CSE 5243 INTRO. TO DATA MINING

Advanced Frequent Pattern Mining
&
Locality Sensitivity Hashing

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Slides adapted from Prof. Jiawei Han @UIUC, Prof. Srinivasan Parthasarathy @OSU
Sequence Mining: Description

Input

A database $D$ of sequences called data-sequences, in which:
- $l = \{i_1, i_2, \ldots, i_n\}$ is the set of items
- each sequence is a list of transactions ordered by transaction-time
- each transaction consists of fields: sequence-id, transaction-id, transaction-time and a set of items.

<table>
<thead>
<tr>
<th>Sequence-Id</th>
<th>Transaction Time</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>Ringworld</td>
</tr>
<tr>
<td>C1</td>
<td>2</td>
<td>Foundation</td>
</tr>
<tr>
<td>C1</td>
<td>15</td>
<td>Ringworld, Second Foundation</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>Foundation, Ringworld</td>
</tr>
<tr>
<td>C2</td>
<td>20</td>
<td>Foundation and Empire</td>
</tr>
<tr>
<td>C2</td>
<td>50</td>
<td>Ringworld Engineers</td>
</tr>
</tbody>
</table>
Sequential Pattern and Sequential Pattern Mining

- **Sequential pattern mining**: Given a set of sequences, find the complete set of frequent subsequences (i.e., satisfying the min_sup threshold)

A *sequence database*

<table>
<thead>
<tr>
<th>SID</th>
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<tbody>
<tr>
<td>10</td>
<td>&lt;a(abc)(ac)d(cf)&gt;</td>
</tr>
<tr>
<td>20</td>
<td>&lt;(ad)c(bc)(ae)&gt;</td>
</tr>
<tr>
<td>30</td>
<td>&lt;(ef)(ab)(df)cb&gt;</td>
</tr>
<tr>
<td>40</td>
<td>&lt;eg(af)cbbc&gt;</td>
</tr>
</tbody>
</table>

A *sequence*: `<(ef)(ab)(df)cb>`

- An *element* may contain a set of *items* (also called *events*)
- Items within an element are unordered and we list them alphabetically

`<a(bc)dc>` is a *subsequence* of `<a(abc)(ac)d(cf)>`
Sequential Pattern and Sequential Pattern Mining

- **Sequential pattern mining**: Given a set of sequences, find the complete set of frequent subsequences (i.e., satisfying the min_sup threshold).

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<tr>
<td>40</td>
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A sequence $a = \langle a_1a_2\cdots a_n \rangle$ is called a subsequence of another sequence $\beta = \langle b_1b_2\cdots b_m \rangle$, and $\beta$ is a supersequence of $\alpha$, denoted as $\alpha \sqsubseteq \beta$, if there exist integers $1 \leq j_1 < j_2 < \cdots < j_n \leq m$ such that $a_1 \subseteq b_{j_1}, a_2 \subseteq b_{j_2}, \ldots, a_n \subseteq b_{j_n}$. For example, if $\alpha = \langle (ab), d \rangle$ and $\beta = \langle (abc), (de) \rangle$, where $a, b, c, d,$ and $e$ are items, then $\alpha$ is a subsequence of $\beta$ and $\beta$ is a supersequence of $\alpha$. 

\(<a(bc)dc>\) is a subsequence of \(<a(abc)(ac)d(cf)\>\)
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<td>&lt;eg(af)cbc&gt;</td>
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A **sequence**: <(ef)(ab)(df)cb>

- An **element** may contain a set of **items** (also called **events**)
- Items within an element are unordered and we list them alphabetically


- Given **support threshold** min_sup = 2, <(ab)c> is a **sequential pattern**
A Basic Property of Sequential Patterns: Apriori

- A basic property: Apriori (Agrawal & Sirkant’94)
  - If a sequence S is not frequent
  - Then none of the super-sequences of S is frequent
  - E.g, <hb> is infrequent $\Rightarrow$ so do <hab> and <(ah)b>
GSP (Generalized Sequential Patterns): Apriori-Based Sequential Pattern Mining

- Initial candidates: All 8-singleton sequences
  - \(<a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>\)
- Scan DB once, count support for each candidate

\(\text{min}_\text{sup} = 2\)

<table>
<thead>
<tr>
<th>Cand.</th>
<th>sup</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;a&gt;</td>
<td>3</td>
</tr>
<tr>
<td>&lt;b&gt;</td>
<td>5</td>
</tr>
<tr>
<td>&lt;c&gt;</td>
<td>4</td>
</tr>
<tr>
<td>&lt;d&gt;</td>
<td>3</td>
</tr>
<tr>
<td>&lt;e&gt;</td>
<td>3</td>
</tr>
<tr>
<td>&lt;f&gt;</td>
<td>2</td>
</tr>
<tr>
<td>&lt;g&gt;</td>
<td>1</td>
</tr>
<tr>
<td>&lt;h&gt;</td>
<td>1</td>
</tr>
</tbody>
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</tr>
<tr>
<td>30</td>
<td>(&lt;ah)(bf)abf&gt;</td>
</tr>
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Initial candidates: All 8-s singleton sequences
- <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>

Scan DB once, count support for each candidate

Generate length-2 candidate sequences

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Why?
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**Why?**

GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT’96
**GSP (Generalized Sequential Patterns):**

**Apriori-Based Sequential Pattern Mining**

- **Initial candidates:** All 8-singleton sequences
  - `<a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>`
- **Scan DB once, count support for each candidate**
- **Generate length-2 candidate sequences**

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- **Without Apriori pruning:**
  - (8 singletons) \(8 \times 8 + 8 \times 7/2 = 92\) length-2 candidates
- **With pruning, length-2 candidates:** 36 + 15 = 51
GSP Mining and Pruning

5th scan: 1 cand. 1 length-5 seq. pat.
4th scan: 8 cand. 7 length-4 seq. pat.
3rd scan: 46 cand. 20 length-3 seq. pat. 20 cand. not in DB at all
2nd scan: 51 cand. 19 length-2 seq. pat. 10 cand. not in DB at all
1st scan: 8 cand. 6 length-1 seq. pat.

Candidates cannot pass min_sup threshold

Candidates not in DB

min_sup = 2

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- Repeat (for each level (i.e., length-k))
  - Scan DB to find length-k frequent sequences
  - Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
  - set k = k+1
  - Until no frequent sequence or no candidate can be found

Candidates cannot pass min_sup threshold
Candidates not in DB

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</tr>
<tr>
<td>40</td>
<td>(be)(ce)d</td>
</tr>
<tr>
<td>50</td>
<td>a(bd)bcb(ade)</td>
</tr>
</tbody>
</table>
GSP: Algorithm

- **Phase 1:**
  - Scan over the database to identify all the frequent items, i.e., 1-element sequences

- **Phase 2:**
  - Iteratively scan over the database to discover all frequent sequences. Each iteration discovers all the sequences with the same length.
  - In the iteration to generate all $k$-sequences
  - Generate the set of all candidate $k$-sequences, $C_k$, by joining two $(k-1)$-sequences
    - Prune the candidate sequence if any of its $k-1$ subsequences is not frequent
    - Scan over the database to determine the support of the remaining candidate sequences
  - Terminate when no more frequent sequences can be found

A detailed example illustration:

http://simpledatamining.blogspot.com/2015/03/generalized-sequential-pattern-gsp.html
Bottlenecks of GSP

- A huge set of candidates could be generated
  - 1,000 frequent length-1 sequences generate length-2 candidates!
    \[1000 \times 1000 + \frac{1000 \times 999}{2} = 1,499,500\]
  
- Multiple scans of database in mining

- Real challenge: mining long sequential patterns
  - An exponential number of short candidates
  - A length-100 sequential pattern needs \(10^{30}\) candidate sequences!

\[\sum_{i=1}^{100} \binom{100}{i} = 2^{100} - 1 \approx 10^{30}\]
GSP: Optimization Techniques

- Applied to phase 2: computation-intensive
- Technique 1: the hash-tree data structure
  - Used for counting candidates to reduce the number of candidates that need to be checked
    - Leaf: a list of sequences
    - Interior node: a hash table
- Technique 2: data-representation transformation
  - From horizontal format to vertical format

<table>
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<th>Transaction-Time</th>
<th>Items</th>
</tr>
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<tbody>
<tr>
<td>10</td>
<td>1, 2</td>
</tr>
<tr>
<td>25</td>
<td>4, 6</td>
</tr>
<tr>
<td>45</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>1, 2</td>
</tr>
<tr>
<td>65</td>
<td>3</td>
</tr>
<tr>
<td>90</td>
<td>2, 4</td>
</tr>
<tr>
<td>95</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
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<th>Item</th>
<th>Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>→ 10 → 50 → NULL</td>
</tr>
<tr>
<td>2</td>
<td>→ 10 → 50 → 90 → NULL</td>
</tr>
<tr>
<td>3</td>
<td>→ 45 → 65 → NULL</td>
</tr>
<tr>
<td>4</td>
<td>→ 25 → 90 → NULL</td>
</tr>
<tr>
<td>5</td>
<td>→ NULL</td>
</tr>
<tr>
<td>6</td>
<td>→ 25 → 95 → NULL</td>
</tr>
<tr>
<td>7</td>
<td>→ NULL</td>
</tr>
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SPADE

- **Problems in the GSP Algorithm**
  - Multiple database scans
  - Complex hash structures with poor locality
  - Scale up linearly as the size of dataset increases

- **SPADE: Sequential PAttern Discovery using Equivalence classes**
  - Use a vertical id-list database
  - Prefix-based equivalence classes
  - Frequent sequences enumerated through simple temporal joins
  - Lattice-theoretic approach to decompose search space

- **Advantages of SPADE**
  - 3 scans over the database
  - Potential for in-memory computation and parallelization

Paper Link:
FINDING SIMILAR ITEMS

MMDS Secs. 3.2-3.4.

FINDING SIMILAR ITEMS

Slides also adapted from Prof. Srinivasan Parthasarathy @OSU
Task: Finding Similar Documents

- **Goal:** Given a large number ($N$ in the millions or billions) of documents, find “near duplicate” pairs

- **Applications:**
  - Mirror websites, or approximate mirrors → remove duplicates
  - Similar news articles at many news sites → cluster
Task: Finding Similar Documents

- **Goal:** Given a large number ($N$ in the millions or billions) of documents, find “near duplicate” pairs

- **Applications:**
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What are the challenges?
Task: Finding Similar Documents

Goal: Given a large number ($N$ in the millions or billions) of documents, find “near duplicate” pairs

Applications:
- Mirror websites, or approximate mirrors → remove duplicates
- Similar news articles at many news sites → cluster

Problems:
- Many small pieces of one document can appear out of order in another
- Too many documents to compare all pairs
- Documents are so large or so many (scale issues)
Two Essential Steps for Similar Docs

1. **Shingling**: Convert documents to sets

2. **Min-Hashing**: Convert large sets to short signatures, while preserving similarity

Host of follow up applications
- e.g. Similarity Search
- Data Placement
- Clustering etc.

The Big Picture

The set of strings of length $k$ that appear in the document

*Signatures*: short integer vectors that represent the sets, and reflect their similarity

**SHINGLING**

**Step 1:** *Shingling* Convert documents to sets

- **Document** → **Shingling** → The set of strings of length $k$ that appear in the document.
Step 1: Shingling: Convert documents to sets

Simple approaches:
- Document = set of words appearing in document
- Document = set of “important” words
- Don’t work well for this application. Why?

Need to account for ordering of words!
A different way: Shingles!

Define: Shingles

- A $k$-shingle (or $k$-gram) for a document is a sequence of $k$ tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples
Define: Shingles

- A *k-shingle* (or *k-gram*) for a document is a sequence of *k* tokens that appears in the document.
  - Tokens can be *characters*, *words* or something else, depending on the application.
  - Assume tokens = characters for examples.

**Example:** $k=2$; document $D_1 = \text{abcab}$

Set of 2-shingles: $S(D_1) = \{\text{ab, bc, ca}\}$
Define: Shingles

- A \( k \)-shingle (or \( k \)-gram) for a document is a sequence of \( k \) tokens that appears in the doc.
  - Tokens can be characters, words or something else, depending on the application.
  - Assume tokens = characters for examples.

Example: \( k = 2 \); document \( D_1 = \text{abcab} \)
Set of 2-shingles: \( S(D_1) = \{ab, bc, ca\} \)

Another option: Shingles as a bag (multiset), count \( ab \) twice: \( S'(D_1) = \{ab, bc, ca, ab\} \)
Shingles: How to treat white-space chars?

Example 3.4: If we use $k = 9$, but eliminate whitespace altogether, then we would see some lexical similarity in the sentences “The plane was ready for touch down”. and “The quarterback scored a touchdown”. However, if we retain the blanks, then the first has shingles touch dow and ouch dow, while the second has touchdown. If we eliminated the blanks, then both would have touchdown. □

It makes sense to replace any sequence of one or more white-space characters (blank, tab, newline, etc.) by a single blank.

This way distinguishes shingles that cover two or more words from those that do not.
How to choose K?

- **Documents that have lots of shingles in common have similar text, even if the text appears in different order**

- **Caveat:** You must pick \( k \) large enough, or most documents will have most shingles
  - \( k = 5 \) is OK for short documents
  - \( k = 10 \) is better for long documents
To **compress long shingles**, we can **hash** them to (say) 4 bytes

- Like a Code Book
- If #shingles manageable $\rightarrow$ Simple dictionary suffices

  e.g., 9-shingle $\Rightarrow$ bucket number $[0, 2^{32} - 1]$
  (using 4 bytes instead of 9)
Compressing Shingles

- To **compress long shingles**, we can **hash** them to (say) 4 bytes
  - Like a Code Book
  - If #shingles manageable → Simple dictionary suffices

- **Doc represented by the set of hash/dict. values of its k-shingles**
  - **Idea:** Two documents could appear to have shingles in common, when the hash-values were shared
Compressing Shingles

- To **compress long shingles**, we can **hash** them to (say) 4 bytes
  - Like a Code Book
  - If #shingles manageable $\rightarrow$ Simple dictionary suffices

- **Doc represented by the set of hash/dict. values of its $k$-shingles**

- **Example**: $k=2$; document $D_1 = \text{abcab}$
  - Set of 2-shingles: $S(D_1) = \{ab, bc, ca\}$
  - Hash the singles: $h(D_1) = \{1, 5, 7\}$
Similarity Metric for Shingles

- **Document** $D_1$ is a set of its $k$-shingles $C_1 = S(D_1)$

- Equivalently, each document is a 0/1 vector in the space of $k$-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse

- **A natural similarity measure is the Jaccard similarity:**

  \[
  \text{sim}(D_1, D_2) = \frac{|C_1 \cap C_2|}{|C_1 \cup C_2|}
  \]
Motivation for Minhash/LSH

- Suppose we need to find similar documents among $N = 1$ million documents.

- Naïvely, we would have to compute pairwise Jaccard similarities for every pair of docs.

  - $N(N - 1)/2 \approx 5 \times 10^{11}$ comparisons
  - At $10^5$ secs/day and $10^6$ comparisons/sec, it would take 5 days

- For $N = 10$ million, it takes more than a year...
Step 2: **Minhashing**: Convert large variable length sets to short fixed-length signatures, while preserving similarity.
Many similarity problems can be formalized as finding subsets that have significant intersection.
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Encode sets using 0/1 (bit, boolean) vectors:

- One dimension per element in the universal set.

Interpret set intersection as bitwise AND, and set union as bitwise OR.
Many similarity problems can be formalized as finding subsets that have significant intersection.

- Encode sets using 0/1 (bit, boolean) vectors
  - One dimension per element in the universal set
- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**

**Example:** $C_1 = 10111; C_2 = 10011$
- Size of intersection $= 3$; size of union $= 4$,
- **Jaccard similarity** (not distance) $= 3/4$
- **Distance:** $d(C_1, C_2) = 1 - \text{(Jaccard similarity)} = 1/4$
From Sets to Boolean Matrices

- **Rows** = elements (shingles)
  - 1 in row $e$ and column $s$ if and only if $e$ is a valid shingle of document represented by $s$

- **Columns** = sets (documents)
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value 1)
  - Typical matrix is sparse!

**Note: Transposed Document Matrix**

```
Shingles          Documents
1 1 1 0
1 1 0 1
0 1 0 1
0 0 0 1
1 0 0 1
1 1 1 0
1 0 1 0
```
Finding Similar Columns

So far:

- A document → a set of shingles
- Represent a set as a boolean vector in a matrix

<table>
<thead>
<tr>
<th>Documents</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>
So far:
- A document $\rightarrow$ a set of shingles
- Represent a set as a boolean vector in a matrix

Next goal: Find similar columns while computing small signatures

Similarity of columns $\leftrightarrow$ similarity of signatures
Outline: Finding Similar Columns

- **Next Goal**: Find similar columns, Small signatures

- **Naïve approach**:
  - 1) **Signatures of columns**: small summaries of columns
Next Goal: Find similar columns, Small signatures

Naïve approach:

1) Signatures of columns: small summaries of columns
2) Examine pairs of signatures to find similar columns
   - Essential: Similarities of signatures and columns are related
3) Optional: Check that columns with similar signatures are really similar
Next Goal: Find similar columns, Small signatures

Naïve approach:

1) Signatures of columns: small summaries of columns
2) Examine pairs of signatures to find similar columns
   - Essential: Similarities of signatures and columns are related
3) Optional: Check that columns with similar signatures are really similar

Warnings:

Comparing all pairs may take too much time: Job for LSH
   - These methods can produce false negatives, and even false positives (if the optional check is not made)
**Key idea**: “hash” each column $C$ to a small signature $h(C)$, such that:

- (1) $h(C)$ is small enough that the signature fits in RAM
- (2) $\text{sim}(C_1, C_2)$ is the same as the “similarity” of signatures $h(C_1)$ and $h(C_2)$
Hashing Columns (Signatures) : LSH principle

- **Key idea:** “hash” each column $C$ to a small signature $h(C)$, such that:
  1. $h(C)$ is small enough that the signature fits in RAM
  2. $\text{sim}(C_1, C_2)$ is the same as the “similarity” of signatures $h(C_1)$ and $h(C_2)$

- **Goal:** Find a hash function $h(\cdot)$ such that:
  1. If $\text{sim}(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  2. If $\text{sim}(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$
Hashing Columns (Signatures) : LSH principle

- **Key idea:** “hash” each column \( C \) to a small signature \( h(C) \), such that:
  - (1) \( h(C) \) is small enough that the signature fits in RAM
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  - If \( \text{sim}(C_1,C_2) \) is low, then with high prob. \( h(C_1) \neq h(C_2) \)

- Hash docs into buckets. Expect that “most” pairs of near duplicate docs hash into the same bucket!

---

Min-Hashing

- **Goal:** Find a hash function $h(\cdot)$ such that:
  - if $\text{sim}(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $\text{sim}(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- **Clearly, the hash function depends on the similarity metric:**
  - Not all similarity metrics have a suitable hash function
Min-Hashing

- **Goal:** Find a hash function $h(\cdot)$ such that:
  - if $\text{sim}(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $\text{sim}(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- Clearly, the hash function depends on the similarity metric:
  - Not all similarity metrics have a suitable hash function

- There is a suitable hash function for the Jaccard similarity: It is called Min-Hashing

Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation $\pi$. 

Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation $\pi$

- Define a “hash” function $h_\pi(C) = \text{the index of the first (in the permuted order $\pi$) row in which column } C \text{ has value } 1$:
  
  $$h_\pi(C) = \min_\pi \pi(C)$$
Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation $\pi$

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- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column
Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation $\pi$

- Define a “hash” function $h_\pi(C) = \text{the index of the first (in the permuted order } \pi) \text{ row in which column } C \text{ has value 1}$:

$$h_\pi(C) = \min_\pi \pi(C)$$

- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column
Zoo example (shingle size $k=1$)

Universe $\rightarrow \{ \text{dog, cat, lion, tiger, mouse} \}$

$\pi_1 \rightarrow [ \text{cat, mouse, lion, dog, tiger}]$

$\pi_2 \rightarrow [ \text{lion, cat, mouse, dog, tiger}]$

$A = \{ \text{mouse, lion} \}$
Zoo example (shingle size k=1)

Universe $\rightarrow \{\text{dog, cat, lion, tiger, mouse}\}$

$\pi_1 \rightarrow [\text{cat, mouse, lion, dog, tiger}]$

$\pi_2 \rightarrow [\text{lion, cat, mouse, dog, tiger}]$

$A = \{\text{mouse, lion}\}$

$mh_1(A) = \min (\pi_1\{\text{mouse, lion}\}) = \text{mouse}$

$mh_2(A) = \min (\pi_2\{\text{mouse, lion}\}) = \text{lion}$
Min-Hashing Example

Permutation $\pi$

Input matrix (Shingles x Documents)

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>7</th>
<th>6</th>
<th>1</th>
<th>5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Signature matrix $M$

\[
\begin{bmatrix}
2 & 1 & 2 & 1
\end{bmatrix}
\]

Min-Hashing Example

Permutation \( \pi \)

Input matrix (Shingles x Documents)

Signature matrix \( M \)

2\textsuperscript{nd} element of the permutation is the first to map to a 1

Permutation \( \pi \):

\[
\begin{array}{ccc}
2 & 3 & 7 \\
6 & 1 & 5 \\
4 & & \\
\end{array}
\]

Input matrix (Shingles x Documents):

\[
\begin{array}{cccc}
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 \\
\end{array}
\]

Signature matrix \( M \):

\[
\begin{array}{cccc}
2 & 1 & 2 & 1 \\
\end{array}
\]

Min-Hashing Example

Permutation $\pi$

<table>
<thead>
<tr>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
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<tr>
<td>1</td>
<td>6</td>
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<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Input matrix (Shingles x Documents)

<table>
<thead>
<tr>
<th>2</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tbody>
</table>

Signature matrix $M$

<table>
<thead>
<tr>
<th>2</th>
<th>1</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Min-Hashing Example

Permutation $\pi$  

Input matrix (Shingles x Documents)  

Signature matrix $M$

2\textsuperscript{nd} element of the permutation is the first to map to a 1

4\textsuperscript{th} element of the permutation is the first to map to a 1

### Min-Hashing Example

#### Signature Matrix $M$

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

#### Input Matrix (Shingles x Documents)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
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</table>

#### Permutation $\pi$

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</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
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<tr>
<td>7</td>
<td>1</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>3</td>
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<tr>
<td>5</td>
<td>7</td>
<td>1</td>
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<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Min-Hashing Example

Permutation $\pi$ | Input matrix (Shingles x Documents) | Signature matrix $M$
---|---|---
2 | 1 0 1 0 | 2 1 2 1
3 | 1 0 0 1 | 2 1 4 1
7 | 0 1 0 1 | 1 2 1 2
6 | 0 1 0 1 |
1 | 0 1 0 1 |
5 | 1 0 1 0 |
4 | 1 0 1 0 |

Note: Another (equivalent) way is to store row indexes or raw shingles (e.g. mouse, lion):
Min-Hash Signatures

- Pick $K=100$ random permutations of the rows
- Think of $\text{sig}(C)$ as a column vector
- $\text{sig}(C)[i] =$ according to the $i$-th permutation, the index of the first row that has a 1 in column $C$
  
  $\text{sig}(C)[i] = \min (\pi_i(C))$

- **Note:** The sketch (signature) of document $C$ is small $\sim 100$ bytes!

- **We achieved our goal!** We “compressed” long bit vectors into short signatures
Key Fact

For two sets A, B, and a min-hash function \( mh_i() \):

\[
Pr[mh_i(A) = mh_i(B)] = Sim(A, B) = \frac{|A \cap B|}{|A \cup B|}
\]

Unbiased estimator for \( Sim \) using \( K \) hashes (notation policy – this is a different \( K \) from size of shingle)

\[
\hat{Sim}(A, B) = \frac{1}{k} \sum_{i=1}^{k} I[mh_i(A) = mh_i(B)]
\]
Min-Hashing Example

Permutation $\pi$

<table>
<thead>
<tr>
<th>2</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
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<tr>
<td>7</td>
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<td>7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Input matrix (Shingles x Documents)

<table>
<thead>
<tr>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0</td>
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</tbody>
</table>

Signature matrix $M$

<table>
<thead>
<tr>
<th>2</th>
<th>1</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Similarities:

<table>
<thead>
<tr>
<th>Col/Col</th>
<th>Sig/Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>2-4</td>
</tr>
<tr>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>0.67</td>
<td>1.00</td>
</tr>
</tbody>
</table>
The Min-Hash Property

- **Choose a random permutation** $\pi$
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$
- **Why?**

<p>| | | | |</p>
<table>
<thead>
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</tr>
</tbody>
</table>

One of the two cols had to have 1 at position $y$
The Min-Hash Property

- Choose a random permutation $\pi$
- Claim: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$
- Why?
  - Let $X$ be a doc (set of shingles), $y \in X$ is a shingle

\[
\begin{array}{c|c}
0 & 0 \\
0 & 0 \\
1 & 1 \\
0 & 0 \\
0 & 1 \\
1 & 0 \\
\end{array}
\]

One of the two cols had to have 1 at position $y$
The Min-Hash Property

- **Choose a random permutation** \( \pi \)
- **Claim:** \( \Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2) \)
- **Why?**
  - Let \( X \) be a doc (set of shingles), \( y \in X \) is a shingle
  - **Then:** \( \Pr[\pi(y) = \min(\pi(X))] = 1/|X| \)
    - It is equally likely that any \( y \in X \) is mapped to the \textit{min} element

One of the two cols had to have 1 at position \( y \)
The Min-Hash Property

- Choose a random permutation \( \pi \)
- **Claim:** \( \Pr[h_{\pi}(C_1) = h_{\pi}(C_2)] = \text{sim}(C_1, C_2) \)
- **Why?**
  - Let \( X \) be a doc (set of shingles), \( y \in X \) is a shingle
  - Then: \( \Pr[\pi(y) = \min(\pi(X))] = 1/|X| \)
    - It is equally likely that any \( y \in X \) is mapped to the min element
  - Let \( y \) be s.t. \( \pi(y) = \min(\pi(C_1 \cup C_2)) \)
  - Then either: \( \pi(y) = \min(\pi(C_1)) \) if \( y \in C_1 \), or
    \( \pi(y) = \min(\pi(C_2)) \) if \( y \in C_2 \)

One of the two cols had to have 1 at position \( y \)
The Min-Hash Property

- **Choose a random permutation** $\pi$
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$
- **Why?**
  - Let $X$ be a doc (set of shingles), $y \in X$ is a shingle
  - **Then:** $\Pr[\pi(y) = \min(\pi(X))] = \frac{1}{|X|}$
    - It is equally likely that any $y \in X$ is mapped to the min element
  - Let $y$ be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - **Then either:**
    - $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, or
    - $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that **both** are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1))=\min(\pi(C_2))] = \frac{|C_1 \cap C_2|}{|C_1 \cup C_2|} = \text{sim}(C_1, C_2)$

One of the two cols had to have 1 at position $y$
The Min-Hash Property (Take 2: simpler proof)

- **Choose a random permutation** $\pi$
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- **Why?**
  - Given a set $X$, the probability that any one element is the min-hash under $\pi$ is $1/|X|$ $\leftarrow (0)$
    - It is equally likely that any $y \in X$ is mapped to the min element
  - Given a set $X$, the probability that one of any $k$ elements is the min-hash under $\pi$ is $k/|X|$ $\leftarrow (1)$
  - For $C_1 \cup C_2$, the probability that any element is the min-hash under $\pi$ is $1/|C_1 \cup C_2|$ (from 0) $\leftarrow (2)$
  - For any $C_1$ and $C_2$, the probability of choosing the same min-hash under $\pi$ is $|C_1 \cap C_2|/|C_1 \cup C_2|$ $\leftarrow$ from (1) and (2)
Similarity for Signatures

- We know: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$
- Now generalize to multiple hash functions

- The similarity of two signatures is the fraction of the hash functions in which they agree

- Note: Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures
Min-Hashing Example

<table>
<thead>
<tr>
<th>Permutation $\pi$</th>
<th>Input matrix (Shingles x Documents)</th>
<th>Signature matrix $M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 4 3</td>
<td>1 0 1 0 1 0</td>
<td>2 1 2 1</td>
</tr>
<tr>
<td>3 2 4</td>
<td>1 0 0 0 1 1</td>
<td>2 1 4 1</td>
</tr>
<tr>
<td>7 1 7</td>
<td>0 1 0 1 1 1</td>
<td>1 2 1 2</td>
</tr>
<tr>
<td>6 3 2</td>
<td>0 1 0 1 1 1</td>
<td></td>
</tr>
<tr>
<td>1 6 6</td>
<td>0 1 0 1 1 1</td>
<td></td>
</tr>
<tr>
<td>5 7 1</td>
<td>1 0 1 0 1 1</td>
<td></td>
</tr>
<tr>
<td>4 5 5</td>
<td>1 0 1 0 1 1</td>
<td></td>
</tr>
</tbody>
</table>

Similarities:

<table>
<thead>
<tr>
<th></th>
<th>1-3</th>
<th>2-4</th>
<th>1-2</th>
<th>3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Col/Col</td>
<td>0.75</td>
<td>0.75</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sig/Sig</td>
<td>0.67</td>
<td>1.00</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Min-Hash Signatures

- Pick $K=100$ random permutations of the rows
- Think of $\text{sig}(C)$ as a column vector
- $\text{sig}(C)[i] =$ according to the $i$-th permutation, the index of the first row that has a 1 in column $C$
  
  $$\text{sig}(C)[i] = \min (\pi_i(C))$$

- Note: The sketch (signature) of document $C$ is small $\sim 100$ bytes!

- We achieved our goal! We “compressed” long bit vectors into short signatures
Implementation Trick

- Permuting rows even once is prohibitive
- Approximate Linear Permutation Hashing
- Pick K independent hash functions (use a, b below)
  - Apply the idea on each column (document) for each hash function and get minhash signature

How to pick a random hash function \( h(x) \)?

Universal hashing:

\[
h_{a,b}(x) = ((a \cdot x + b) \mod p) \mod N
\]

where:
- \( a, b \) … random integers
- \( p \) … prime number (\( p > N \))
Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID

- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used *similarity preserving hashing* to generate signatures with property $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$
  - We used hashing to get around generating random permutations

Backup slides
Outline: Finding Similar Columns

- **So far:**
  - Documents → Sets of shingles
  - Represent sets as boolean vectors in a matrix

- **Next goal:** Find similar columns while computing small signatures
  - Similarity of columns == similarity of signatures
Next Goal: Find similar columns, Small signatures

Naïve approach:
1) Signatures of columns: small summaries of columns
2) Examine pairs of signatures to find similar columns
   - Essential: Similarities of signatures and columns are related
3) Optional: Check that columns with similar signatures are really similar

Warnings:
Comparing all pairs may take too much time: Job for LSH
- These methods can produce false negatives, and even false positives (if the optional check is not made)

Hashing Columns (Signatures) : LSH principle

- **Key idea**: “hash” each column $C$ to a small signature $h(C)$, such that:
  - (1) $h(C)$ is small enough that the signature fits in RAM
  - (2) $\text{sim}(C_1, C_2)$ is the same as the “similarity” of signatures $h(C_1)$ and $h(C_2)$
Hashing Columns (Signatures) : LSH principle

- **Key idea:** “hash” each column $C$ to a small signature $h(C)$, such that:
  1. $h(C)$ is small enough that the signature fits in RAM
  2. $\text{sim}(C_1, C_2)$ is the same as the “similarity” of signatures $h(C_1)$ and $h(C_2)$

- **Goal:** Find a hash function $h(\cdot)$ such that:
  1. If $\text{sim}(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  2. If $\text{sim}(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- Hash docs into buckets. Expect that “most” pairs of near duplicate docs hash into the same bucket!
Min-Hashing

- **Goal:** Find a hash function $h(\cdot)$ such that:
  - if $\text{sim}(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $\text{sim}(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- Clearly, the hash function depends on the similarity metric:
  - Not all similarity metrics have a suitable hash function

- There is a suitable hash function for the Jaccard similarity: It is called **Min-Hashing**
Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation $\pi$

- Define a “hash” function $h_\pi(C) = \text{the index of the first (in the permuted order } \pi) \text{ row in which column } C \text{ has value } 1$:

  $$h_\pi(C) = \min_\pi \pi(C)$$

- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column
Zoo example (shingle size k=1)

Universe $\rightarrow \{ \text{dog, cat, lion, tiger, mouse} \}$

$\pi_1 \rightarrow [ \text{cat, mouse, lion, dog, tiger}]$

$\pi_2 \rightarrow [ \text{lion, cat, mouse, dog, tiger}]$

$A = \{ \text{mouse, lion} \}$

$mh_1(A) = \min ( \pi_1 \{ \text{mouse, lion} \} ) = \text{mouse}$

$mh_2(A) = \min ( \pi_2 \{ \text{mouse, lion} \} ) = \text{lion}$
Key Fact

For two sets $A$, $B$, and a min-hash function $mh_i()$:

$$Pr[mh_i(A) = mh_i(B)] = Sim(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

Unbiased estimator for $Sim$ using $K$ hashes (notation policy – this is a different $K$ from size of shingle)

$$\hat{Sim}(A, B) = \frac{1}{k} \sum_{i=1}^{k} I[mh_i(A) = mh_i(B)]$$
Min-Hashing Example

Permutation $\pi$

Input matrix (Shingles x Documents)

Signature matrix $M$

Note: Another (equivalent) way is to store row indexes or raw shingles (e.g. mouse, lion):

The Min-Hash Property

- Choose a random permutation $\pi$

- Claim: $\Pr[h_{\pi}(C_1) = h_{\pi}(C_2)] = \text{sim}(C_1, C_2)$

- Why?
  - Let $X$ be a doc (set of shingles), $y \in X$ is a shingle
  - Then: $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$  
    - It is equally likely that any $y \in X$ is mapped to the min element
  - Let $y$ be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - Then either:  
    - $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, or  
    - $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that both are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1)) = \min(\pi(C_2))] = |C_1 \cap C_2| / |C_1 \cup C_2| = \text{sim}(C_1, C_2)$
Choose a random permutation $\pi$

Claim: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$

Why?

- Given a set $X$, the probability that any one element is the min-hash under $\pi$ is $1/|X|$ $\leftarrow (0)$
  - It is equally likely that any $y \in X$ is mapped to the min element
- Given a set $X$, the probability that one of any $k$ elements is the min-hash under $\pi$ is $k/|X|$ $\leftarrow (1)$
- For $C_1 \cup C_2$, the probability that any element is the min-hash under $\pi$ is $1/|C_1 \cup C_2|$ (from 0) $\leftarrow (2)$
- For any $C_1$ and $C_2$, the probability of choosing the same min-hash under $\pi$ is $|C_1 \cap C_2|/|C_1 \cup C_2| \leftarrow$ from (1) and (2)
Similarity for Signatures

- We know: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = \text{sim}(C_1, C_2)$
- Now generalize to multiple hash functions

The similarity of two signatures is the fraction of the hash functions in which they agree

- Note: Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures
### Min-Hashing Example

**Permutation \( \pi \)**

<table>
<thead>
<tr>
<th>2</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

**Input matrix (Shingles x Documents)**

\[
\begin{array}{cccc}
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0
\end{array}
\]

**Signature matrix \( M \)**

\[
\begin{array}{cccc}
2 & 1 & 2 & 1 \\
2 & 1 & 4 & 1 \\
1 & 2 & 1 & 2
\end{array}
\]

**Similarities:**

<table>
<thead>
<tr>
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<th>1-2</th>
<th>3-4</th>
</tr>
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<tbody>
<tr>
<td>0.75</td>
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Backup slides
Sequential Pattern Mining in Vertical Data Format: The SPADE Algorithm

- A sequence database is mapped to: <SID, EID>
- Grow the subsequences (patterns) one item at a time by Apriori candidate generation

<table>
<thead>
<tr>
<th>SID</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(&lt;a(abc)(ac)d(cf)&gt;)</td>
</tr>
<tr>
<td>2</td>
<td>&lt;(ad)c(bc)(ae)&gt;</td>
</tr>
<tr>
<td>3</td>
<td>&lt;(ef)(ab)(df)c (cb)&gt;)</td>
</tr>
<tr>
<td>4</td>
<td>&lt;eg(af)c (bc)&gt;)</td>
</tr>
</tbody>
</table>

\(\text{min} \_\text{sup} = 2\)

Ref: SPADE (Sequential PAtrn Discovery using Equivalent Class) [M. Zaki 2001]
PrefixSpan: A Pattern-Growth Approach

PrefixSpan Mining: Prefix Projections

- **Step 1:** Find length-1 sequential patterns
  - <a>, <b>, <c>, <d>, <e>, <f>

- **Step 2:** Divide search space and mine each projected DB
  - <a>-projected DB,
  - <b>-projected DB,
  - ...
  - <f>-projected DB, ...

- **Prefix and suffix**
  - Given <a(abc)(ac)d(cf)>
  - **Prefixes:** <a>, <aa>, <a(ab)>, <a(abc)>, ...
  - **Suffix:** Prefixes-based projection

<table>
<thead>
<tr>
<th>SID</th>
<th>Sequence</th>
<th>Prefix</th>
<th>Suffix (Projection)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>&lt;a(abc)(ac)d(cf)&gt;</td>
<td>&lt;a&gt;</td>
<td>&lt;a(abc)(ac)d(cf)&gt;</td>
</tr>
<tr>
<td>20</td>
<td>&lt;ad)c(bc)(ae)</td>
<td>&lt;aa&gt;</td>
<td>(_bc)(ac)d(cf)</td>
</tr>
<tr>
<td>30</td>
<td>&lt;ef)(ab)(df)(\varepsilon)b&gt;</td>
<td>&lt;ab&gt;</td>
<td>(_c)(ac)d(cf)</td>
</tr>
<tr>
<td>40</td>
<td>&lt;eg)(af)cbc&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

PrefixSpan (Prefix-projected Sequential pattern mining)
Pei, et al. @TKDE’04
### PrefixSpan: Mining Prefix-Projected DBs

<table>
<thead>
<tr>
<th>SID</th>
<th>Sequence</th>
<th>min_sup = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>&lt;a(abc)(ac)d(cf)&gt;</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>&lt;(ad)c(bc)(ae)&gt;</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>&lt;(ef)(ab)(df)cb&gt;</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>&lt;eg(af)cbc&gt;</td>
<td></td>
</tr>
</tbody>
</table>

**Length-1 sequential patterns:**
- <a>, <b>, <c>, <d>, <e>, <f>

**Length-2 sequential patterns:**
- <aa>, <ab>, <(ab)>, <ac>, <ad>, <af>

**Major strength of PrefixSpan:**
- No candidate subsequences to be generated.
- Projected DBs keep shrinking.

**PrefixSpan: Mining Prefix-Projected DBs**

- **Prefix <a>**
  - <a>-projected DB
  - <(abc)(ac)d(cf)>
  - <(_d)c(bc)(ae)>
  - <(_b)(df)cb>
  - <(_f)cbc>

- **Prefix <aa>**
  - <aa>-projected DB
  - ...<a>-projected DB

- **Prefix <af>**
  - <af>-projected DB
  - ...<a>-projected DB

- **Prefix <b>**
  - <b>-projected DB
  - ...<b>-projected DB

- **Prefix <c>, ..., <f>**
  - ...<c>, ..., <f>
Consideration:
Pseudo-Projection vs. Physical Projection

- Major cost of PrefixSpan: Constructing projected DBs
  - Suffixes largely repeating in recursive projected DBs
- When DB can be held in main memory, use pseudo projection
  - No physically copying suffixes
  - Pointer to the sequence
  - Offset of the suffix
  - But if it does not fit in memory
    - Physical projection
    - Suggested approach:
      - Integration of physical and pseudo-projection
      - Swapping to pseudo-projection when the data fits in memory
CloSpan: Mining Closed Sequential Patterns

- A closed sequential pattern $s$: There exists no superpattern $s'$ such that $s' \supseteq s$, and $s'$ and $s$ have the same support

- Which ones are closed? $<\text{abc}>: 20$, $<\text{abcd}>: 20$, $<\text{abcde}>: 15$

- Why directly mine closed sequential patterns?
  - Reduce # of (redundant) patterns
  - Attain the same expressive power

- Property P$_1$: If $s \supseteq s_1$, $s$ is closed iff two project DBs have the same size

- Explore Backward Subpattern and Backward Superpattern pruning to prune redundant search space

- Greatly enhances efficiency (Yan, et al., SDM’03)
CloSpan: When Two Projected DBs Have the Same Size

- If \( s \supseteq s_1 \), \( s \) is closed iff two project DBs have the same size
  - When two projected sequence DBs have the same size?
    - Here is one example:

  ![Diagram](image)

  - **Backward subpattern pruning**
  - **Backward superpattern pruning**

<table>
<thead>
<tr>
<th>ID</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt;aefbcg&gt;</td>
</tr>
<tr>
<td>2</td>
<td>&lt;afegb(ac)&gt;</td>
</tr>
<tr>
<td>3</td>
<td>&lt;(af)ea&gt;</td>
</tr>
</tbody>
</table>

\( \min_{\text{sup}} = 2 \)
Chapter 7: Advanced Frequent Pattern Mining

- Mining Diverse Patterns
- Sequential Pattern Mining
- Constraint-Based Frequent Pattern Mining
- Graph Pattern Mining
- Pattern Mining Application: Mining Software Copy-and-Paste Bugs
- Summary
Constraint-Based Pattern Mining

- Why Constraint-Based Mining?
- Different Kinds of Constraints: Different Pruning Strategies
  - Constrained Mining with Pattern Anti-Monotonicity
  - Constrained Mining with Pattern Monotonicity
  - Constrained Mining with Data Anti-Monotonicity
  - Constrained Mining with Succinct Constraints
  - Constrained Mining with Convertible Constraints
- Handling Multiple Constraints
- Constraint-Based Sequential-Pattern Mining
Why Constraint-Based Mining?

- Finding all the patterns in a dataset autonomously?—unrealistic!
  - Too many patterns but not necessarily user-interested!

- Pattern mining in practice: Often a user-guided, interactive process
  - User directs what to be mined using a data mining query language (or a graphical user interface), specifying various kinds of constraints

- What is constraint-based mining?
  - Mine together with user-provided constraints

- Why constraint-based mining?
  - User flexibility: User provides constraints on what to be mined
  - Optimization: System explores such constraints for mining efficiency
    - E.g., Push constraints deeply into the mining process
Various Kinds of User-Specified Constraints in Data Mining

- **Knowledge type constraint**—Specifying what kinds of knowledge to mine
  - Ex.: Classification, association, clustering, outlier finding, ...

- **Data constraint**—using SQL-like queries
  - Ex.: Find products sold together in NY stores this year

- **Dimension/level constraint**—similar to projection in relational database
  - Ex.: In relevance to region, price, brand, customer category

- **Interestingness constraint**—various kinds of thresholds
  - Ex.: Strong rules: min_sup ≥ 0.02, min_conf ≥ 0.6, min_correlation ≥ 0.7

- **Rule (or pattern) constraint**
  - Ex.: Small sales (price < $10) triggers big sales (sum > $200)

The focus of this study
Pattern Space Pruning with Pattern Anti-Monotonicity

A constraint $c$ is **anti-monotone**
- If an itemset $S$ violates constraint $c$, so does any of its superset
- That is, mining on itemset $S$ can be terminated

**Ex. 1:** $c_1: \text{sum}(S.\text{price}) \leq v$ is anti-monotone
**Ex. 2:** $c_2: \text{range}(S.\text{profit}) \leq 15$ is anti-monotone
- Itemset $ab$ violates $c_2$ (range(ab) = 40)
- So does every superset of $ab$

**Ex. 3:** $c_3: \text{sum}(S.\text{Price}) \geq v$ is **not** anti-monotone
**Ex. 4.** Is $c_4: \text{support}(S) \geq \sigma$ anti-monotone?
- Yes! Apriori pruning is essentially pruning with an anti-monotonic constraint!

Note: item.price > 0
Profit can be negative
Pattern Monotonicity and Its Roles

- A constraint $c$ is monotone: If an itemset $S$ satisfies the constraint $c$, so does any of its superset.
  - That is, we do not need to check $c$ in subsequent mining.
  - Ex. 1: $c_1$: $\text{sum}(S.Price) \geq v$ is monotone.
  - Ex. 2: $c_2$: $\text{min}(S.Price) \leq v$ is monotone.
  - Ex. 3: $c_3$: $\text{range}(S.profit) \geq 15$ is monotone.
    - Itemset $ab$ satisfies $c_3$.
    - So does every superset of $ab$.

<table>
<thead>
<tr>
<th>TID</th>
<th>Transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>a, b, c, d, f, h</td>
</tr>
<tr>
<td>20</td>
<td>b, c, d, f, g, h</td>
</tr>
<tr>
<td>30</td>
<td>b, c, d, f, g</td>
</tr>
<tr>
<td>40</td>
<td>a, c, e, f, g</td>
</tr>
</tbody>
</table>

| min_sup = 2 |

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>b</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>150</td>
<td>-20</td>
</tr>
<tr>
<td>d</td>
<td>35</td>
<td>-15</td>
</tr>
<tr>
<td>e</td>
<td>55</td>
<td>-30</td>
</tr>
<tr>
<td>f</td>
<td>45</td>
<td>-10</td>
</tr>
<tr>
<td>g</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>h</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: item.price > 0
Profit can be negative
Data Space Pruning with Data Anti-Monotonicity

A constraint \( c \) is data anti-monotone: In the mining process, if a data entry \( t \) cannot satisfy a pattern \( p \) under \( c \), \( t \) cannot satisfy \( p \)'s superset either.

- Data space pruning: Data entry \( t \) can be pruned.

- Ex. 1: \( c_1 \): sum(S.Profit) ≥ \( v \) is data anti-monotone
  - Let constraint \( c_1 \) be: sum(S.Profit) ≥ 25
    - \( T_{30} : \{b, c, d, f, g\} \) can be removed since none of their combinations can make an \( S \) whose sum of the profit is ≥ 25.

- Ex. 2: \( c_2 \): min(S.Price) ≤ \( v \) is data anti-monotone
  - Consider \( v = 5 \) but every item in a transaction, say \( T_{50} \), has a price higher than 10.

- Ex. 3: \( c_3 \): range(S.Profit) > 25 is data anti-monotone

<table>
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</tr>
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- \( \text{min\_sup} = 2 \)

<table>
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<tr>
<th>Item</th>
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<td>40</td>
</tr>
<tr>
<td>b</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>150</td>
<td>-20</td>
</tr>
<tr>
<td>d</td>
<td>35</td>
<td>-15</td>
</tr>
<tr>
<td>e</td>
<td>55</td>
<td>-30</td>
</tr>
<tr>
<td>f</td>
<td>45</td>
<td>-10</td>
</tr>
<tr>
<td>g</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>h</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: item.price > 0
Profit can be negative
Expressing Patterns in Compressed Form: Closed Patterns

How to handle such a challenge?

Solution 1: Closed patterns: A pattern (itemset) X is closed if X is frequent, and there exists no super-pattern Y ⊇ X, with the same support as X

- Let Transaction DB TDB₁: T₁: {a₁, ..., a₅₀}; T₂: {a₁, ..., a₁₀₀}
- Suppose \( \text{minsup} = 1 \). How many closed patterns does TDB₁ contain?
  - Two: \( P₁: \{a₁, ..., a₅₀\}: 2 \); \( P₂: \{a₁, ..., a₁₀₀\}: 1 \)

Closed pattern is a lossless compression of frequent patterns

- Reduces the # of patterns but does not lose the support information!
- You will still be able to say: \( \{a₂, ..., a₄₀\}: 2 \), \( \{a₅, a₅₁\}: 1 \)
Expressing Patterns in Compressed Form: Max-Patterns

- **Solution 2: Max-patterns:** A pattern X is a maximal frequent pattern or max-pattern if X is frequent and there exists no frequent super-pattern Y ⊆ X

- **Difference from close-patterns?**
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB₁:  \( T_1: \{a_1, ..., a_{50}\}; \ T_2: \{a_1, ..., a_{100}\} \)
  - Suppose \( \text{minsup} = 1 \). How many max-patterns does TDB₁ contain?
    - One: \( P: \{a_1, ..., a_{100}\}: 1 \)

- Max-pattern is a lossy compression!
  - We only know \( \{a_1, ..., a_{40}\} \) is frequent
  - But we do not know the real support of \( \{a_1, ..., a_{40}\}, ..., \) any more!
  - Thus in many applications, close-patterns are more desirable than max-patterns
Scaling FP-growth by Item-Based Data Projection

- What if FP-tree cannot fit in memory?—Do not construct FP-tree
  - “Project” the database based on frequent single items
  - Construct & mine FP-tree for each projected DB

- Parallel projection vs. partition projection
  - Parallel projection: Project the DB on each frequent item
    - Space costly, all partitions can be processed in parallel
  - Partition projection: Partition the DB in order
    - Passing the unprocessed parts to subsequent partitions

Trans. DB

f₂ f₃ f₄ g h
f₃ f₄ i j
f₂ f₄ k
f₁ f₃ h
...

Parallel projection

f₄-proj. DB

f₂ f₃
f₃
f₂
...

f₃-proj. DB

f₂ f₃
f₃
f₁
...

Partition projection

f₄-proj. DB

f₂ f₃
f₃
...

f₃-proj. DB

f₁
...

f₂ will be projected to f₃-proj. DB only when processing f₄-proj. DB
Analysis of DBLP Coauthor Relationships

- **DBLP**: Computer science research publication bibliographic database
- > 3.8 million entries on authors, paper, venue, year, and other information

<table>
<thead>
<tr>
<th>ID</th>
<th>Author A</th>
<th>Author B</th>
<th>$s(A \cup B)$</th>
<th>$s(A)$</th>
<th>$s(B)$</th>
<th>Jaccard</th>
<th>Cosine</th>
<th>Kulc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hans-Peter Kriegel</td>
<td>Martin Ester</td>
<td>28</td>
<td>146</td>
<td>54</td>
<td>0.163 (2)</td>
<td>0.315 (7)</td>
<td>0.355 (9)</td>
</tr>
<tr>
<td>2</td>
<td>Michael Carey</td>
<td>Miron Livny</td>
<td>26</td>
<td>104</td>
<td>58</td>
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Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related?
  - Use Kulc to find Advisor-advisee, close collaborators
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What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
  - Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers; ......

- Null-invariance is an important property

- Lift, $\chi^2$ and cosine are good measures if null transactions are not predominant
  - Otherwise, Kulczynski + Imbalance Ratio should be used to judge the interestingness of a pattern

- Exercise: Mining research collaborations from research bibliographic data
  - Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
  - Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
  - Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD’10
Mining Compressed Patterns

- Why mining compressed patterns?
  - Too many scattered patterns but not so meaningful
- Pattern distance measure
  \[ \text{Dist}(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|} \]
- \(\delta\)-clustering: For each pattern \(P\), find all patterns which can be expressed by \(P\) and whose distance to \(P\) is within \(\delta\) (\(\delta\)-cover)
- All patterns in the cluster can be represented by \(P\)
- Method for efficient, direct mining of compressed frequent patterns (e.g., D. Xin, J. Han, X. Yan, H. Cheng, "On Compressing Frequent Patterns", Knowledge and Data Engineering, 60:5-29, 2007)

<table>
<thead>
<tr>
<th>Pat-ID</th>
<th>Item-Sets</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>{38,16,18,12}</td>
<td>205227</td>
</tr>
<tr>
<td>P2</td>
<td>{38,16,18,12,17}</td>
<td>205211</td>
</tr>
<tr>
<td>P3</td>
<td>{39,38,16,18,12,17}</td>
<td>101758</td>
</tr>
<tr>
<td>P4</td>
<td>{39,16,18,12,17}</td>
<td>161563</td>
</tr>
<tr>
<td>P5</td>
<td>{39,16,18,12}</td>
<td>161576</td>
</tr>
</tbody>
</table>

- Closed patterns
  - P1, P2, P3, P4, P5
  - Emphasizes too much on support
  - There is no compression
- Max-patterns
  - P3: information loss
  - Desired output (a good balance):
    - P2, P3, P4
Redundancy-Aware Top-k Patterns

- Desired patterns: high significance & low redundancy

- Method: Use MMS (Maximal Marginal Significance) for measuring the combined significance of a pattern set

- Xin et al., Extracting Redundancy-Aware Top-K Patterns, KDD’06
Redundancy Filtering at Mining Multi-Level Associations

- Multi-level association mining may generate many redundant rules
- Redundancy filtering: Some rules may be redundant due to “ancestor” relationships between items

  - milk ⇒ wheat bread [support = 8%, confidence = 70%] (1)
  - 2% milk ⇒ wheat bread [support = 2%, confidence = 72%] (2)
    - Suppose the “2% milk” sold is about “1/4” of milk sold
      - Does (2) provide any novel information?

- A rule is redundant if its support is close to the “expected” value, according to its “ancestor” rule, and it has a similar confidence as its “ancestor”
  - Rule (1) is an ancestor of rule (2), which one to prune?
Succinctness

- Succinctness:
  - Given $A_1$, the set of items satisfying a succinctness constraint $C$, then any set $S$ satisfying $C$ is based on $A_1$, i.e., $S$ contains a subset belonging to $A_1$.
  - Idea: Without looking at the transaction database, whether an itemset $S$ satisfies constraint $C$ can be determined based on the selection of items.
    - $\min(S.Price) \leq v$ is succinct
    - $\sum(S.Price) \geq v$ is not succinct
  - Optimization: If $C$ is succinct, $C$ is pre-counting pushable.
## Which Constraints Are Succinct?

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Succinct</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v \in S )</td>
<td>yes</td>
</tr>
<tr>
<td>( S \supseteq V )</td>
<td>yes</td>
</tr>
<tr>
<td>( S \subseteq V )</td>
<td>yes</td>
</tr>
<tr>
<td>( \min(S) \leq v )</td>
<td>yes</td>
</tr>
<tr>
<td>( \min(S) \geq v )</td>
<td>yes</td>
</tr>
<tr>
<td>( \max(S) \leq v )</td>
<td>yes</td>
</tr>
<tr>
<td>( \max(S) \geq v )</td>
<td>yes</td>
</tr>
<tr>
<td>( \sum(S) \leq v \ (a \in S, a \geq 0) )</td>
<td>no</td>
</tr>
<tr>
<td>( \sum(S) \geq v \ (a \in S, a \geq 0) )</td>
<td>no</td>
</tr>
<tr>
<td>( \text{range}(S) \leq v )</td>
<td>no</td>
</tr>
<tr>
<td>( \text{range}(S) \geq v )</td>
<td>no</td>
</tr>
<tr>
<td>( \text{avg}(S) \theta v, \theta \in {=, \leq, \geq} )</td>
<td>no</td>
</tr>
<tr>
<td>( \text{support}(S) \geq \xi )</td>
<td>no</td>
</tr>
<tr>
<td>( \text{support}(S) \leq \xi )</td>
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</tbody>
</table>
Push a Succinct Constraint Deep

Database D

<table>
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<tr>
<th>TID</th>
<th>Items</th>
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<tbody>
<tr>
<td>100</td>
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<tr>
<td>200</td>
<td>2 3 5</td>
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<td>1 2 3 5</td>
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<td>400</td>
<td>2 5</td>
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Scan D

C₁

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<tr>
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<th>sup.</th>
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<tbody>
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</tr>
<tr>
<td>{2}</td>
<td>3</td>
</tr>
<tr>
<td>{3}</td>
<td>3</td>
</tr>
<tr>
<td>{4}</td>
<td>1</td>
</tr>
<tr>
<td>{5}</td>
<td>3</td>
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</table>

L₁

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>{1}</td>
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<tr>
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<tr>
<td>{3}</td>
<td>3</td>
</tr>
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<td>3</td>
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</table>

C₂

<table>
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<tr>
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<tbody>
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<td>1</td>
</tr>
<tr>
<td>{1 3}</td>
<td>2</td>
</tr>
<tr>
<td>{1 5}</td>
<td>1</td>
</tr>
<tr>
<td>{2 3}</td>
<td>2</td>
</tr>
<tr>
<td>{2 5}</td>
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L₂

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C₃

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<td>{2 3 5}</td>
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</table>

Constraint:
\[ \min\{S.\text{price} \leq 1\} \]
Sequential Pattern Mining

- Sequential Pattern and Sequential Pattern Mining
- GSP: Apriori-Based Sequential Pattern Mining
- SPADE: Sequential Pattern Mining in Vertical Data Format
- PrefixSpan: Sequential Pattern Mining by Pattern-Growth
- CloSpan: Mining Closed Sequential Patterns
The sequence $< (1,2) (3) (5) >$ is dropped in the pruning phase, since its contiguous subsequence $< (1) (3) (5) >$ is not frequent.
The apriori-generate function takes as argument \( L_{k-1} \), the set of all large \((k - 1)\)-sequences. The function works as follows. First, join \( L_{k-1} \) with \( L_{k-1} \):

\[
\begin{align*}
\text{insert into } & C_k \\
\text{select } & p \cdot \text{items} \_1, \ldots, p \cdot \text{items} \_k, q \cdot \text{items} \_k \\
\text{from } & L_{k-1} p, L_{k-1} q \\
\text{where } & p \cdot \text{items} \_1 = q \cdot \text{items} \_1, \ldots, \\
& p \cdot \text{items} \_k = q \cdot \text{items} \_k;
\end{align*}
\]

Next, delete all sequences \( c \in C_k \) such that some \((k - 1)\)-subsequence of \( c \) is not in \( L_{k-1} \).

<table>
<thead>
<tr>
<th>Large 3-Sequences</th>
<th>Candidate 4-Sequences (after join)</th>
<th>Candidate 4-Sequences (after pruning)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {1 , 2 , 3} )</td>
<td>( {1 , 2 , 3 , 4} )</td>
<td>( {1 , 2 , 3 , 4} )</td>
</tr>
<tr>
<td>( {1 , 2 , 4} )</td>
<td>( {1 , 2 , 4 , 3} )</td>
<td></td>
</tr>
<tr>
<td>( {1 , 3 , 4} )</td>
<td>( {1 , 3 , 4 , 5} )</td>
<td></td>
</tr>
<tr>
<td>( {1 , 3 , 5} )</td>
<td>( {1 , 3 , 5 , 4} )</td>
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</tr>
<tr>
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<td></td>
</tr>
</tbody>
</table>

Figure 7: Candidate Generation

Mining Sequential Patterns, Agrawal et al., ICDE'95