Assignment #3: Reasoning About Programs — Safety & Progress

DUE: in class, Monday, February 9th.

1. Safety (6 points)
   Recall the program \textit{Strange} from an earlier assignment, where the constants \((M\text{ and }N)\) satisfy \(M \geq 0\) and \(N > 0\):

   \begin{verbatim}
   Program Strange
   var x, y, z, k : \mathbb{Z}
   initially x = 0 \land y = M \land z = N \land k = 1
   assign
       y \geq 2 \times z \rightarrow z, k := 2 \times z, 2 \times k
       y < 2 \times z \rightarrow z, k := N, 1
       y \geq z \rightarrow x, y := x + k, y - z
   \end{verbatim}

   Prove the following invariant for program \textit{Strange}:

   \[ x \times N + y = M \land y \geq 0 \land z = k \times N \land k \geq 1 \]

   If this program reaches a fixed point, what has it computed?

2. Progress (6 points)
   Consider the following program (the actions are labelled for your convenience):

   \begin{verbatim}
   assign
       a_1 : x := x + 1
       a_2 : y := y + 1
       a_3 : x \neq y \rightarrow z := z + 1
   \end{verbatim}

   Are the following properties of this program? Justify your answer with a proof or counter-example, as appropriate.

   (a) \textbf{transient}.(x + y = k)
   (b) \textbf{transient}.(x \leq k)
   (c) \(x = k \leadsto x > k\)
   (d) \textbf{transient}.(z = k)
   (e) \(z = k \leadsto z \neq k\)
3. **Termination** (6 points)

Given an array $A[0..N-1]$ of distinct natural numbers, consider the following program:

$$\text{assign} ( \quad \mid x : 0 \leq x < N \mid 0 \leq A[x] < N \rightarrow \ A[x], A[A[x]] := A[A[x]], A[x] )$$

Does this program terminate? If so, prove your claim. If not, provide a counter-example.