Assignment #4: Termination and Logical Time

DUE: in class, Monday, February 16th.

1. **The Coffee Bean Problem** (11 points)

   Consider a can of coffee beans. Each bean is either white or black. We are told the can is initially nonempty. Now consider the program that consists of a single action:

   Choose two beans from the can; if they are the same color, toss them out and put in a white bean (there is a sufficient supply); if they are different colors, toss them out and put in a black bean (there is a sufficient supply).

   This action is repeated.

   (a) (2 points) Let \( w \) be the number of white beans in the can and let \( b \) be the number of black beans. Write the program corresponding to the previous informal description. (Your program should have only two variables, \( w \) and \( b \)).

   (b) (2 points) Prove that \( \{ b \geq 0 \} \) is an invariant of this program.

   (c) (3 points) Give a variant function for this program and use it to prove the program terminates.

   (d) (4 points) If the program terminates with \( b = 1 \), what must have been true of the beans in the can at the beginning of the computation? (Hint: find an invariant on \( w \) and/or \( b \).)

2. **Vector Clocks** (15 points)

   Consider the implementation of vector clocks given in the notes:

   ```
   Program VectorClock
   var
   j, k : processes,
   ch.j.k : channel from j to k,
   m : message,
   A : event,
   vclock.j : vector time of j,
   vtime.A : vector time of A,
   
   initially
   vclock.j.j = 1
   \wedge (\forall k : k \neq j : vclock.j.k = 0)
   
   assign
   local event A \rightarrow
   vclock.j.j := clock.j.j + 1
   ; vtime.A := vclock.j
   
   \parallel send event A \rightarrow
   vclock.j.j := vclock.j.j + 1
   (to k)
   ; vtime.A, vtime.m := vclock.j, vclock.j
   ```
Prove the following invariants:

(a) \( \forall j, k :: \text{velock}.k.j \leq \text{velock}.j.j \)

(b) \( \forall A_j, B_k :: \text{vtime}.A_j < \text{vtime}.B_k \Rightarrow A_j \rightarrow B_k \) (where \( A_j \) denotes event \( A \) that occurred at process \( j \)).