

# Heterogeneous Delay Tolerant Task Scheduling and Energy Management in the Smart Grid with Renewable Energy

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**Abstract**—The smart grid is the new generation of electricity grid that can efficiently utilize new distributed sources of energy (e.g., harvested renewable energy), and allow for dynamic electricity price. In this paper, we investigate the cost minimization problem for an end-user, such as a home, community, or a business, which is equipped with renewable energy devices when electrical appliances allow different levels of delay tolerance. The varying price of electricity presents an opportunity to reduce the electricity bill from an end-user’s point of view by leveraging the flexibility to schedule operations of various appliances and HVAC systems. We assume that the end user has an energy storage battery as well as an energy harvesting device so that harvested renewable energy can be stored and later used when the price is high. The energy storage battery can also draw energy from the external grid. The problem we formulate here is to minimize the cost of the energy drawn from the external grid while usage of appliances are subject to individual delay constraints and a long-term average delay constraint. The resulting algorithm requires some future information regarding electricity prices, but it achieves provable performance without requiring future knowledge of either the power demands or the task arrival process. Moreover, we analyze the influence of the assumption that energy can be sold from the battery to the grid. An alternative algorithm is proposed to take advantage of the ability to sell energy. The performance gap between our proposed algorithm and the optimum is shown to diminish as energy selling price approaches the electricity price.

## I. INTRODUCTION

The next-generation electricity grid, known as the “smart grid”, provides both suppliers and consumers with full visibility and pervasive control over their assets and services in order to achieve economy and sustainability [2]. Being able to incorporate renewable energy sources (e.g., solar or wind) is one of the key objectives of the smart grid [3]. In addition, the utility companies are allowed to dynamically adjust the electricity price in order to control the power usage. For example, price of electricity increases during high demand periods, and decreases during low demand periods. Consumers

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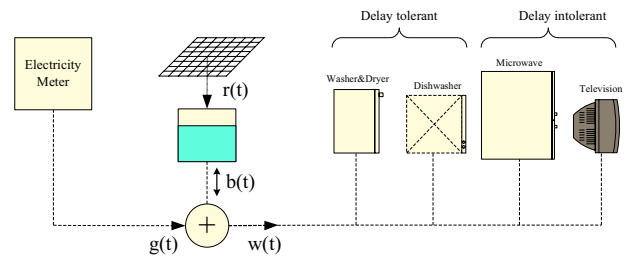


Fig. 1. Demand and Supply

thus can avoid the premium for using electricity at high price periods when they are aware of the price for some future period.

In this paper, we consider an end-user equipped with renewable energy devices in smart grid, where the electricity price is time varying. The renewable energy devices consist of an energy storage battery and an energy harvesting device. Renewable energy can be harvested and stored in the battery. We assume that the arrivals of demands for electrical appliances is a stochastic process (from now on, we use the terms appliance and task interchangeably). Fig. 1 shows some typical appliances at an end user. We assume that some tasks are delay tolerant, that is, they do not need to be activated immediately upon their arrival, such as washer and dish washer. They can be opportunistically scheduled when the electricity price is relatively low in order to reduce cost. For instance, if the price is high around 7pm and low around 2am, then some delay-tolerant tasks, such as dish washer, can be postponed to be scheduled around 2am. The power demand is met by drawing energy from the battery, or purchasing extra energy from the outside grid. We also allow the battery to charge energy from the grid, because the battery can purchase and store energy when the price is low, and discharge when the price is high.

In this work, we are interested in developing an optimal task scheduling algorithm that minimizes the total price cost of the energy drawn from the external grid subject to delay constraints. The customer has full control over all electricity appliances. The algorithm can exploit the delay flexibility and take advantage of time-varying prices.

### A. State-of-the-art

In power networks, there have been some literature that has focused on scheduling delay-tolerant tasks. Koutsopoulos

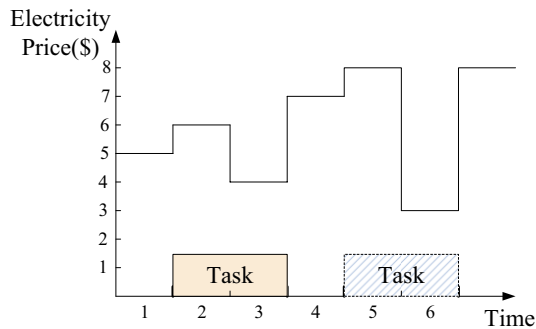


Fig. 2. An example of battery's influence on task scheduling

and Tassiulas [4] investigate an off-line version and an on-line version of the task scheduling problem. The authors propose two algorithms under these two cases, respectively, and provide a provable performance bound. However, these finite-horizon problems are proven to be NP-hard, and the algorithms only achieve optimality when the delay constraint is arbitrarily loose. In [5], the authors develop an energy allocation algorithm to minimize the total electricity cost. However, they do not allow renewable energy to be saved for future use, which our work takes into account. In addition, we provide individual delay constraints to all tasks, instead of an universal worst-case delay constraint which is likely to be very large. Some works adopt dynamic programming techniques, e.g. [6]. They can achieve optimality only if the distribution of the power demand is known a priori. There are also some other works that have formulated problems using game theory, e.g., [7]. The authors in [8] [9] develop a scheduling scheme to achieve an optimized upper bound on the power peak load. In [10], we investigate an energy trading problem in the smart grid. The energy selling price is assumed to be a fraction of the electricity price. Under the assumption that the energy demand process is an exogenous input process, an asymptotically optimal energy trading scheme is developed. However, in this paper, energy usage is one of our control variables. In [1], we only consider task scheduling problem, yet we do not allow selling energy from the battery to the grid.

### B. Our Contributions

In this paper, we address the task scheduling problem while tasks are subject to individual hard delay constraints and average delay constraints. To the best of our knowledge, it is the first work that takes into account these two different types of delay constraints in the area of smart grid. If there is no average delay constraint, a greedy algorithm could achieve the optimal solution. We, however, also take into account the average delay constraint, which is an important quality of service metric, but makes the problem challenging. Further, having a battery brings about significant differences. The reason is that the battery can draw energy from external grid when the electricity price is low and discharges energy when the price is high. Fig. 2 shows a simple example, where a task (the boxes illustrated in the figure) requires a service period of two slots. If there is no battery, we can see that the optimal

way is to schedule during time slot 2 and slot 3 (the red box), resulting in a total cost of 10 dollars. However, with the help of battery, we can store some energy in time slot 3 since the electricity price is low during this time slot. For simplicity of exposition, we assume that the maximum energy that is stored in one slot can be used to support up to one-slot service. Now, let us consider an alternate schedule, where the power demand during time slot 5 is met from the stored energy in slot 3 and the demand during time slot 6 is met from the external grid, as shown by the shadowed blue box. It can be seen that the total cost under this scheduling policy is 7 dollars, which is the optimal. Moreover, if energy can be sold from the battery to the grid, it implies higher flexibility in energy management and may lead to a further cost reduction.

We summarize our main contributions as follows:

- 1) We consider different types of delay constraints in our model. First, each task has a hard delay constraint, which cannot be violated. Further, there is a “dissatisfaction” function of delay for each task, and we require the long-term average dissatisfaction to be less than a threshold. This is a generalization of the average delay constraint.
- 2) We propose a simple algorithm that can achieve provable performance, which is within a bounded distance of the optimum. Note that our algorithm does not require future knowledge of the power demand and the task arrival process.
- 3) We revisit the cost minimization problem if selling energy from the battery to the grid is allowed. An alternative algorithm is proposed to take advantage of the ability to sell energy. The performance gap between our proposed algorithm and the optimum is shown to diminish as energy selling price approaches the electricity price.
- 4) We validate our algorithm using real electricity price traces to compute realistic savings. We show that our algorithm can indeed reduce cost under various system parameter settings.

Our paper is organized as follows: In Section II, we discuss our system model. In Section III, we formulate our cost minimization problem with various delay constraints. In Section IV, we develop our task scheduling algorithm and show its performance. Energy selling situation is discussed in Section V. After presenting simulation results in Section VI, we conclude our paper in Section VII.

## II. SYSTEM MODEL

We consider a set of appliances connected to the external smart grid. Time is assumed to be slotted. The price of electricity is time varying and denoted by  $P(t)$  in time slot  $t$ . As an example, Fig. 3 shows the average five-minute spot market prices for the Columbus area obtained from CAISO [11]. Let  $N_t$  represent the set of tasks that arrive in time slot  $t$ , while  $n_t$  represents the number of tasks in  $N_t$ , i.e.,  $n_t = |N_t|$ , where  $|\cdot|$  denotes the cardinality of a set. For simplicity of exposition, we assume that all tasks arrive in the beginning of each slot.

We note that there are two types of tasks, delay-tolerant and delay-intolerant tasks. Let  $c_i^t$  denote the required service

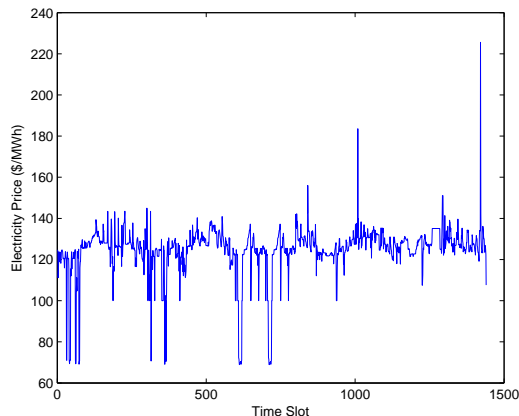


Fig. 3. 5-minute average spot market price during the week of 10/10/2011-10/14/2011 for Columbus Area from CAISO [11]

time for each task  $i \in N_t$ . Also, there is a deadline associated with each task  $i \in N_t$ , i.e., the maximum number of time slots allowed for finishing the job from its arrival time  $t$ , denoted by  $d_i^t$ . The deadline is a hard constraint, namely the task needs to be completed before time  $t + d_i^t$ . We call the task delay-intolerant if  $c_i^t = d_i^t$ , and delay-tolerant if  $c_i^t < d_i^t$ . For a delay-intolerant task, the only choice that we have is to activate it immediately upon its arrival. However, for delay-tolerant tasks, we can opportunistically schedule them in order to make use of the fluctuating nature of the electricity price. Our goal here is to find the optimal “postponing” time  $s_i^t$  so that the total cost is minimized subject to the delay constraints. Clearly, for the delay-intolerant tasks, we have to set  $s_i^t = 0$ . Let  $d_{max}$  denote the maximum delay allowed for any task, i.e.,  $d_{max} \triangleq \max_{t,i} d_i^t$ . Note that  $(c_i^t, s_i^t, d_i^t), \forall t, \forall i$  are integers. It is assumed that we have an accurate short-term estimation of the electricity price. More precisely, we know  $\vec{P}_t \triangleq P(t), P(t+1), \dots, P(t+d_{max})$ . It is worth pointing out that this is a reasonable assumption because the short-term estimation of electricity price can be obtained from the history [12].

Let  $h(t)$  denote the harvested renewable energy in time slot  $t$ , and let  $r(t)$  denote our energy storage decision, i.e., the actual energy that is stored into the battery. For simplicity of exposition, we assume that  $r(t)$  amount of energy is stored in the battery at the end of slot  $t$ . First, it is convenient for us to assume that battery has infinite capacity. We will show later that our algorithm only requires a reasonable sized finite battery. A natural constraint of  $r(t)$  is

$$r(t) \leq h(t). \quad (1)$$

The reason that we keep  $r(t)$  and  $h(t)$  different is due to some technical issues used in our proof. We assume that  $[n_t, c_i^t, h(t)]$  is i.i.d. over slots.

Let  $w(t)$  represent the total power demand in time slot  $t$ . We assume that each task  $i \in N_t$  consumes energy at a constant rate  $\pi_i^t$ , namely the power consumption for task  $i$  stays the same during its activation period. In this paper, we only consider the case where the activation period of any task

TABLE I  
NOTATIONS

Control var.	
$s_i^t$	Delay for task $i \in N_t$
$b(t)$	Energy drawn from (stored in) the battery
$r(t)$	Actual energy stored in the battery in slot $t$
Grid var.	
$P(t)$	Electricity price in time slot $t$
$g(t)$	Energy drawn from the grid in time slot $t$
Internal var.	
$c_i^t$	Required service time for task $i \in N_t$
$d_i^t$	Deadline for task $i \in N_t$
$w(t)$	Power demand in time slot $t$
$h(t)$	Harvested renewable energy in time slot $t$
$B(t)$	Battery level in time slot $t$
$\pi_i(t)$	Power consumption for task $i$
$U_i^t(s)$	Dissatisfaction function for delay $s$ for task $i$

is a contiguous chunk of time, and we do not consider the case where the activation period of tasks can be interrupted and resumed. We notice that part of  $w(t)$  is met by utilizing energy from the battery, while the other part will be drawn from the grid. Let  $g(t)$  and  $b(t)$  represent the amounts of energy that are drawn from the outside grid and the battery in time slot  $t$ , respectively. Because the supply always needs to balance the demand, we have  $w(t) = g(t) + b(t)$  as shown in Fig. 1. In addition, we also allow the battery to charge energy from the grid, which means that  $b(t)$  could be negative. In particular, the battery discharges/charges energy if we have  $b(t) \geq 0$ . We denote  $b_{max}$  as a maximal amount of energy either charging or discharging from the battery in one time slot. We use  $B(t)$  to denote the battery level at the beginning of time slot  $t$ , and the energy dynamics can be formulated as follows:

$$B(t+1) = B(t) + r(t) - b(t). \quad (2)$$

Since we have  $g(t) \geq 0$ , it follows that  $b(t) \leq w(t)$ . Therefore, the constraints on  $b(t)$  are given by

$$|b(t)| \leq b_{max} \quad (3)$$

$$b(t) \leq B(t), \quad (4)$$

$$b(t) \leq w(t), \quad (5)$$

where the second constraint means that the allocated energy from the battery should be less than or equal to the current available energy in the battery. For storing energy in the battery,  $b(t)$  could be negative and thus Eqn. (4) and (5) do not constrain the storage process.

Note that  $w(t)$  depends on the decisions made during time slot  $t$  up to time slot  $t - d_{max} + 1$ , we have

$$w(t) = \sum_{\tau=t-d_{max}+1}^t \sum_{i=1}^{n_\tau} \pi_i^\tau \mathbf{1}(\tau + s_i^\tau + c_i^\tau > t \ \& \ \tau + s_i^\tau \leq t), \quad (6)$$

where  $\mathbf{1}(\tau + s_i^\tau + c_i^\tau > t \ \& \ \tau + s_i^\tau \leq t)$  is the indicator function. The term  $\tau + s_i^\tau + c_i^\tau > t \ \& \ \tau + s_i^\tau \leq t$  means that a task  $i$  started before time slot  $t$  and will finish after time slot  $t$ . Thus, this task has energy consumption in time slot  $t$ .

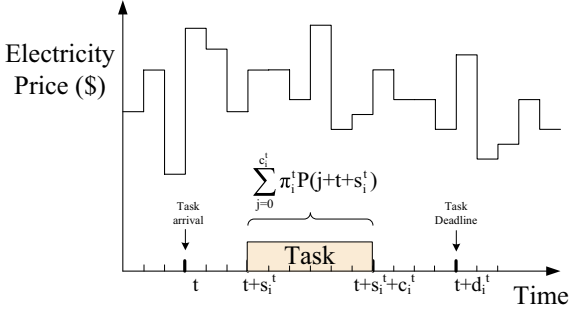


Fig. 4. Example of the scheduling of one task  $i \in N_t$

Our goal is decide  $(r(t), s_i^t, b(t))$  at each time slot such that the total price cost of the energy drawn from the external grid is minimized. We do not explicitly consider some practical issues, such as energy leakage in the battery or DC/AC conversion loss, but we can readily incorporate them into our model. We summarize the notations in Table I.

### III. PROBLEM FORMULATION

Suppose that there is an increasing convex function  $U_i^t(s)$ , satisfying  $U_i^t(0) = 0$ , which reflects the dissatisfaction associated with delay  $s$  for task  $i \in N_t$ . The convexity models a typical user for whom the rate of increase in dissatisfaction increases with delay. Notice that  $U_i^t(\cdot)$  is different for heterogeneous tasks. We assume that the long-term average dissatisfaction should be no greater than some threshold  $\alpha$ , that is,

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^{n_t} U_i^t(s_i^t) \leq \alpha. \quad (7)$$

For any task  $i \in N_t$ , since we have to finish it before the deadline, it yields

$$s_i^t + c_i^t \leq d_i^t.$$

Therefore, the constraint for the postponing time  $s_i^t$  is given by

$$0 \leq s_i^t \leq d_i^t - c_i^t. \quad (8)$$

Hence, the cost minimization problem can be formulated as

$$\text{Problem A: } \min_{r(t), s_i^t, b(t)} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}[g(t)P(t)] \quad (9)$$

s.t. (1), (2), (3), (4), (5), (7), (8),

where  $P(t)g(t)$  represents the total price of the energy drawn from the grid during time slot  $t$ .

Since  $g(t) = w(t) - b(t)$ , we can rewrite Problem A as follows:

$$\min_{r(t), s_i^t, b(t)} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}[w(t)P(t) - b(t)P(t)] \quad (10)$$

s.t. (1), (2), (3), (4), (5), (7), (8).

Notice that  $\lim_{T \rightarrow \infty} \sum_{t=1}^T w(t)P(t)$  represents the total cost of the power demand from the time horizon, while it

can be also derived by simply adding the cost for all tasks one by one. Thus, we have the following equation

$$\lim_{T \rightarrow \infty} \sum_{t=1}^T w(t)P(t) = \lim_{T \rightarrow \infty} \sum_{t=1}^T \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j+t+s_i^t), \quad (11)$$

where  $\sum_{j=0}^{c_i^t-1} \pi_i^t P(j+t+s_i^t)$  is the cost of task  $i \in N_t$  as depicted in Fig. 4.

Now, we can reformulate the optimization problem as follows:

$$\text{Problem B: } \min_{r(t), s_i^t, b(t)} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j+t+s_i^t) - P(t)b(t) \right] \quad (12)$$

s.t. (1), (2), (3), (4), (5), (7), (8).

Now we focus on Problem B and adopt the Lyapunov optimization approach [5] to solve it.

### IV. TASK SCHEDULING POLICY

In this section, we propose a task scheduling policy and show that its performance is within a bounded distance of the optimum as  $T$  tends to infinity.

#### A. Virtual Queue

Let us construct an auxiliary virtual queue  $Q(t)$ , whose input and output are  $\sum_{i=1}^{n_t} U_i^t(s_i^t)$  and  $\alpha$  respectively. The queueing dynamics is depicted as

$$Q(t+1) = \max\{Q(t) + \sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha, 0\} \quad (13)$$

**Lemma 1:** If the virtual queue is rate stable, i.e.,  $\limsup_{T \rightarrow \infty} Q(T)/T = 0$  with probability 1, then the constraint (7) is satisfied.

*Proof:* Suppose that the virtual queue is rate stable. Then we have

$$\limsup_{T \rightarrow \infty} \mathbb{E}[Q(T)]/T = 0. \quad (14)$$

Note that for any time  $T$ , by adding Eqn. (13) from slot 0 to slot  $T-1$ , the following inequality always holds:

$$Q(T) \geq Q(0) - T\alpha + \sum_{t=0}^{T-1} \sum_{i=1}^{n_t} U_i^t(s_i^t).$$

Dividing by  $T$  and taking expectation yields:

$$\mathbb{E}[Q(T)]/T \geq \mathbb{E}[Q(0)]/T - \alpha + \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^{n_t} \mathbb{E}[U_i^t(s_i^t)].$$

We take the limsup for both sides, it yields:

$$\alpha \geq \limsup_{T \rightarrow \infty} \frac{1}{T} \left( \sum_{t=1}^T \sum_{i=1}^{n_t} U_i^t(s_i^t) \right).$$

■

### B. Lower Bound the Minimum Cost

In this subsection, we will obtain a lower bound on the minimum cost of Problem **B**. The following lemma shows that the performance achieved by using a stationary and randomized algorithm forms a lower bound.

Let  $C^{opt}$  be the minimum cost to Problem **B**. And let  $\tilde{C}$  be the minimum cost to the following Problem **C**.

$$\text{Problem C: } \min_{r(t), s_i^t, b(t)} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^t + t) - P(t)b(t) \right]$$

s.t. (1), (2), (3), (7), (8).

Note that Problem **C** and **C** have the same objective function, but Problem **C** has fewer constraints. Thus, we know that  $C^{opt}$  is lower bounded by  $\tilde{C}$ , i.e.,  $\tilde{C} \leq C^{opt}$ .

**Lemma 2:**  $\tilde{C}$  can be achieved by an optimal stationary and randomized policy, that is, the control action  $(\tilde{r}(t), \tilde{s}_i^t, \tilde{b}(t))$  in each time slot is only a function of  $[n_t, c_i^t, h(t)]$ . In particular, we have

$$\mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + t + \tilde{s}_i^t) - P(t)\tilde{b}(t) \right] = \tilde{C}, \quad (15)$$

$$\mathbb{E} \left[ \sum_{i=1}^{n_t} U_i^t(\tilde{s}_i^t) - \alpha \right] \leq 0, \quad (16)$$

$$\mathbb{E}[\tilde{r}(t) - \tilde{b}(t)] \geq 0, \quad (17)$$

*Proof:*  $\tilde{C}$  is achieved over all possible control policies, not just stationary and randomized policies. However, we apply Theorem 4.5 in [13] in order to prove our result, that is,  $\tilde{C}$  can be achieved by a stationary and randomized policy  $\tilde{b}(t)$ .

We will show how to project Problem **C** to Eqn. (4.31)-(4.35) in [13]. First,  $[n_t, c_i^t, h(t)]$  corresponds to the i.i.d. state  $w(t)$ . And  $\frac{1}{T} \sum_{t=1}^T \mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^t + t) - P(t)b(t) \right]$  corresponds to  $\bar{y}_0(t)$ . Eqn. (16) means that the long-term average dissatisfaction achieved by the stationary policy is no greater than  $\alpha$ . Eqn. (17) implies that the average allocated energy from the battery is no greater than the stored energy. Eqns. (16) and (17) imply the battery level  $B(t)$  and the virtual queue  $Q(t)$  are required to be mean rate stable. Therefore, the result that  $\tilde{C}$  can be achieved by an optimal stationary and randomized policy  $(\tilde{r}(t), \tilde{s}_i^t, \tilde{b}(t))$  holds directly by applying Theorem 4.5 in [13].  $\blacksquare$

### C. HTSA: Heterogeneous Task Scheduling Algorithm

We define the Lyapunov function  $L(t) = \frac{1}{2}(Q(t)^2 + (B(t) - \theta)^2)$ , where  $\theta$  is a parameter specified later. The intuition behind it is that, by minimizing the drift of the Lyapunov function, we force  $B(t)$  to approach  $\theta$ . We also define several constants  $n_{max} = \max_t n_t$ ,  $h_{max} = \max_t h(t)$ ,  $c_{max} = \max_{t,i} c_i^t$ , and  $U_{max} = \max_{t,i} U_i^t(d_i^t)$ , where  $U_{max}$  reflects the maximum dissatisfaction among all tasks.

Let  $Z(t) = (Q(t), B(t))$ . The conditional Lyapunov drift is given by  $\mathbb{E}\{(L(t+1) - L(t)|Z(t))\}$ .

We will show some properties of the drift via the following lemma.

**Lemma 3:** The conditional Lyapunov drift satisfies that

$$\mathbb{E}\{(L(t+1) - L(t)|Z(t))\} \leq D + Q(t)\mathbb{E} \left[ \sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha | Z(t) \right] + (B(t) - \theta)\mathbb{E}[r(t) - b(t)|Z(t)], \quad (18)$$

where  $D \triangleq \frac{1}{2}(n_{max}^2 U_{max}^2 + \alpha^2 + r_{max}^2 + b_{max}^2)$ .

*Proof:* We refer to Appendix for the proof.  $\blacksquare$

By adding  $V\mathbb{E}[\sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^t + t) - P(t)b(t)|Z(t)]$  on both sides of Eqn. (18), we have

$$\begin{aligned} & \mathbb{E}\{(L(t+1) - L(t)|Z(t))\} \\ & + V\mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^t + t) - P(t)b(t) | Z(t) \right] \\ & \leq D + Q(t)\mathbb{E} \left[ \sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha | Z(t) \right] \\ & + (B(t) - \theta)\mathbb{E}[r(t) - b(t)|Z(t)] \\ & + V\mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^t + t) - P(t)b(t) | Z(t) \right] \\ & = D - \alpha Q(t) + (B(t) - \theta)\mathbb{E}[r(t)|Z(t)] + \\ & \sum_{i=1}^{n_t} \mathbb{E}[Q(t)U_i^t(s_i^t) + V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^t + t) | Z(t)] \\ & + (\theta - B(t) - VP(t))\mathbb{E}[b(t)|Z(t)], \end{aligned} \quad (19)$$

where  $V$  is a control parameter.

We now describe our scheme, *heterogeneous task scheduling algorithm (HTSA)*. The idea of HTSA is to minimize the right-hand side (RHS) of Eqn. (19) subject to the energy-availability constraint (4) and (5).

**Heterogeneous task scheduling algorithm (HTSA):**

- In each time slot  $t$ , the harvested energy  $r^*(t)$  is determined by

$$r^*(t) = \begin{cases} h(t), & \text{if } B(t) - \theta < 0, \\ 0, & \text{otherwise.} \end{cases} \quad (20)$$

- In each time slot  $t$ , the postponing time  $s_i^t$  for task  $i \in N_t$  is determined by:

$$s_i^{t*} = \arg \min_{0 \leq s_i^t \leq d_i^t - c_i^t} Q(t)U_i^t(s_i^t) + V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + t + s_i^t). \quad (21)$$

- In each time slot  $t$ , the battery charge/discharge is given by:

$$b^*(t) = \begin{cases} \min\{b_{max}, w(t)\}, & \text{if } \theta - B(t) - VP(t) < 0, \\ -b_{max}, & \text{otherwise,} \end{cases} \quad (22)$$

where  $w(t)$  is determined by Eqn. (6).

Define a constant  $P_{max}$  as the highest electricity price, i.e.,  $P_{max} = \max_t P(t)$ . By setting  $\theta = b_{max} + VP_{max}$ , from Eqn. (22), we can see that when  $B(t) < b_{max}$ , it always has  $\theta - B(t) - VP(t) > 0$ . In other words, the battery always draws energy from the grid, namely  $b(t) = -b_{max}$ , when the battery level is less than  $b_{max}$ . This implies that when the battery discharges, there is always enough energy in the battery, i.e.,  $B(t) > b_{max}$ . Therefore, the energy constraint of Eqn. (4) is indeed *redundant*.

#### D. Performance Analysis

In this subsection, we will prove that *HTSA* achieves a performance that is within a bounded distance of the optimum via the following theorem.

**Theorem 1:** By setting  $\theta = b_{max} + VP_{max}$  and  $B(0) = \theta$ , *HTSA* has the following property:

- 1) The battery level  $B(t)$  satisfies:

$$B(t) \leq \theta + b_{max} + h_{max}. \quad (23)$$

- 2) There exists  $M > 0$ , such that  $Q(t)$  is bounded by  $M$  for all  $t$ , where  $M$  is a constant.
- 3) The cost achieved by *HTSA* satisfies:

$$\begin{aligned} & \limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^* + t) - P(t)b^*(t) \right] \\ & \leq C^{opt} + P_{max}b_{max} + \frac{D + (b_{max} + h_{max})^2}{V}. \end{aligned}$$

*Proof:* 1). We will use mathematical induction to prove it. i). We have  $B(0) = \theta < \theta + b_{max} + h_{max}$ . ii). Assume that  $B(t) \leq \theta + b_{max} + h_{max}$ . iii). For time slot  $t+1$ , let us consider two subcases. First, if  $B(t) \leq \theta$ , we can see that the maximum increased energy in the battery during one time slot is  $h_{max} + b_{max}$ , which is under the case  $r(t) = h_{max}$  and  $b(t) = -b_{max}$ . Thus, we have  $B(t+1) \leq \theta + b_{max} + h_{max}$ . Second, if  $B(t) > \theta$ , from Eqn. (22), we can see that  $b(t) > 0$  and  $r(t) = 0$  when  $B(t) > \theta$ , that is, as long as the battery level is greater than  $\theta$ , it discharges and there is no energy replenishment. Therefore, it follows that  $B(t+1) \leq B(t) \leq \theta + h_{max} + b_{max}$ . Hence, we conclude that  $B(t+1) \leq \theta + h_{max} + b_{max}$ , which means that under *HTSA*, the battery level is always bounded. Therefore, the required battery size is finite.

2). Without loss of generality, we assume that  $U_i^t(1) \neq 0$  and denote  $U_{min} \triangleq \min_{t,i} U_i^t(1)$ . Note that  $Q(t)U_i^t(s_i^t)$  is an increasing function of  $s_i^t$  and  $U_i^t(0) = 0$ . Consider Eqn. (21), when  $s_i^{t*} = 0$ , we have the value of Eqn. (21) to be  $V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + t)$ . Thus, if we have  $Q(t)U_i^t(1) \geq V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + t)$ , that is, the cost when  $s_i^t = 1$  is higher than the cost when  $s_i^t = 0$ , it follows that  $s_i^{t*} = 0$ . This means that when  $Q(t) > \frac{Vc_{max}\pi_{max}P_{max}}{U_{min}}$ , we have the input of  $Q(t)$ , i.e.,  $U_i^t(s_i^t)$ , equals 0. Similarly to part 1), we can show that  $Q(t) \leq M \triangleq \frac{Vc_{max}\pi_{max}P_{max}}{U_{min}} + n_{max}U_{max}$ .

3). Recall that *HTSA* minimizes the RHS of Eqn. (19). However, the existence of constraint Eqn. (5) has prevented  $b(t)$  being selected in  $(0, b_{max})$ . Thus, the term  $(\theta - B(t) -$

$VP(t))\mathbb{E}[b(t)|Z(t)]$  is not maximized. We compare the stationary and randomized policy in Lemma 2 and *HTSA*. In particular, we have that

$$\begin{aligned} & D - \alpha Q(t) + (B(t) - \theta)\mathbb{E}[r^*(t)|Z(t)] + \\ & \sum_{i=1}^{n_t} \mathbb{E}[Q(t)U_i^t(s_i^{t*}) + V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^{t*} + t)|Z(t)] \\ & + (\theta - B(t) - VP(t))\mathbb{E}[b^*(t)|Z(t)] \\ & \leq D - \alpha Q(t) + (B(t) - \theta)\mathbb{E}[\tilde{r}(t)|Z(t)] + \\ & \sum_{i=1}^{n_t} \mathbb{E}[Q(t)U_i^t(\tilde{s}_i^t) + V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + \tilde{s}_i^t + t)|Z(t)] \\ & + (\theta - B(t) - VP(t))\mathbb{E}[\tilde{b}(t)|Z(t)] \\ & + b_{max}|\theta - B(t) - VP(t)|, \end{aligned} \quad (24)$$

where the last term is an upper bound on the term  $(\theta - B(t) - VP(t))\mathbb{E}[b^*(t)|Z(t)]$ , since we only need to consider the case  $\theta - B(t) - VP(t) < 0$

From the fact that  $B(t) < \theta + b_{max} + h_{max}$ , we have  $\theta \geq \theta - B(t) - VP(t) \geq -(b_{max} + h_{max} + VP_{max}) = -(\theta + h_{max})$ . It follows that  $|\theta - B(t) - VP(t)| \leq b_{max} + h_{max} + VP_{max}$ .

Rearranging the RHS of Eqn. (24), it yields:

$$\begin{aligned} & D + Q(t)\mathbb{E} \left[ \sum_{i=1}^{n_t} U_i^t(\tilde{s}_i^t) - \alpha |Z(t) \right] \\ & + (B(t) - \theta)\mathbb{E}[\tilde{r}(t) - \tilde{b}(t)|Z(t)] \\ & + V\mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} P(j + \tilde{s}_i^t + t) - P(t)\tilde{b}(t) |Z(t) \right] \\ & + b_{max}|\theta - B(t) - VP(t)| \\ & \leq D + V\tilde{C} + (b_{max} + h_{max} + VP_{max})b_{max}. \end{aligned} \quad (25)$$

where for the last inequality, we have used the following expressions:

$$\mathbb{E} \left[ \sum_{i=1}^{n_t} U_i^t(\tilde{s}_i^t) - \alpha |Z(t) \right] = \mathbb{E} \left[ \sum_{i=1}^{n_t} U_i^t(\tilde{s}_i^t) - \alpha \right] \leq 0 \quad (26)$$

$$\begin{aligned} & (B(t) - \theta)\mathbb{E}[\tilde{r}(t) - \tilde{b}(t)|Z(t)] = (B(t) - \theta)\mathbb{E}[\tilde{r}(t) - \tilde{b}(t)] \\ & \leq (B(t) - \theta)\mathbb{E}[\tilde{r}(t)] \leq (b_{max} + h_{max})h_{max}. \end{aligned} \quad (27)$$

Eqn. (26) is derived from Eqn. (16) because  $(\tilde{s}_i^t, \tilde{b}(t))$  is a stationary policy which is independent of  $Z(t)$ . Similarly, Eqn. (27) is from Eqn. (17) and  $B(t) < \theta + b_{max} + h_{max}$ .

Thus, combining Eqn. (24) and (25), we have

$$\begin{aligned} & \mathbb{E}[(L(t+1) - L(t)|Z(t))] \\ & + V\mathbb{E} \left[ \sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^{t*} + t) - P(t)b^*(t) |Z(t) \right] \\ & \leq D + (b_{max} + h_{max})^2 + V\tilde{C} + VP_{max}b_{max} \\ & \leq D + (b_{max} + h_{max})^2 + VC^{opt} + VP_{max}b_{max}, \end{aligned} \quad (28)$$

where the last inequality holds because  $\tilde{C}$  is a lower bound of  $C^{opt}$ .

By taking the expectation with respect to  $Z(t)$  on both sides of Eqn. (28) and take the summation from  $t = 0$  to  $T$ , it yields that

$$\begin{aligned} & \mathbb{E}[(L(T+1) - L(0))] \\ & + V \sum_{t=1}^T \mathbb{E}[\sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^{t*} + t) - P(t)b^*(t)] \\ & \leq TD + T(b_{max} + h_{max})^2 + VTC^{opt} + VTP_{max}b_{max}. \end{aligned} \quad (29)$$

If we set  $B(0) = \theta$ , we have  $L(0) = 0$ . Rearranging Eqn. (29) and dividing by  $VT$  on both sides, we have

$$\begin{aligned} & \frac{1}{T} \sum_{t=1}^T \mathbb{E}[\sum_{i=1}^{n_t} \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + s_i^{t*} + t) - P(t)b^*(t)] \\ & \leq C^{opt} + P_{max}b_{max} + \frac{D + (b_{max} + h_{max})^2}{V}. \end{aligned} \quad (30)$$

Taking the limsup as  $T \rightarrow \infty$  yields our result.  $\blacksquare$

From part (2) in Theorem 1, since  $Q(t)$  is bounded, combining with lemma 1, we can see that the average delay constraint, i.e., Eqn. (7), is satisfied.

Eqn. (30) shows that the cost induced by our algorithm is within a bounded distance of the optimum by setting the parameter  $V$  to be sufficiently large. It is worth pointing out that the algorithm does not require the future knowledge of the statistics of power demand and the task arrival process.

*Discussion:* Since our focus here is a family or a community, it is assumed here that the scheduling actions will not influence the electricity price. However, if the scheduling policy is adopted for a large scale of power grid, it will lead to an impact on the electricity price, which will form the basis of our future work.

## V. ENERGY SELLING

In this section we allow the system to sell energy back to the grid. Note that in our previous discussion,  $g(t) \geq 0$  always holds due to the constraint in Eqn. (5), i.e.,  $b(t) \leq w(t)$ . However, If we allow energy selling, this constraint is relaxed, which implies that  $g(t) < 0$  is possible. We present our corresponding algorithm as follows:

### Joint task scheduling and energy selling algorithm (JTSES):

- In each time slot  $t$ , the harvested energy  $r^*(t)$  is determined by

$$r^*(t) = \begin{cases} h(t), & \text{if } B(t) - \theta < 0, \\ 0, & \text{otherwise.} \end{cases} \quad (31)$$

- In each time slot  $t$ , the postponing time  $s_i^t$  for task  $i \in N_t$  is determined by:

$$s_i^{t*} = \arg \min_{0 \leq s_i^t \leq d_i^t - c_i^t} Q(t)U_i^t(s_i^t) + V \sum_{j=0}^{c_i^t-1} \pi_i^t P(j + t + s_i^t). \quad (32)$$

- In each time slot  $t$ , the battery charge/discharge is given by:

$$b^*(t) = \begin{cases} b_{max}, & \text{if } \theta - B(t) - VP(t) < 0, \\ -b_{max}, & \text{otherwise.} \end{cases} \quad (33)$$

Notice that the only difference between *JTSES* and *HTSA* is  $b(t)$ , where  $b(t)$  can be larger than  $w(t)$  in *JTSES*.

Surprisingly, we can show that under this algorithm, our scheme can actually achieve asymptotic optimality as shown by the following theorem.

**Theorem 2:** If the user is allowed to sell energy to the grid at the price of  $P(t)$ , by setting  $\theta = b_{max} + VP_{max}$  and  $B(0) = \theta$ , *JTSES* achieves a performance that could be arbitrarily close to the optimum as  $T$  tends to infinity.

*Proof:* Notice that when the constraint Eqn. (5) does not exist,  $b(t)$  thus can be selected in  $(0, b_{max})$ . Therefore, RHS of Eqn. (19) is maximized by *JTSES*. It can be seen that the extra term  $b_{max}|\theta - B(t) - VP(t)|$  in Eqn. (24) no longer exists. Following the same line of the proof of Theorem 1, it yields the conclusion.  $\blacksquare$

Theorem 2 implies that the gap  $P_{max}b_{max}$  diminishes if energy selling is allowed. Furthermore, it is worth pointing out that in *JTSES* the task scheduling and energy management have been decoupled due to the removed Eqn. (5).

In [10], we investigate an energy trading problem in the smart grid. The energy selling price is assumed to be  $\beta P(t)$ , where  $\beta$  is a constant between 0 and 1. Under the assumption that the energy demand process  $w(t)$  is an exogenous input process, an asymptotically optimal energy trading scheme is developed. However, in this work, due to the fact that  $w(t)$  is determined by the task scheduling decision, the asymptotic optimality is only achieved under the case when the energy selling price is equal to the energy buying price, i.e.,  $\beta = 1$ . Furthermore, we can show the performance of *JTSES* under the selling price  $\beta P(t)$  via the following theorem.

**Theorem 3:** If the energy selling price is  $\beta P(t)$ , by setting  $\theta = b_{max} + VP_{max}$  and  $B(0) = \theta$ , *JTSES* achieves an average cost that is within a bound of  $(1-\beta)P_{max}b_{max}$  of the optimum by setting  $V$  to be sufficiently large.

*Proof:* As discussed in Theorem 2, when  $\beta = 1$ , *JTSES* achieves the optimal value, which is denoted as  $J_1^*$ . Note that when  $\beta < 1$ , the optimum  $J_\beta^*$  will be higher since the benefit brought about by selling the energy to the grid is reduced, i.e.,  $J_\beta^* \geq J_1^*$ .

Note that *JTSES* is sub-optimal for the case of  $\beta < 1$ , therefore the cost  $J_\beta$  achieved by *JTSES* for the case  $\beta < 1$  is higher than  $J_\beta^*$ , i.e.,  $J_\beta \geq J_\beta^*$ . Also note that the gap between the two cases  $\beta = 1$  and  $\beta < 1$  is bounded by  $(1-\beta)P_{max}b_{max}$  under the same scheme *JTSES*, namely  $J_\beta \leq J_1^* + (1-\beta)P_{max}b_{max}$ . Thus, we have  $J_\beta^* \leq J_\beta \leq J_1^* + (1-\beta)P_{max}b_{max} \leq J_\beta^* + (1-\beta)P_{max}b_{max}$ , which proves our results.  $\blacksquare$

## VI. CASE STUDY

The remainder of the paper evaluates the algorithms presented in the previous section.

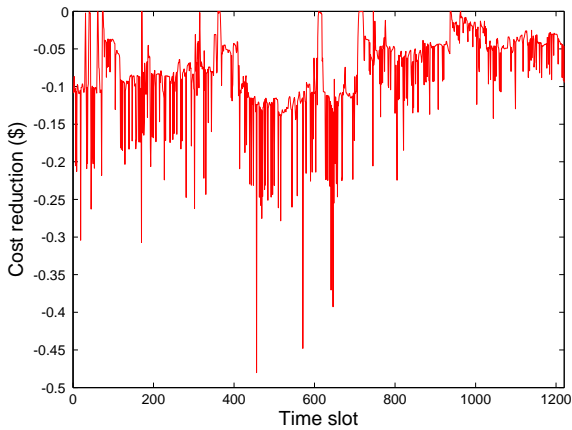


Fig. 5. Reduction in cost for Class I appliances

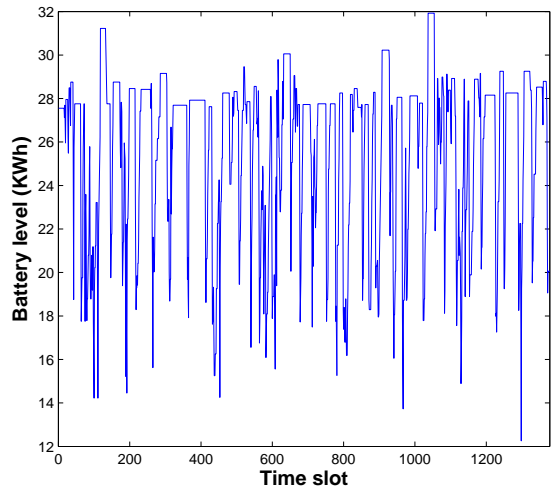


Fig. 7. Battery level in each time slot

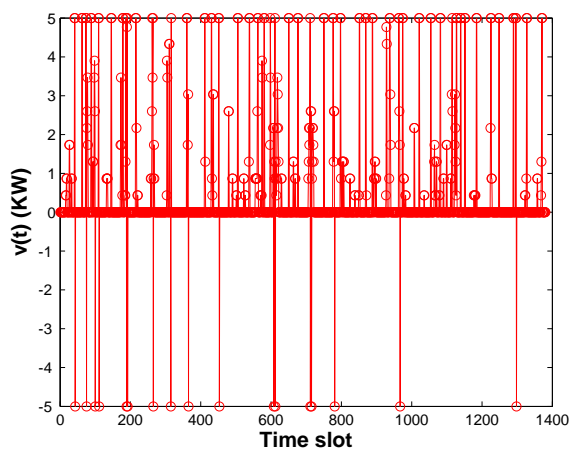


Fig. 6. Energy drawn from the battery in each time slot

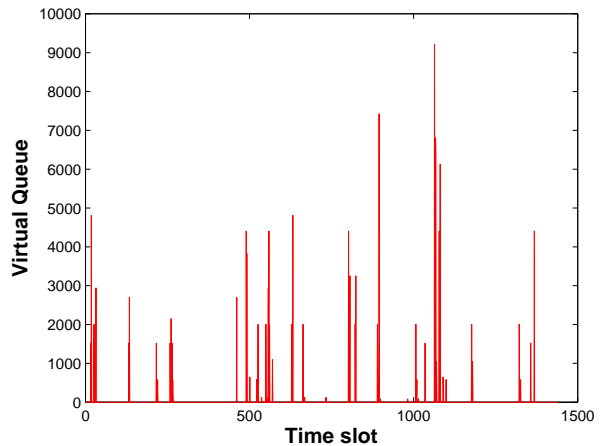


Fig. 8. Virtual queue length in each time slot

### A. Experiment Setup

We adopt the 5-minute average spot market prices for Columbus Area from CAISO [11]. The profile depicted in Fig. 3 shows the electricity price for the period 10/10/2011-10/14/2011. The arrival process of all tasks here are assumed to be Poisson process with different intensity  $\lambda_i$ , although Theorem 1 holds for any general arrival process. Without loss of generality, we consider four types of appliances in our simulations. The first three tasks are delay-tolerant, while the last one is delay-intolerant. The arrival intensities for these tasks are set to be 2, 0.5, 0.035 and 100, respectively. And the energy consumption rate for these tasks  $\pi_i^t$  are set to be 5.2kw, 3.5kw, 2.4kw and 60w. The “dissatisfaction” functions are assumed to be  $U(x) = x^2$ . The average delay constraint threshold  $\alpha$  is set to be 10000, and the parameter  $V$  is set to be 100.

### B. Performance Evaluation

We start by comparing our algorithm and a naive scheme, which activates the task immediately upon its arrival. Consider the first type of delay-tolerant task, which has a deadline of 100 slots, while the required service time is two slots. Fig. 5

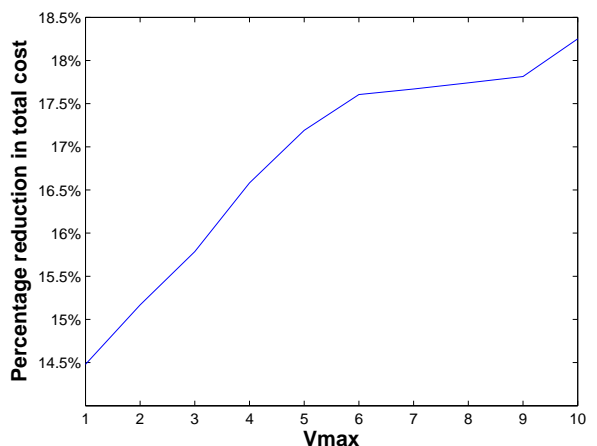


Fig. 9. Reduction in cost versus the battery size

shows the reduction in cost for scheduling this type of task using our algorithm. The total cost saved in these five days is



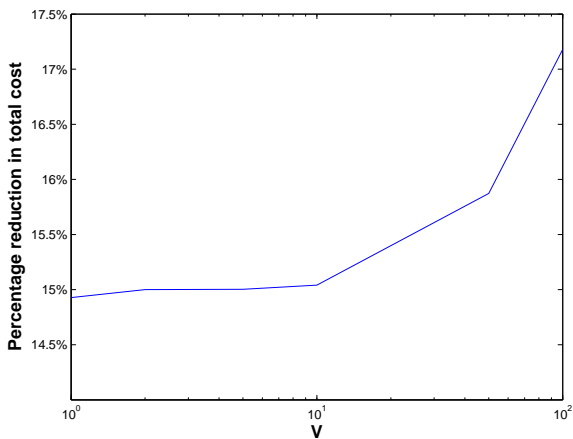


Fig. 10. Reduction in Cost versus parameter  $V$

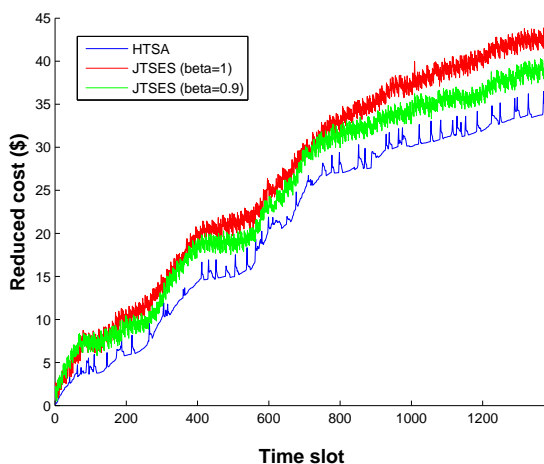


Fig. 11. Reduced Cost for both *HTSA* and *JTSES*

\$35.40, which is 19.82% of the total cost. If we extend the hard delay deadline to 200 slots, the corresponding percentage of saved cost increases to 27.20%. This is because if we have a less stringent delay constraint, we can gain more benefit.

Next, we will show how the battery influences the performance. The deadline for other two types of delay-tolerant tasks are set to be 10 and 20, respectively. We set the battery size to be  $2b_{max} + VP_{max} + h_{max}$ . Fig. 6 shows the energy drawn from the battery, i.e.,  $b(t)$ , versus time in the whole period. Fig. 7 and Fig. 8 illustrate the energy level  $B(t)$  and virtual queue length  $Q(t)$ , respectively, both of which are bounded. These results conform to our analytical result in the previous section.

Fig. 9 depicts the percentage reduction in cost versus  $b_{max}$ . We can see that the percentage reduction in cost increases as  $b_{max}$  grows. This is because a large battery maximum output can lead to a higher shaved cost which can be seen from Eqn. (22).

In Fig. 10, we illustrate the relationship between the percentage of reduced cost and the parameter  $V$ . It can be seen that when  $V$  is small, the reduced cost is less than the counterpart

when  $V$  is large. The reason is that the term  $\frac{D+(b_{max}+h_{max})^2}{V}$  in Eqn. (30) cannot be neglected when  $V$  is small.

Fig. 11 shows the cost reduction under both *HTSA* and *JTSES* with different selling price. The parameter  $\beta$  in *JTSES* are assumed to be 1 and 0.9, respectively. We can see that when  $\beta = 1$ , i.e., the selling price is equal to the buying price, *JTSES* always outperforms *HTSA*. The reason is that selling energy is allowed in *JTSES*, which leads to further cost reduction. On the other hand, if  $\beta$  becomes smaller, the reduced cost also decreases. This observation is consistent with our theoretical result.

## VII. CONCLUSION

In this paper, we investigate the cost minimization problem for an end-user, which is equipped with renewable energy devices when electrical appliances allow different levels of delay tolerance. The varying price of electricity implies an opportunity to reduce the electricity cost by utilizing the flexibility to schedule various appliances. We assume that the end user has an energy storage battery and an energy harvesting device so that harvested renewable energy can be stored and used when the price is high. The problem we formulate here is to minimize the cost of the energy from the external grid while usage of appliances are subject to individual delay constraints and a long-term average delay constraint. Our proposed algorithm, *HTSA*, requires some future information of the electricity price, but achieves provable performance without requiring future knowledge of either the power demands or the task arrival process. Further, when energy can be sold from the battery to the grid, we develop an alternate algorithm *JTSES*. The performance gap between *JTSES* and the optimum is shown to diminish as energy selling price approaches the electricity price.

## APPENDIX

*Proof:* First, By squaring Eqn. (13) and noting that  $\max[x, 0]^2 \leq x^2$ , we have

$$\begin{aligned} & \frac{1}{2}Q(t+1)^2 - \frac{1}{2}Q(t)^2 \\ & \leq \frac{1}{2}\left(\sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha\right)^2 + Q(t)\left(\sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha\right) \\ & \leq \frac{1}{2}n_{max}^2 U_{max}^2 + \frac{1}{2}\alpha^2 + Q(t)\left(\sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha\right). \end{aligned}$$

Similarly, by Eqn. (2), we have

$$\begin{aligned} & \frac{1}{2}(B(t+1) - \theta)^2 - \frac{1}{2}(B(t) - \theta)^2 \\ & \leq \frac{1}{2}r(t)^2 + \frac{1}{2}b(t)^2 + (B(t) - \theta)(r(t) - b(t)) \\ & \leq \frac{1}{2}r_{max}^2 + \frac{1}{2}b_{max}^2 + (B(t) - \theta)(r(t) - b(t)). \end{aligned}$$

Thus, we have

$$\begin{aligned} L(t+1) - L(t) &\leq \frac{1}{2}n_{max}^2 U_{max}^2 + \frac{1}{2}\alpha^2 + Q(t)\left(\sum_{i=1}^{n_t} U_i^t(s_i^t) - \alpha\right) \\ &\quad + \frac{1}{2}r_{max}^2 + \frac{1}{2}b_{max}^2 + (B(t) - \theta)(r(t) - b(t)). \end{aligned}$$

Taking expectations on both sides conditioning on  $Z(t)$ , it yields the result. ■

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