CSE 2331/5331 Homework 3
Autumn, 2019
(75 points)

**Remark 1.** For each of the following problems, express your answer as \( \Theta(n^k) \) or \( \Theta(n^k \log n) \) wherever possible. But if the asymptotic running time is exponential, then just give exponential lower bounds.

**Remark 2.** \( \text{Random}(n) \) generates a random number between 1 and \( n \) with uniform distribution (every integer between 1 and \( n \) is equally likely.) \( \text{CoinFlip()} \) returns heads or tails with equal probability.

1. [15 points] Consider the following function:

   ```java
   Func1(A, n)
   /* A is an array of integers */
   1  s ← 0;
   2  for m ← 1 to 10 do
   3     k ← Random(n);
   4     for i ← 1 to \( \lfloor \frac{n}{k} \rfloor \) do
   5         j ← 1;
   6         while (j < n) do
   7             s ← s + A[i] * A[j];
   8             j ← 2 * j;
   9         end
   10     end
   11 end
   12 return (s);
   ```

   (a) What is the asymptotic worst case running time of \( \text{Func1} \)?
   (b) What is the asymptotic expected running time of \( \text{Func1} \)? Justify your solution.

2. [20 points] Consider the following function:

   ```java
   Func2(A, n)
   /* A is an array of integers */
   1  s ← A[1];
   2  k ← Random(n);
   3  if (k < \log_2(n)) then
   4      for i ← 1 to n do
   5          j ← 1;
   6          while (j < n) do
   7              s ← s + A[i] * A[j];
   8              j ← 2 * j;
   9          end
   10      end
   11 else
   12    j ← n;
   13    while (j ≥ 1) do
   14        s ← s + A[i] * A[j];
   15        j ← j - 3;
   16    end
   17 end
   18 return (s);
   ```

   (a) What is the asymptotic worst case running time of \( \text{Func2} \)?
   (b) What is the asymptotic expected running time of \( \text{Func2} \)? Justify your solution.
3. [20 points] Consider the following function:

```cpp
Func3(A, n)
/* A is an array of integers */
1 if (n ≤ 10) then return (A[1]);
2 c₁ ← CoinFlip();
3 c₂ ← CoinFlip();
4 s ← 0;
5 for i ← 1 to n do
6   s ← s + A[i];
7 end
8 if (c₁ = c₂ = heads) then
9   s ← s + Func3(A, n − 4) + Func3(A, n − 7);
10 end
11 return (s);
```

(a) What is the asymptotic worst case running time of Func3? Justify your solution.
(b) What is the asymptotic expected running time of Func3? Justify your solution.

4. [20 points] Consider the following function:

```cpp
Func4(A, n)
/* A is an array of integers */
1 if (n ≤ 10) then return (A[1]);
2 s ← 0;
3 for i ← 1 to n do
4   s ← s + A[i];
5 end
6 for i ← 1 to 4 do
7   coin ← CoinFlip();
8   if (coin = heads) then
9     s ← s + Func4(A, ⌊n/4⌋);
10 end
11 end
12 return (s);
```

(a) What is the asymptotic worst case running time of Func4? Justify your solution.
(b) What is the asymptotic expected running time of Func4? Justify your solution.