Question 1. A coordinate system (or a space) is specified by four vectors: the origin, the X axis, the Y axis, and the Z axis. Let OpenGL be under the MODELVIEW matrix mode.

a. (1 point) Describe the local space of vertex \( p_1 \) in the eye space. (In other words, what are the origin and the three axes of \( p_1 \)'s local space in the eye space? \textbf{Hint: check the MODELVIEW matrix.}) What are \( p_1 \)'s coordinate values in the eye space?

```plaintext
glLoadIdentity();
glTranslatef(3, 0, 0);
glRotatef(90, 1, 0, 0);
glBegin(GL_POINTS);
glVertex3f(3, 3, 3);  // \( p_1 \)
glEnd();
glTranslatef(0, 2, 0);
glScalef(1, 2, 1);
glBegin(GL_POINTS);
glVertex3f(2, 2, 0);  // \( p_2 \)
glEnd();
```

b. (1 point) Describe the local space of vertex \( p_2 \) in the eye space. (In other words, what are the origin and the three axes of \( p_2 \)'s local space in the eye space?) What are \( p_2 \)'s coordinate values in the eye space?
Question 2. Let the current MODELVIEW matrix represent a 2D local space 1 as Figure 1 shows.

Figure 1: The transformation between two local spaces.

a. (0.5 points) Write the OpenGL code in 2D that can convert the MODELVIEW matrix from Space 1 to Space 2.

b. (0.5 points) Let p be a vertex drawn in Space 2 and its coordinate values are (2, 1). What are its coordinate values in Space 1?

c. (0.5 points) Assuming that the current MODELVIEW matrix is representing Space 2 instead. Write the OpenGL code in 2D that converts the matrix from Space 2 to Space 1. (Hint: how to invert a transformation?)
Question 3. Let $M$ be a projection matrix and $p = (x, y, z)$ be a 3D vertex in the eye space. The projection process converts $p$ into another 3D vector $p' = (x', y', z')$, in which $x'$ and $y'$ determines the image location of this vertex, and $z'$ indicates the depth of this vertex. An interesting question is why cannot we use $z$ as the depth value instead.

a. (0.5 point) Let $M = \begin{bmatrix} 2 & 2 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}$ be a projection matrix, $p_0 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, $p_1 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$, and $p_2 = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$ be three vertices, what are the projected vertices $p_0'$, $p_1'$, and $p_2'$ respectively?

b. (0.5 point) $p_0$, $p_1$, and $p_2$ are on the same line in 3D. Are $v_0'$, $v_1'$, and $v_2'$ on the same line as well? (Hint: three vertices $p_0$, $p_1$, and $p_2$ are on the same line if and only if $(p_1 - p_0) \times (p_2 - p_0) = 0$.)

c. (0.5 point) If we formulate three new vertices $p_0'' = (x_0', y_0', z_0)$, $p_1'' = (x_1', y_1', z_1)$, and $p_2'' = (x_2', y_2', z_2)$, will they be on the same line?

Submission Guideline  Please submit your solution either electronically or in person to our grader: Xiaoyin Ge (gex AT cse.ohio-state.edu).