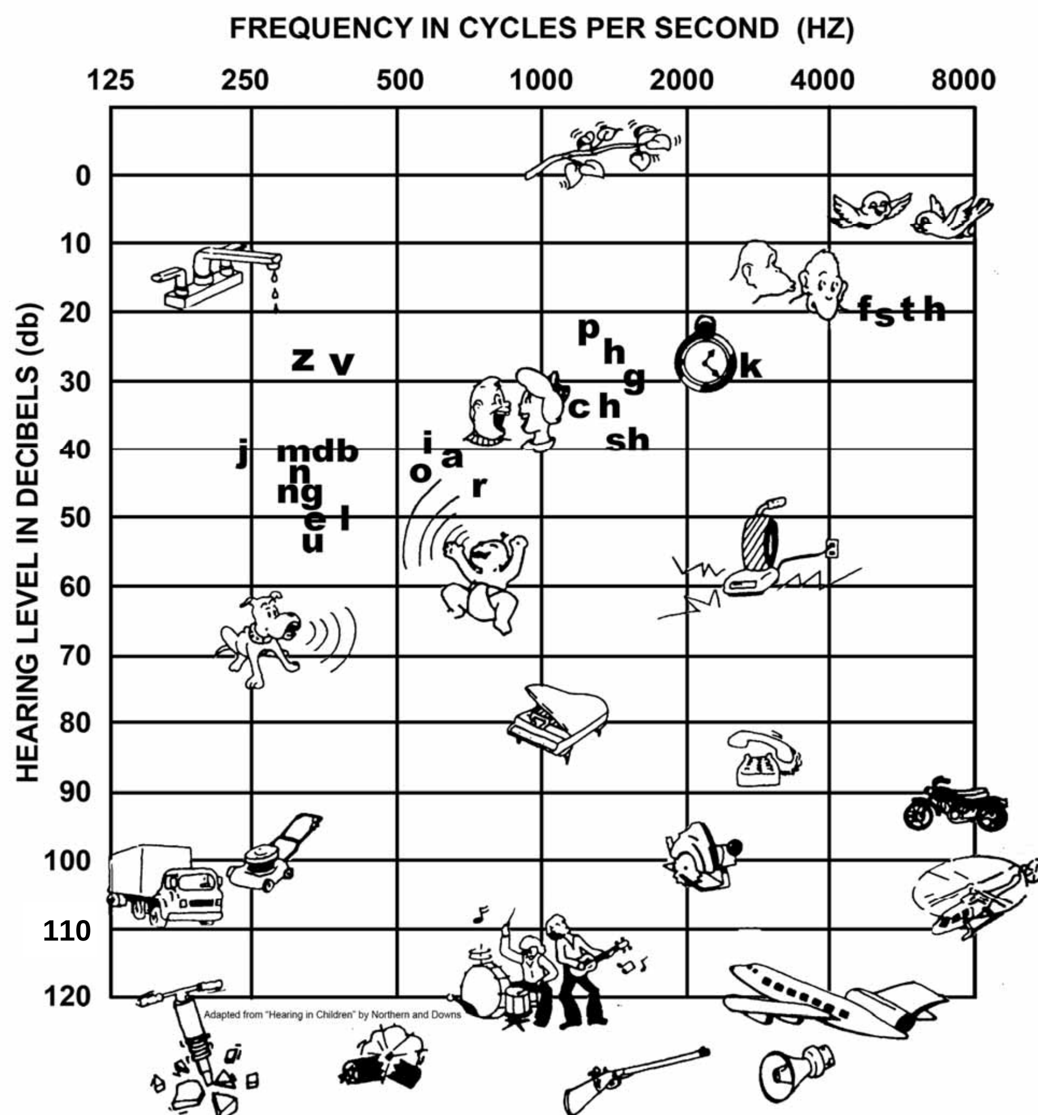
Sound level

- **Ratio of one sound to another (baseline), expressed as decibels (dB)**

$$L_2 - L_1(\text{decibels}) = 10\log_{10}(I_2 / I_1)$$

  - Note the use of common logarithm
  - Double intensity leads to 3 dB, and double amplitude leads to 6 dB
  - SNR: *signal-to-noise ratio*
  - Conversational speech is about 65 dB. Above 100 dB is damaging to the ear

How loud are sounds?



Source: http://www.handsandvoices.org/resource_guide/055_audiogram.html

Spectrum

- **Fourier Series: For any periodic function of time, *x*(*t*), with period *T*, i.e.**

$$x(t+mT) = x(t), \qquad \text{for all integer } m$$

- ***x*(*t*) can be represented as a Fourier series like this**

$$x(t) = A_0 + \sum_{n=1}^{\infty}[A_n \cos(\omega_n t) + B_n \sin(\omega_n t)]$$

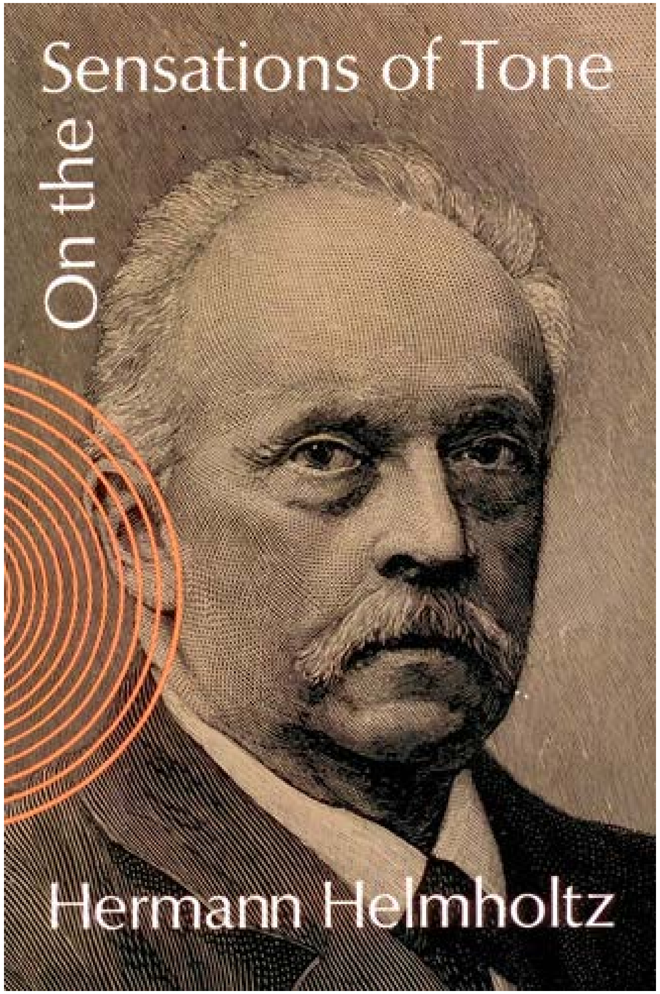
- **Furthermore,**

$$\omega_n = n\omega_0 = 2\pi n / T$$

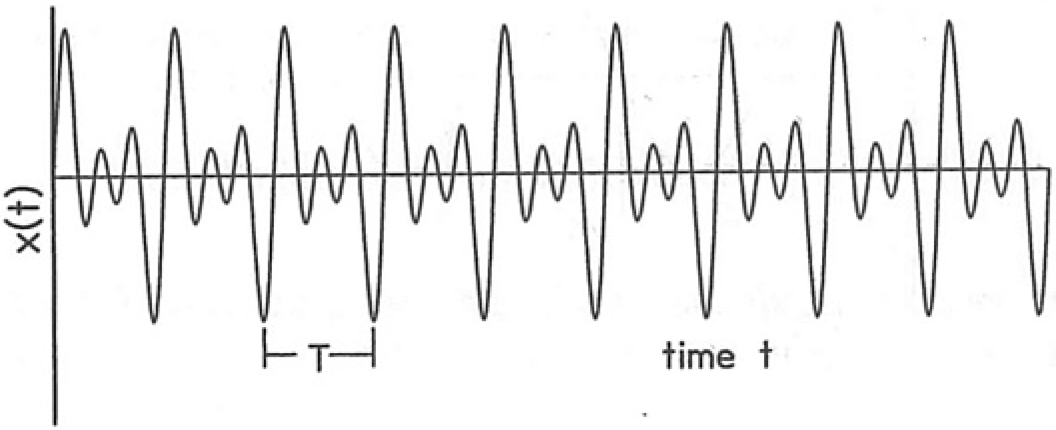
  - n is integer and ω_0 is the fundamental frequency

Spectrum (cont.)

- **"The multiplicity of vibrational forms which can be thus produced by the composition of simple pendular vibrations is not merely extraordinarily great; it is so great that it can not be greater." (H. Holmholtz, 1863)**

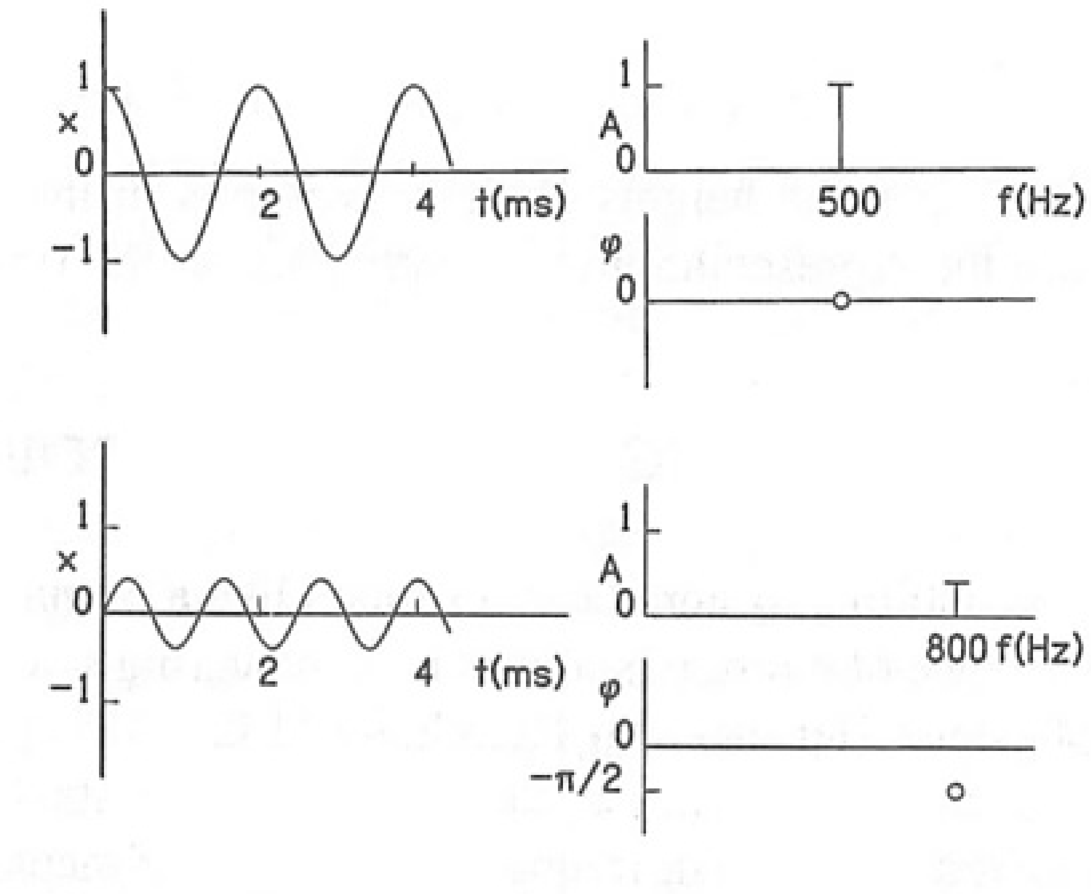


Waveform illustration



A periodic waveform

Spectrum illustration



Spectrum illustration

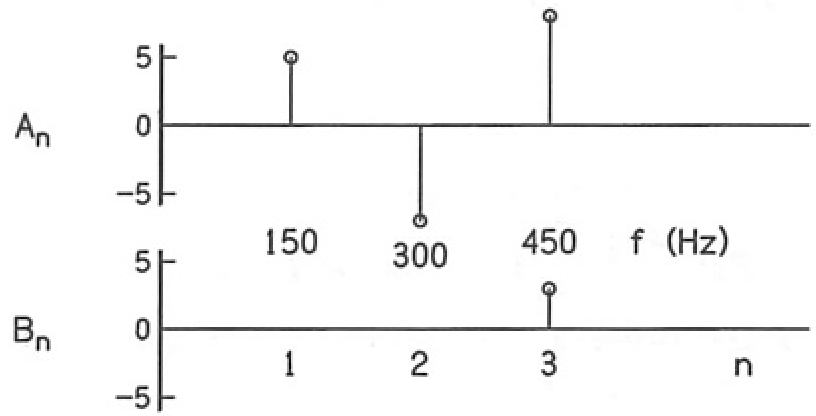
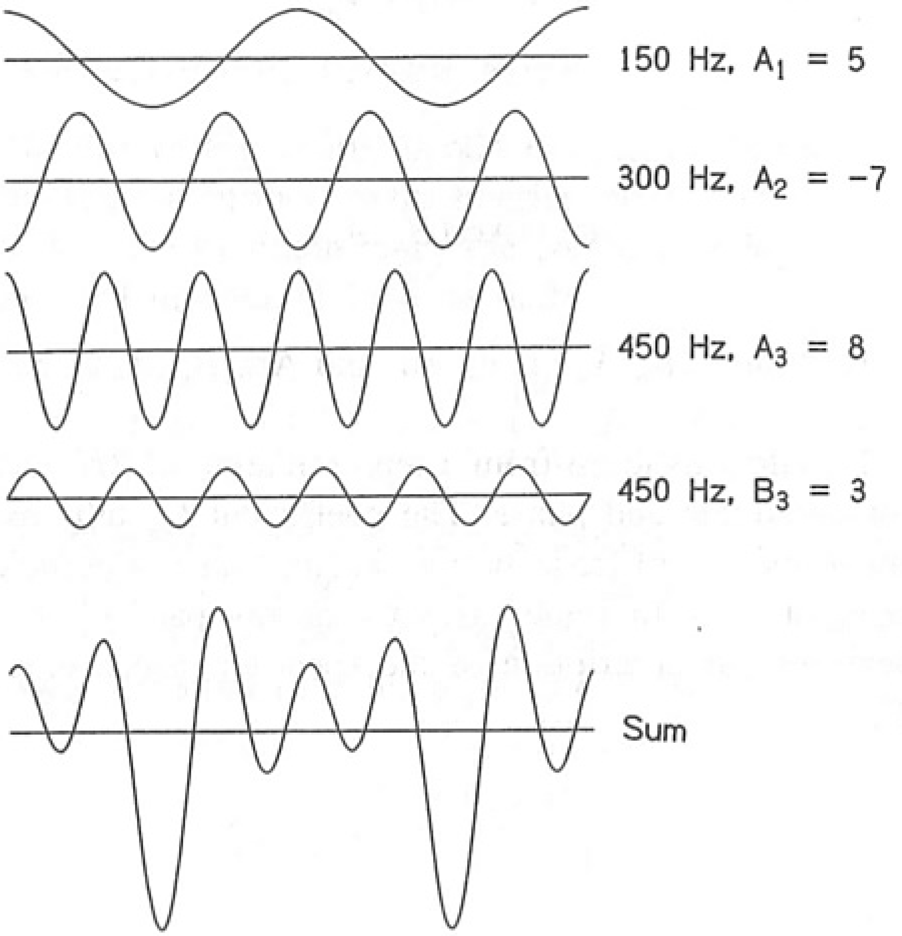


FIGURE 5.4. Coefficients A_n and B_n as a function of harmonic number n (or of frequency nf_0) constitute the Fourier spectrum of $x(t)$

Spectrum illustration



Fourier transform

- **For any function of time, *x*(*t*), the Fourier transform *X*(*ω*) of *x*(*t*) is defined in terms of the Fourier integral:**

$$X(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} x(t)\,dt$$

- **The Fourier transform converts a function of time to a function of frequency**
- **Inverse Fourier transform**

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} X(\omega)\,d\omega$$

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$