Basics of sound

• Mathematics of the pure tone

\[ x(t) = A \sin\left(\frac{2\pi t}{T} + \phi\right) \]

or \[ x(t) = A \sin(2\pi ft + \phi) \]

• \( A \): amplitude
• \( \phi \): phase
• \( T \): period
• \( f \): frequency
**Phase lead – phase lag**

**FIGURE 1.3.** The wave shown by the solid line is said to lead the wave shown by the dashed line because every waveform feature—peak, positive-going zero crossing etc.—occurs at an earlier time for the solid line. Alternatively the dashed-line wave can be said to lag the solid-line wave. Both waves have the same frequency and amplitude, but their starting phases are different.
Power, intensity, and decibels

- Treat signal $x(t)$ as voltage
- By Ohm's law, the current $i(t) = x(t)/R$
- Then the instantaneous power is

$$P(t) = x(t)i(t) = \frac{x^2(t)}{R}$$

- Energy is the integrated power over a certain time period (e.g. kilowatt vs. kilowatthour)
Power, intensity, and decibels (cont.)

- Treat signal $x(t)$ as sound pressure
- Then the instantaneous intensity is

$$I(t) = \frac{x^2(t)}{(\rho c)}$$

- $I(t)$ is measured in watts/m$^2$ ($x(t)$: pressure, Newtons/m$^2$ or pascals)
- $\rho$: the density of the medium
- $c$: speed of sound
Sound level

• Ratio of one sound to another (baseline), expressed as decibels (dB)

\[ L_2 - L_1 \text{ (decibels)} = 10 \log_{10} \left( \frac{I_2}{I_1} \right) \]

• Note the use of common logarithm
• Double intensity leads to 3 dB, and double amplitude leads to 6 dB
• SNR: signal-to-noise ratio
• Conversational speech is about 65 dB. Above 100 dB is damaging to the ear
How loud are sounds?

Source:
http://www.handsandvoices.org/resource_guide/055_audio gram.html
Spectrum

- **Fourier Series:** For any periodic function of time, \( x(t) \), with period \( T \), i.e.

\[
x(t + mT) = x(t), \quad \text{for all integer } m
\]

- \( x(t) \) can be represented as a Fourier series like this

\[
x(t) = A_0 + \sum_{n=1}^{\infty} \left[ A_n \cos(\omega_n t) + B_n \sin(\omega_n t) \right]
\]

- Furthermore,

\[
\omega_n = n \omega_0 = \frac{2\pi n}{T}
\]

- \( n \) is integer and \( \omega_0 \) is the fundamental frequency
"The multiplicity of vibrational forms which can be thus produced by the composition of simple pendular vibrations is not merely extraordinarily great; it is so great that it can not be greater." (H. Helmholtz, 1863)
Waveform illustration

A periodic waveform
Spectrum illustration
**Spectrum illustration**

![Spectrum illustration](image)

**FIGURE 5.4.** Coefficients $A_n$ and $B_n$ as a function of harmonic number $n$ (or of frequency $nf_0$) constitute the Fourier spectrum of $x(t)$.
Spectrum illustration

150 Hz, $A_1 = 5$

300 Hz, $A_2 = -7$

450 Hz, $A_3 = 8$

450 Hz, $B_3 = 3$

Sum
Fourier transform

- For any function of time, $x(t)$, the Fourier transform $X(\omega)$ of $x(t)$ is defined in terms of the Fourier integral:

$$X(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} x(t) dt$$

- The Fourier transform converts a function of time to a function of frequency

- Inverse Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} X(\omega) d\omega$$

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$