CHAPTER 6

SUMMARY AND FUTURE WORK

6.1 Summary

Synchronous neural activity has been observed in many brain regions [Singer and Gray, 1995] and understanding this phenomena may be fundamental to understanding the brain. In order to understand synchronous neural activity we have explored the dynamics of locally coupled populations of neural oscillators. We have been interested specifically in synchrony and desynchrony, and we have studied these topics in the context of how they may be used for information processing.

We first examined integrate-and-fire oscillators [Peskin, 1975], a one variable oscillator and perhaps the simplest model of neuronal behavior. Our numerical data indicate that one- and two-dimensional noiseless networks of identical oscillators synchronize at times proportional to the logarithm of the system size. We gave a heuristic explanation for this behavior. We also demonstrated a neurally plausible network of integrate-and-fire oscillators that performs oscillatory correlation, i.e. groups of oscillators that are desynchronized from each other, while synchrony is maintained within each group. We used this oscillator network to perform image segmentation with real images.

We also examined networks of relaxation oscillators, a more complicated, two variable system of equations that model neuronal behavior [Fitzhugh, 1961, Nagumo et al., 1962, Morris and Lecar, 1981]. Through appropriate parameter choices, these oscillators can exhibit either sinusoidal type, relaxation type, or integrate-and-fire type oscillations. By studying relaxation oscillators, we revealed their intrinsic links to these two other classes of oscillators. In one-dimensional noiseless networks of identical relaxation oscillators we have examined synchrony, fractured synchrony, and travelling waves. Our analysis has yielded conditions necessary for formation of each of these three behaviors. Note that the basin of attraction for synchrony is much larger than the other two types of solutions when random initial conditions are used. Our numerical evidence indicates that the time to synchrony increases as n^p, where n is the system size and p is calculated from the data. We have suggested what the causes of this scaling relationship might be. For two-dimensional networks there exist parameter regimes in which the time to synchrony scales as $\log_{10}(n)$, and we have shown other, more complex relationships between the time to synchrony and the system size. Rotating waves and other types of patterns can exist in two-dimensional networks and we have suggested parameters regimes and initial conditions which hinder their formation.

We have examined how the time to synchrony is affected by the type of oscillator and the type of interaction used. Our data suggest that a discontinuous Heaviside type interaction leads to synchronization times proportional to n, independent of whether the oscillations are sinusoidal or relaxation like. When a smooth interaction is used, the time to synchrony scales as n^2. We conjecture that discontinuous interactions in general have better properties of synchronization in networks of oscillators than smooth interactions. In support of this conjecture is the work in Chapter 2, in which a discontinuous interaction leads to much faster rate of synchrony than a diffusive interaction. Also in support of this conjecture is the work of Daido [Daido, 1993b], who noted that a discontinuous interaction resulted in the synchronization of a significant fraction of oscillators in a globally coupled network of phase oscillators with a normal distribution of frequencies. If a smooth interaction was used, this same network would not exhibit any synchronization. This conjecture may have practical applications in synchronizing chaotic systems, or arrays of Josephson junctions.

We also examined relaxation oscillators with time delays in the interaction. We analytically derived results for a pair of oscillators indicating that perfect synchrony was no longer attained, but that the oscillators asymptotically approached a solution such that their time difference was less than or equal to the time delay - a solution we called loose synchrony. In one- and two-dimensional networks of locally coupled oscillators, simulations revealed similar behaviors. We suggested a measure of synchrony for networks of such oscillators and presented evidence indicating how this measure increases as the network size increases. A range of initial conditions was proposed in which the degree of synchrony does not degrade as the system evolves. If the time delay becomes larger than the critical time delay, we find desynchronous solutions of high frequency form in both pairs and networks of oscillators.

Finally, we examined networks of Wilson-Cowan oscillators. These oscillators are based on populations of interacting excitatory and inhibitory neurons [Wilson and Cowan, 1972]. We proved that one-dimensional networks consisting of piece-wise linear approximation to Wilson-Cowan oscillators achieved synchrony if the coupling strength was large enough. In order to achieve synchrony quickly (in one or two cycles), we used a diffusive type coupling and chose parameters so that the system was near a bifurcation point. Because of this, the interaction resulted in the formation and destruction of fixed points that occurred in such a fashion that synchrony was facilitated. Also, we created a mechanism for desynchronization in these networks. Up to nine different groups of oscillators could be desynchronized without destroying synchrony within each group. To our knowledge this is one of a few networks in which desynchronization of more than two oscillator groups can be achieved. Two other networks are described in Chapter 2 and in [Terman and Wang, 1995].

In summary we have investigated several classes of oscillators and how locally coupled networks of these oscillators synchronize with respect to n, the size of the network. We have observed four different scaling relationships between the average time to synchrony and n; $\langle T_S \rangle \sim \log_{10}(n)$, $\langle T_S \rangle \sim n^p \;\; with \;\; p < 0.5$, $\langle T_S \rangle \sim n$, and $\langle T_S \rangle \sim n^2$. To the best of our knowledge, the first two relationships are new to the literature of coupled oscillators. Also, we have used numerical simulations to explore how the interaction alters the rate of synchronization. Our data indicate that the scaling relation $\langle T_S \rangle \sim n$ arises when a discontinuous interaction is used, and is independent of whether the oscillator is relaxation or sinusoidal type. Our work has also demonstrated the versatility of relaxation oscillator networks, which through proper alteration of parameters can exhibit all four scaling relationships.

6.2 Future Work

While we have understood and characterized synchrony and desynchrony in several types of oscillator networks, there are many questions which remain unanswered. Foremost among these questions is "What are the fundamental causes of the four different scaling relationships we observed in the time to synchrony as a function of the system size?" We list Chapter by Chapter some more specific directions for future work.

In Chapter 2 we examined integrate-and-fire oscillators with a specific reset rule and a specific type of interaction. How much can one alter these rule and these interactions and still observe the scaling relation $\langle T_S \rangle \sim \log_{10}(n)$? Networks of these oscillators achieve synchrony in the presence of some forms of noise. This raises several questions. How much noise can these systems tolerate? What kind of noise can these systems tolerate? How does the rate of synchrony change in the presence of noise?

In Chapter 3 we examined relaxation oscillators and found a variety of phenomena. In one-dimensional networks that exhibit fractured synchrony, it is unknown how the block size is related to the coupling strength. The formation of fractured synchrony in two-dimensional systems is also of interest. Preliminary work indicates that the average block size approaches the system size at much smaller coupling strengths than in one-dimensional networks. Many similar questions can also be asked about travelling waves. Given random initial conditions, and a network with periodic boundary conditions, how frequently do travelling waves form? What is the average block size within the travelling waves? These are two specific questions that are part of the same general question that arises when several basins of attraction exist between units in an interacting system. A detailed understanding of these questions may provide some insight for similar questions in other dynamical systems.

Spatiotemporal pattern formation is also an interesting topic in these relaxation oscillator networks. One of the first questions that should be examined is whether or not these patterns are stable in other types of relaxation oscillators (the specific model we studied resulted in patterns that were neutrally stable). If these patterns are stable, then each separate pattern has a basin of attraction and estimating the stability and the number of patterns then becomes an interesting issue. If many stable patterns can form, then this system might be useful as an associative memory.

In two-dimensional networks of relaxation oscillators we have even more questions. We have suggested that restricting the initial conditions to the lower left branch prevents spatiotemporal pattern formation, but we have not proven this. Also, we have noticed an unusual phenomenon in the synchronization time for two-dimensional networks; a non-monotonic relationships between the systems size and the time to synchrony. This is counter-intuitive and we do not yet have an understanding of its cause.

In relaxation oscillators with time delays (Chapter 4) there are many unresolved issues. One of these issues is the speed with which the network attains the loosely synchronous solution. Is the time needed to achieve a loosely synchronous solution related to the time to synchrony in networks without time delays? It appears that all the behaviors seen in relaxation oscillator networks without time delays also arise in networks with time delays. It would be interesting to see if the properties of fractured synchrony, travelling waves, and spatiotemporal pattern formation observed in networks without time delays, have similar properties when time delays are included. Also, numerical simulations indicate that with small alterations in the defining equations, the synchronous solution becomes stable. Investigating this phenomena might reveal general properties of how to maintain synchrony in the presence of time delays.

In networks Wilson-Cowan oscillators (Chapter 5) we used diffusive coupling and chose parameters such that the system was near a bifurcation point. We do not know how the time to synchrony scales as a function of the system size. On a different note, König and Schillen [König and Schillen, 1991] showed that synchrony arises in a network of Wilson-Cowan type oscillators with time delay coupling. It would be interesting to understand how synchrony in these networks is related to synchrony in relaxation oscillator systems with time delays.

In this work we have described some properties of locally coupled networks for a few types of oscillators. However, these systems have not been fully examined with global coupling. It would be most interesting to see what similarities and differences arise as the nature of the coupling changes. As we begin to understand the differences between locally coupled systems and globally coupled systems, then one may gain substantial insight into how to modify the analytical tools that have been created for globally coupled networks so that they can be used in locally coupled systems.