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Finite-Horizon Energy Allocation and Routing Scheme in Rechargeable Sensor Networks

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***Abstract*—In this paper, we investigate the problem of maximizing the throughput over a finite-horizon time period for a sensor network with energy replenishment. The finite-horizon problem is important and challenging because it necessitates optimizing metrics over the short term rather than metrics that are averaged over a long period of time. Unlike the infinite-horizon problem, the fact that inefficiencies cannot be made to vanish to infinitesimally small values, means that the finite-horizon problem requires more delicate control. The finite-horizon throughput optimization problem can be formulated as a convex optimization problem, but turns out to be highly complex. The complexity is brought about by the "time coupling property," which implies that current decisions can influence future performance. To address this problem, we employ a three-step approach. First, we focus on the throughput maximization problem for a single node with renewable energy assuming that the replenishment rate profile for the entire finite-horizon period is known in advance. An energy allocation scheme that is equivalent to computing a shortest path in a simply-connected space is developed and proven to be optimal. We then relax the assumption that the future replenishment profile is known and develop an online algorithm. The online algorithm guarantees a fraction of the optimal throughput. Motivated by these results, we propose a low-complexity heuristic distributed scheme, called NetOnline, in a rechargeable sensor network. We prove that this heuristic scheme is optimal under homogeneous replenishment profiles. Further, in more general settings, we show via simulations that NetOnline significantly outperforms a state-of-the-art infinite-horizon based scheme, and for certain configurations using data collected from a testbed sensor network, it achieves empirical performance close to optimal.**

I. Introduction

Many new applications of wireless sensor networks have been identified in recent years for monitoring phenomena such as earthquakes and fires. But limited lifetimes of batteries have hampered the deployment of such networks. Recent developments in technologies for harvesting various forms of renewable energy, such as solar and vibration [1], [2], [3], have the potential to alleviate this problem by allowing sensor nodes to replenish their batteries, leading to significant improvement in network lifetime and connectivity. Due to the limited energy typically available from such sources, energy management is critical for deploying real systems based on renewable energy.

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Joint energy allocation and routing problems have been extensively investigated in wireless networks. A low-complexity and distributed algorithm is developed in [4], for which the total energy consumption is at most twice as large as the energy consumption of the optimal solution. The authors in [5] propose two cross-layer algorithms, a primal-based and a dual-based algorithm. Both schemes allow each node to adjust the transmission attempt probability to maximize the utility function. However, in these works, energy replenishment is not considered. There have been some recent works that have studied rechargeable sensor networks. For example, a dynamic programming approach is studied in [6], for a satellite with energy replenishment. Routing schemes in sensor networks with renewable energy are explored in [7]. The authors propose two heuristic solutions. The first allows the source node and one intermediate node to be different from the shortest path and opportunistically forward packets to a neighboring node with energy replenishment. The second scheme chooses the shortest path with the minimum number of nodes that run solely on their batteries. In [8], a discharge/recharge model is adopted, and a simple threshold activation policy is proven to achieve a performance of at least 3/4 of the optimal performance, where the metric of performance is the utility associated with the sensing behavior. An adaptive energy allocation scheme is developed for a single node with rechargeable energy in [10], and the scheme is proved to be order optimal with respect to the battery size. The authors in [11] explore a power-aware routing policy that computes a path with the least cost to accommodate each task in the network, the throughput of which is proven to achieve asymptotically optimal competitive ratio as the number of nodes in the network grows to infinity. A policy in [12] with decoupled admission control and energy allocation is proposed to maximize a function of the long-term rate achieved per link. The scheme is proven to be asymptotically optimal when all nodes have sufficiently large battery capacities.

In this paper, we are interested in the finite-horizon throughput maximization problem for a rechargeable sensor network. The finite-horizon problem is important and challenging because it necessitates optimizing performance metrics that are exhibited in the short term rather than metrics that are averaged over a long period of time. The difference between the finite-horizon problem here and the infinite-horizon problem in

[12] is that, unlike the infinite-horizon problem, inefficiencies cannot be made to vanish to infinitesimally small values. This implies that new control techniques need to be developed to mitigate these inefficiencies. In fact, as we show in the paper, infinite-horizon based solutions could be highly inefficient, especially in the context of networks with energy replenishment. The reason is that the replenishment profiles are time varying and may not even be stationary and ergodic. In this paper, instead of using a linear function as in [9], [11], we assume a strictly concave rate-power function $\mu(P)$, where $\mu(P)$ represents the amount of data that can be transmitted by using P units of energy in a slot under a given physical layer modulation and coding strategy. This variation is closer to reality because it appropriately reflects the law of diminishing returns with increases in power, but it results in a significantly more complex problem in designing the energy allocation scheme. Note that for a linear rate-power function, optimality can be preserved as long as no energy is wasted. However, this does not hold when a strictly concave rate-power function is adopted, because the concavity property also requires that the energy be spent "smoothly." For instance, if energy is overused in a previous period, the total throughput will decrease as a result. On the other hand, if energy is underused in a previous period, the total throughput will also decrease, even though there is no wasted energy. This is the so-called "time coupling property" brought about by the strictly concave rate-power function, which implies that current decisions can dramatically influence future performance. To address this problem, we employ a three-step approach:

- We first focus on the finite-horizon throughput optimization problem for a single node with energy replenishment assuming that the replenishment rate profile for the entire finite-horizon period is known in advance. A scheme that is equivalent to computing a shortest path in a simply-connected space, is developed and proven to be optimal.
- We then relax the assumption that the future replenishment profile is known. In this case, an online algorithm for a single node is developed for estimation with deviation on the future replenishment rate. The performance of the online algorithm is proven to guarantee a fraction of the optimal performance.
- Finally, we propose a distributed heuristic scheme, called NetOnline, in which each node follows the energy allocation scheme computed using the single node formulation.. We prove that the heuristic scheme is optimal under homogeneous replenishment profiles and via simulations demonstrate the efficacy of NetOnline in a general setting.

II. System Model

We consider a static sensor network, denoted by $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ is the set of links. We assume a time-slotted system with a finite-time operation of T time slots. Let M_n denote the battery size at node n. Let $r_n(t)$ denote the amount of replenishment energy that arrives at node n in slot t, while $e_n(t)$ denotes the allocated energy in slot t. For simplicity of exposition, we assume that the harvested energy arrives at the beginning of each slot and is immediately stored in the battery. We also assume that the initial battery is empty. At all times, the stored energy is never allowed to exceed M_n.

Let $R_n(t)$ represent the cumulative harvested energy of node n from slot 1 to slot t, i.e.,

$$R_n(t) = \sum_{i=1}^{t} r_n(i), \quad \text{for all } t \in (1, \cdots, T). \tag{1}$$

For each t, $R_n(t)$ can be viewed as representing a point on a graph with time as the x-axis and the cumulated harvested energy at node n as the y-axis. We connect all the neighboring points $R_n(t)$ and $R_n(t+1)$, for all $t \in (1, \cdots, T-1)$ with line segments. It immediately follows that $R_n(t)$ is a continuous, nondecreasing function of t that passes through points (0,0) and (T, K_n), where $K_n = R_n(T)$.

Similarly, we define $E_n(t)$ as the cumulative energy consumption from time slot 1 to slot t, i.e.,

$$E_n(t) = \sum_{i=1}^{t} e_n(i), \quad \text{for all } t \in (1, \cdots, T). \tag{2}$$

We assume that $R_n(0) = E_n(0) = 0$.

We define $\vec{e}_n = \big(e_n(1), e_n(2), \cdots, e_n(T)\big)$ and $\vec{E}_n = \big(E_n(1), E_n(2), \cdots, E_n(T)\big)$. Note that $\vec{E}_n$ and $\vec{e}_n$ are related by a 1-1 mapping because $e_n(t) = E_n(t) - E_n(t-1)$, for all $t \in (1, \cdots, T)$. Henceforth, we will interchangeably call both $\vec{E}_n$ and $\vec{e}_n$, the energy allocation scheme for node n.

III. Finite-horizon Optimal Energy Allocation Scheme for a Single Node

In this section, we investigate the finite-horizon throughput maximization problem for a single node assuming that the replenishment rate profile for the entire finite-horizon period is known in advance. *Note that, in this section and the next, the subscript n is omitted from all the notations* defined in the previous section, since the results in these sections are based on a single node.

A. Problem Formulation

During a time slot, the throughput of the node is characterized by a nondecreasing and strictly concave rate-power function $\mu(P)$, satisfying $\mu(0) = 0$. Recall that $\mu(P)$ represents the amount of data that can be transmitted using P units of energy in a slot under a given physical layer modulation and coding strategy.

We are interested in finding an energy allocation policy $\vec{e} = \big(e(1), e(2), \cdots, e(T)\big)$ that maximizes the throughput during T time slots. Since the cumulative used energy cannot be greater than the cumulative harvested energy for any slot t, a natural constraint is given as follows:

$$E(t) \leq R(t), \quad \text{for all } t = 1, \ldots, T. \tag{3}$$

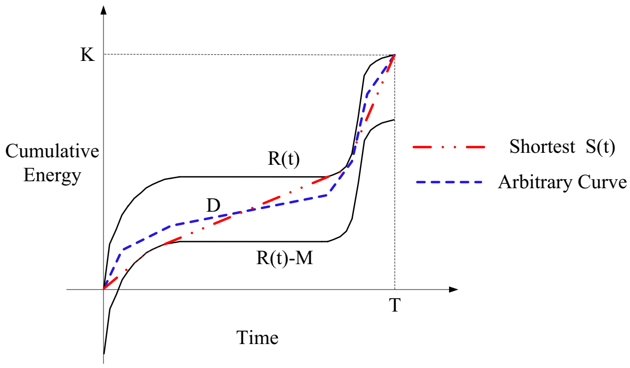

Fig. 1. The feasible domain D and the shortest curve $S(t)$

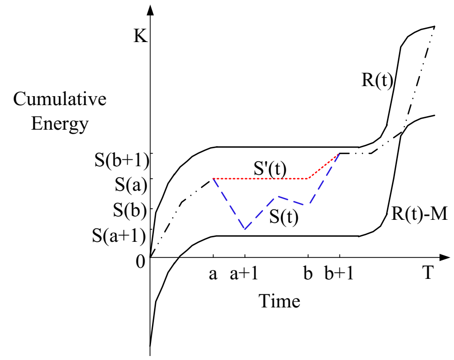

Fig. 2. Illustration of $\vec{S}$ and $\vec{S}'$ in Lemma 1

Since computing $\vec{E}$ is equivalent to computing $\vec{e}$, the problem is formulated as follows:

$$\text{Problem } \mathbf{A}: \quad \max_{\vec{e}} \sum_{t=1}^{T} \mu(e(t))$$
$$\text{subject to} \quad E(t) \le R(t), \quad \text{for all } t \in (1, \cdots, T). \quad (4)$$

Note that an optimal solution $\vec{E}^*$ must satisfy that at all time slots, the residual energy, $R(t) - E^*(t)$, is no greater than the battery capacity M. Otherwise, some energy will be lost due to the finite battery size, and we can easily find another energy allocation that achieves a greater throughput than $\vec{E}^*$, contradicting the optimality of $\vec{E}^*$. Hence, together with Eqn. (3), we obtain

$$R(t) - M \le E(t) \le R(t), \quad \text{for all } t \in (1, \cdots, T). \quad (5)$$

Therefore, Problem **A** can be reformulated as:

$$\text{Problem } \mathbf{B}: \quad \max_{\vec{e}} \sum_{t=1}^{T} \mu(e(t))$$
$$\text{subject to} \quad R(t) - M \le E(t) \le R(t), \quad \text{for all } t. \quad (6)$$

B. Definitions and Preliminary Results

Let domain D denote all possible values that $\vec{E}$ can take such that Eqn. (5) is satisfied, in other words, the area that is surrounded by the curves $R(t)$, $R(t) - M$, the two vertical lines crossing node (0,0) and (T, K), as shown in Fig. 1. Note that D is a simply-connected space.

***Definition 1** (Feasible curve):* Any nondecreasing curve, defined on integer-valued t and located in the domain D, is said to be a *feasible curve*. From Eqn. (5), it can be seen that there is a 1-1 mapping between any feasible curve in D and an energy allocation scheme $\vec{E}$. For example, in Fig. 1, the dashed curve and the dot-and-dash curve represent two different energy allocation schemes. Furthermore, we consider two feasible curves f and g to be identical, if they have the same value at every integer point, i.e., $f(t) = g(t)$ for all $t = 1, \ldots, T$. Also the length of a curve $f(t)$ in an interval $t \in [a, b]$ is defined as the sum of Euclidean lengths of $\{(x, f(x)), (x+1, f(x+1))\}$ in the interval, i.e., $\sum_{x=a}^{b-1} \sqrt{1 + (f(x+1) - f(x))^2}$.

***Definition 2** (Shortest path):* A curve that connects two points $(0, 0)$ and (T, K) in the domain D is said to be the *shortest path*, if its Euclidean length is the smallest among all feasible curves.

We denote the shortest path by $\vec{S}$. In Fig. 1, it is depicted by the dot-and-dash curve. In the following Lemma, we show the existence and feasibility of the shortest path.

***Lemma 1**:* The shortest path $\vec{S}$ exists in domain D, and is feasible.

Proof: Since D is a simply-connected space, for the two points $(0, 0)$, (T, K) in it, it follows from [16] that there exists a unique shortest path $\vec{S}$ connecting them inside D.

Next, we will show that $\vec{S}$ is a feasible curve, which is equivalent to proving that it is nondecreasing, based on Definition 1. Without loss of generality, we assume that $T \ge 2$. Assume that there exists an interval $[a, a+1]$, for some $a \in (1, \cdots, T-1)$, where $S(t)$ is decreasing, i.e., $S(a) > S(a+1)$. Since $S(a+1) < S(a) \le S(T)$, we know that $a + 1 \ne T$. Thus, there exists an integer b in the interval $[a+1, T]$, such that $S(b+1) \ge S(a) > S(b)$. As illustrated in Fig. 2, we define a new curve $\vec{S}'$, such that

$$S'(t) = \begin{cases} S(a), & t \in [a, b], \\ S(t), & \text{otherwise}, \end{cases}$$

First, we will show that $\vec{S}'$ is located in the domain D. Since $\vec{S}'$ and $\vec{S}$ are different only in the interval $[a, b]$, it suffices to show that

$$R(t) - M \le S'(t) \le R(t), \quad \text{for all } t \in [a, b]. \quad (7)$$

Since $R(t)$ is nondecreasing, for any t in the interval $[a, b]$, we have that

$$R(t) - M \le R(b) - M \le S(b) < S(a) \le R(a) \le R(t).$$

Hence, together with the fact that $S'(t) = S(a)$ for all t in the interval $[a, b]$, Eqn. (7) is proved.

Next, we will show that the length of $S'(t)$ is shorter than the length of $S(t)$. The straight line segment between $S'(a)$ and $S'(b)$ is shorter than the curve between $S(a)$ and $S(b)$. Furthermore, the line segment between $S'(b)$ and $S'(b+1)$ is shorter than the line segment between $S(b)$ and $S(b+1)$,

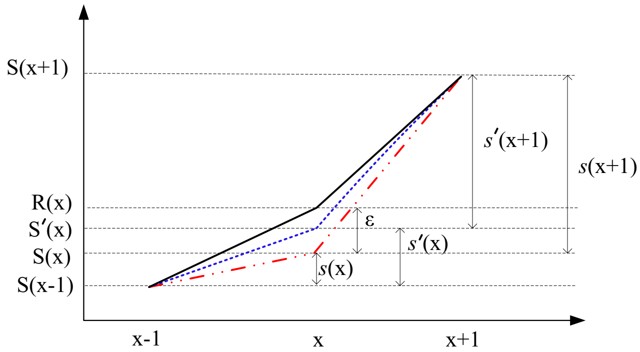

Fig. 3. Illustration of area around point x in Lemma 2

because $S'(b+1) - S'(b) < S(b+1) - S(b)$. Hence, $S'(t)$ is shorter than $S(t)$, which contradicts the fact that $S(t)$ is the shortest path. Therefore, it follows that $S(t)$ is nondecreasing, and is feasible by Definition 1. □

C. The Optimality of the Shortest-Path Solution

Let $s(t) = S(t) - S(t-1)$. We know that $\vec{s} = \big(s(1), s(2), \cdots, s(T)\big)$ is a feasible energy allocation scheme by Lemma 1.

We first prove a property of the shortest path $S(t)$.

Lemma 2: The shortest path $S(t)$ is concave at any point t in the set $\{t : S(t) < R(t)\}$, and is convex at any point t in the set $\{t : S(t) > R(t) - M\}$, except for the boundary points $t = 0$ and $t = T$.

Proof: Assume that $S(t)$ is not concave at a non-boundary point x, which is in the set $\{t : S(t) < R(t)\}$, i.e., $S(x) < R(x)$. Note that no concavity implies strict convexity, since $S(t)$ only takes value on integer-valued t. Hence, we have $S(x) < (S(x-1) + S(x+1))/2$.

We define a new curve $S'(t)$, such that

$$S'(t) = \begin{cases} S(x) + \epsilon/2, & t = x, \\ S(t), & \text{otherwise,} \end{cases}$$

where $\epsilon \triangleq \min\{R(x), (S(x-1) + S(x+1))/2\} - S(x) > 0$. Note that $S'(t)$ is within the domain D from $R(x) - M < S'(x) < R(x)$, and is feasible, since $S'(t)$ is nondecreasing. Also it follows that

$$s'(x) \triangleq S'(x) - S'(x-1) = s(x) + \epsilon/2, \tag{8}$$

$$s'(x+1) \triangleq S'(x+1) - S'(x) = s(x+1) - \epsilon/2. \tag{9}$$

Then, we can obtain that

$$\begin{aligned} & s'(x) - s'(x+1) = s(x) - s(x+1) + \epsilon \\ & = \min\{R(x), (S(x-1) + S(x+1))/2\} \\ & \quad + S(x) - S(x-1) - S(x+1) \\ & < S(x) - (S(x-1) + S(x+1))/2 < 0, \end{aligned} \tag{10}$$

where the last inequality comes from the convexity of $S(t)$ at x. Hence, combing with Eqn. (8) and Eqn. (9), we have

$$s(x) < s'(x) < s'(x+1) < s(x+1), \tag{11}$$

and $s(t) = s'(t)$ for all $t \neq x, x+1$.

Next, we will show that $S'(t)$ is shorter than $S(t)$. Since $S'(t)$ consists of T line segments, the Euclidean length of $S'(t)$ can be represented by $\sum_{t=1}^{T} L(s'(t))$, where $L(x) = \sqrt{1+x^2}$. Similarly, the length of $S(t)$ is $\sum_{t=1}^{T} L(s(t))$. Note that from the mean value theorem, there exists $c_x \in (s(x), s'(x))$ such that $dL(c_x)\frac{\epsilon}{2} = L(s'(x)) - L(s(x))$, and also $c_{x+1} \in (s'(x+1), s(x+1))$ such that $dL(c_{x+1})\frac{\epsilon}{2} = L(s(x+1)) - L(s'(x+1))$, where $dL(x)$ is the derivative of $L(x)$. Note that $L(\cdot)$ is a differentiable convex function whose derivative is increasing and $c_{x+1} > c_x$, we can obtain that

$$\begin{aligned} & \sum_{t=1}^{T} L(s(t)) - \sum_{t=1}^{T} L(s'(t)) \\ & = \{L(s(x+1)) - L(s'(x+1))\} - \{L(s'(x)) - L(s(x))\} \\ & = \{dL(c_{x+1}) - dL(c_x)\}\epsilon/2 > 0, \end{aligned} \tag{12}$$

which implies that $S'(t)$ is shorter than $S(t)$. This contradicts the fact that $S(t)$ is the shortest path. Therefore, we can conclude that $S(t)$ is concave at any point t in the set $\{t : S(t) < R(t)\}$.

Similarly, it can be shown that $S(t)$ is convex at any point t in the set $\{t : S(t) > R(t) - M\}$. □

Now, we claim the optimality of the energy allocation scheme $\vec{s}$ via the following theorem:

Theorem 1: The energy allocation scheme $\vec{s}$, each element of which satisfies $s(t) = S(t) - S(t-1)$, maximizes the throughput of a single node with rechargeable energy, resulting in an optimal solution to Problem **B**.

Proof: Please refer to Appendix A for the proof.

D. Discussion

In this subsection, we claim several observations related to the shortest-path solution.

1) Although Theorem 1 is proven for a discrete-time system, the shortest-path solution also holds for a continuous-time system. Under the continuous time setting, Eqn. (1) becomes $R(t) = \int_0^t r(s)ds$. As a result, the edge of domain D, $R(t)$ becomes a smooth continuous curve rather than line segments connecting neighboring points of $R(t)$.
2) When the initial battery in the buffer is not empty, say M', Eqn. (5) will become $R(t) - M + M' \leq E(t) \leq R(t) + M'$. This leads to a new feasible domain D'. The shortest path connecting $(0, 0)$ and $(T, K + M')$ in D' will be the optimal solution.
3) We can see how the battery size M influences the optimal energy allocation solution. If M is large enough, we can see that $R(t) - M$ will always be less than 0. As a result, the domain D only has an upper bound $R(t)$. On the other hand, if M is very small, in particular, when $M = 0$, then $R(t)$ and $R(t) - M$ will coincide to become one curve, which is also the only feasible curve. The corresponding energy allocation scheme is then to spend all the energy harvested in the current time slot. This corresponds to the correct intuition that

if the energy buffer size is zero, the best scheme is to spend all the harvested energy, since no energy can be stored.

4) Note that Theorem 1 holds for any nondecreasing and concave function $\mu(P)$. We can also incorporate the energy cost for sensing data. Let $\varphi(P)$ represent the amount of data generated using P units of energy for sensing. Typically, $\varphi(P)$ is assumed to be linear, i.e., $\varphi(P) = \gamma P$, where γ is a constant scaler. We can prove that the amount of data generated by sensing and then transmitting is also a concave function of P. Therefore, the shortest-path scheme is still the solution to the problem of maximizing the amount of data sensed and then transmitted in the period $[0, T]$. Please refer to our technical report [17] for details.

IV. An Online Algorithm with Estimation

As claimed in Theorem 1, the shortest-path solution maximizes the throughput of a single node. However, this solution is not causal because it requires the knowledge of the entire replenishment profile ahead of time. Replenishment rates can often be estimated, but there could be deviations. Under such case, we propose an online algorithm, which guarantees a fraction of the optimal performance when the deviations are bounded.

Assume that we can estimate the replenishment profile for the period $[0, T]$ ahead of time, and let $\hat{r}(t)$ and $r(t)$ represent the estimated replenishment rate and the actual replenishment rate at time slot t. Assume that $r(t)$ is lower bounded by $\underline{r}(t)$ and upper bounded by $\overline{r}(t)$, where $\underline{r}(t)$ and $\overline{r}(t)$ satisfy

$$\begin{aligned}\underline{r}(t) &= (1-\beta_1)\hat{r}(t)\\ \overline{r}(t) &= (1+\beta_2)\hat{r}(t),\end{aligned} \tag{13}$$

where, β_1 and β_2 are two constants that describe the inaccuracy of estimation. Based on the estimated replenishment profile, we can calculate the optimal energy allocation at time slot t, which is denoted as $\hat{e}(t)$. Let $\vec{\underline{e}} = (\underline{e}(1), \cdots, \underline{e}(T))$ represent the optimal energy allocation corresponding to the lower-bounded replenishment profile $\vec{\underline{r}} = (\underline{r}(1), \cdots, \underline{r}(T))$.

Now we describe the online algorithm as follows:

- *Step 1*: Calculate $\underline{e}(t)$ from the lower-bound of the estimated replenishment profile $\vec{\underline{r}}$ via the shortest-path solution.
- *Step 2*: The allocated energy is determined as follows: $e(t) = \underline{e}(t) + r(t) - \underline{r}(t)$.

Note that $\underline{r}(t)$ for each t is known from the estimation, and $\underline{e}(t)$ is also based on the estimation. Hence, the current power allocation $e(t)$ depends on the estimations and the current recharging energy $r(t)$, and thus the algorithm is causal.

We first show that the proposed online algorithm always provides an achievable resource allocation satisfying Eqn. (3).

Lemma 3*:* The energy allocation $\vec{E}$ of the online algorithm satisfies that $E(t) \le R(t)$ for all t.

Proof: Note that $e(t) \ge 0$, since $r(t) \ge \underline{r}(t)$ and $\underline{e}(t) \ge 0$. Furthermore, $\sum_{i=1}^{t} e(i) = \sum_{i=1}^{t} r(i) + \sum_{i=1}^{t}[\underline{e}(i) - \underline{r}(i)] \le \sum_{i=1}^{t} r(i)$ for all t, since $\sum_{i=1}^{t} \underline{e}(i) \le \sum_{i=1}^{t} \underline{r}(i)$. □

The following proposition shows that the online algorithm can guarantee a fraction of the optimal performance.

Proposition 2*:* The total throughput $U \triangleq \sum_{t=1}^{T} \mu(e(t))$ is lower bounded by $\frac{(1-\beta_1)}{(1+\beta_2)} \sum_{t=1}^{T} \mu(e^*(t))$, where $\vec{e}^* = \big(e^*(1), e^*(2), \cdots, e^*(T)\big)$ is the optimal energy allocation scheme.

Proof: Please refer to Appendix B for the proof.

V. Optimal Joint Energy Allocation and Routing Scheme in Rechargeable Sensor Networks

Consider a sensor network with replenishment nodes, modeled by a graph $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ is the set of links. There are some flows in the network and each flow s is associated with a source node f_s and a destination node d_s. Assume that $\mathcal{S}$ is the set of source nodes and we call all the other nodes relaying nodes, although source nodes can relay data as well. Let $x^s(t)$ be the amount of data that is sent from the source node $f_s \in \mathcal{S}$ to the destination node d_s in slot t over possibly multiple paths and multiple hops.

Our goal is to maximize the total throughput of the network over some finite-horizon period $[0, T]$. To simplify the analysis, we make the same assumption as in [11] that the reduction of energy is instantaneous for all the nodes along the path, since the rate of energy replenishment is usually much slower than that of energy consumption. Energy cost of sensing data at the source nodes and receiving data at the relaying nodes can be also incorporated into our model. According to the discussion in Section III.D, if we assume that sensing and receiving data have the same linear energy cost function, for either the source nodes or the relaying nodes, the amount of data that can be supported ("supported" means sensed and then transmitted for source nodes, or received and then transmitted for relaying nodes) using P units of energy is still a concave function of P, denoted as $\phi(P)$. Since the focus of this paper is energy allocation and routing for a finite-horizon period, we do not explicitly consider interference here. There are many excellent works on scheduling for interference, which can be incorporated into our solution. But this is beyond the scope of this paper and will form the basis of our future work.

We formulate the problem as follows:

$$\text{Problem } \mathbf{C}: \qquad \max \quad \sum_{s} \sum_{t=1}^{T} x^s(t).$$

The solution to the above problem will provide an answer to the following questions.

- How much energy $e_n(t)$ should be spent for each node $n \in \mathcal{N}$ in time slot t.
- For each node i, how to choose $w_{ij}^d(t)$, where $w_{ij}^d(t)$ represents the amount of data in the outgoing links $(i, j) \in \mathcal{L}$ for destination node d in time slot t.

Before we provide a solution to Problem **C**, we first introduce the definition of *rate region* as follows:

Definition 3 *(Rate region):* The rate region Λ_n, for node $n \in \mathcal{N}$, is defined as the set of all available vectors $\vec{v}_n = (v_n(1), v_n(2), \cdots, v_n(T))$, such that for any $\vec{v}_n \in \Lambda_n$, there exists some energy allocation scheme $\vec{e}_n$ that can achieve $\vec{v}_n$, i.e., $v_n(t) = \phi(e_n(t))$, for all $t \in (1, \cdots T)$.

We will show that Λ_n is convex for each node $n \in \mathcal{N}$ by the following lemma.

Lemma 4*:* The rate region Λ_n for each node $n \in \mathcal{N}$ is convex.

Proof: Please refer to our technical report [17] for the proof.

Note that each node should balance the incoming and outgoing data in each slot t. Thus, Problem **C** can then be reformulated as follows:

$$\text{Problem } \mathbf{D}: \quad \max \sum_s \sum_{t=1}^{T} x^s(t)$$

$$\text{subject to} \quad w_{ij}^d(t) \geq 0, \ \text{ for all } t, \text{ for all } d, \text{ for all } (i,j) \in \mathcal{L},$$
$$\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{s:f_s=i,d_s=d} x^s(t) \geq 0,$$
$$\text{for all } t, \text{ for all } d, \text{ and for all } i \neq d,$$
$$\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i, \ \text{ for all node } i \in \mathcal{N}, \tag{14}$$

where $\vec{w}_{ij} = (\sum_d w_{ij}^d(1), \sum_d w_{ij}^d(2), \cdots, \sum_d w_{ij}^d(T))$.

A. Solution to the Optimization Problem

It is worthwhile to note that, since Problem **D** is a convex optimization problem, we can use duality to get its optimal solution [14]. The procedure is then summarized as follows:

- The generating data rate of the source nodes are determined by

$$\max_{0 \leq x_s \leq \phi(R_{f_s}(t))} \sum_{t=1}^{T} x^s(t)\big(1 - q_{f_s}^{d_s}(t)\big), \tag{15}$$

  where $q_{f_s}^{d_s}(t)$ is the associated Lagrange multiplier for each constraint in Eqn. (14).
- The data rate at each link is determined by

$$\max_{0 \leq w_{ij}^d(t) \leq \phi(R_i(t))} \sum_{(i,j)\in\mathcal{L}} \sum_{d\neq i} \sum_{t=1}^{T} w_{ij}^d(t)(q_i^d(t) - q_j^d(t))$$
$$\text{subject to} \quad \sum_{j:(i,j)\in\mathcal{L}} \vec{w}^{ij} \in \Lambda_i, \ \text{ for all node } i \in \mathcal{N}. \tag{16}$$

- The Lagrange multipliers are updated by

$$q_i^d(t, \tau+1) = \big[q_i^d(t,\tau) - h\big(\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t,\tau) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t,\tau) - \sum_{f_s=i,d_s=d} x^s(t,\tau)\big)\big]^+, \tag{17}$$

  where h is the step size.

After solving $w_{ij}^d(t)$, the energy allocation for node i in slot t is determined by

$$e_n(t) = \phi^{-1}\Big(\sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t)\Big). \tag{18}$$

B. Discussion

From Eqn. (15) and (16), we observe that the solution to Problem **D** can be decomposed into a series of sub-optimization problems for each node $n \in \mathcal{N}$. However, these operations are highly complex, thus motivating the need for simple solutions.

We notice that the last expression in Eqn. (14) represents the "time coupling property" due to the battery energy constraint and the concave rate-power function.

VI. A Heuristic Distributed Joint Energy Allocation and Routing Scheme in Rechargeable Sensor Networks

A. A Heuristic Scheme

As mentioned earlier, although Problem **D** is a convex optimization problem, the solution is highly complex and cumbersome to implement. Here, we present a simple heuristic distributed scheme, which we call *NetOnline*. NetOnline is based on the single node results (Section IV). It exploits the fact that the replenishment rate profiles for all nodes in a rechargeable sensor network are likely to be similar due to spatial locations (e.g., solar energy for an outdoor sensor network will result in similar replenishment rate at each node).

- *Step 1*: Each node $n \in \mathcal{N}$ follows the allocation scheme by the previous online algorithm $\vec{e}_n^* = \big(e_n^*(1), e_n^*(2), \cdots, e_n^*(T)\big)$ according to its estimated replenishment profile in the period $[0, T]$.
- *Step 2*: The routing part in each slot t is determined by solving the following problem:

$$\text{Problem } \mathbf{E}: \quad \max \sum_s x^s(t)$$
$$\text{subject to} \quad w_{ij}^d(t) \geq 0, \ \text{ for all } d, \text{ for all } (i,j) \in \mathcal{L},$$
$$\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t) \geq 0,$$
$$\text{for } \forall d, \text{ and for } i \neq d$$
$$\sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t) \leq \phi(e_i^*(t)), \ \text{ for all node } i \in \mathcal{N}. \tag{19}$$

Note that Problem **E** above is a simple linear programming (LP) problem. In contrast to the last expression in Eqn. (14), the last expression in Eqn. (19) excludes the "time coupling property", which therefore has much lower complexity.

Since Problem **E** is also a convex optimization problem, we can use duality to get its optimal solution [14]. We assign a Lagrangian multiplier q_i^d to each second constraint and Q_i to each third constraint in Eqn. (19). The procedure is then summarized as follows:

- The generating data rate of the source nodes $\vec{x}^s(t)$ are determined by

$$\max_{0 \leq x_s \leq \phi(e_{f_s}^*(t))} \big[x^s(t) - x^s(t) q_{f_s}^{d_s}\big], \tag{20}$$

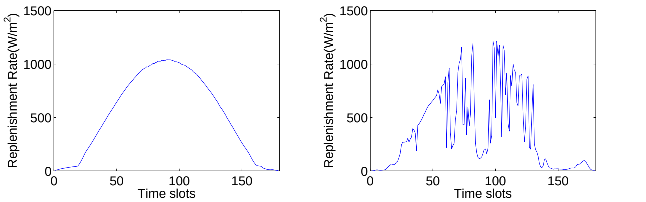

Fig. 4. Replenishment profiles for the sunny day (left) and the cloudy day (right)

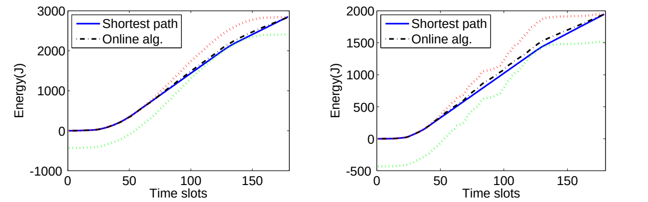

Fig. 5. Resource allocations of shortest-path solution and the online algorithm

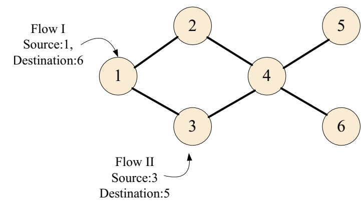

Fig. 6. A simple network topology with two flows

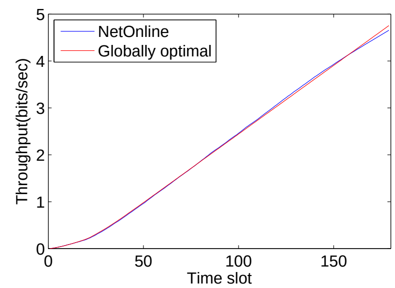

Fig. 7. The performance by NetOnline and the optimal solution

- The data rate at each link $\vec{w}_{ij}^d(t)$ is determined by

$$\max_{0\le w_{ij}^d(t)\le \phi(e_i^*(t))} \sum_d \sum_{(i,j)\in\mathcal{L}} w_{ij}^d(t)(q_i^d - q_j^d - Q_i), \quad (21)$$

- The Lagrange multipliers are updated by

$$q_i^d(\tau+1) = \Big[q_i^d(\tau) - h\big(\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t,\tau) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t,\tau) - \sum_{f_s=i,d_s=d} x^s(t,\tau)\big)\Big]^+,$$
$$Q_i(\tau+1) = \Big[Q_i(\tau) + h\big(\sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t,\tau) - \phi(e_i^*(t))\big)\Big]^+. \quad (22)$$

where h is the step size.

Note that the algorithm above only needs local information, and therefore can be implemented fully distributedly. We show in our technical report [17] that the algorithm converges.

B. Optimality conditionally preserved for NetOnline

In the following proposition, we show that NetOnline, which has much lower complexity, is still optimal under homogeneous replenishment profiles for all nodes with perfect estimation.

***Proposition 3**:* The heuristic scheme, NetOnline, is optimal under homogeneous replenishment profiles with perfect estimation for all nodes. In other words, the solution to Problem **E** is equivalent to the solution to Problem **D** under the condition that all nodes have the same replenishment profiles.

Proof: Please refer to our technical report [17] for the proof.

VII. Numerical Evaluation

We now evaluate our schemes through simulations. We use the Baseline Measurement System at the National Renewable Energy Laboratory [15] as the recharging profiles of nodes. The data set used here is Global 40-South LI-200, which measures the solar resource for collectors tilted 40 degrees from the horizontal and optimized for year-round performance. We set a sunny day (June 28th, 2010) and a cloudy day (June 26th, 2010) as the finite-horizon periods. Each time slot is set to five minutes. Fig. 4 illustrates these two replenishment profiles.

Fig. 5 shows energy allocations of a single node under the shortest-path solution in Section III and the online solution in Section IV for both the sunny day and the cloudy day. The node is assumed to have a solar panel with a dimension ($10mm \times 10mm$). We set the rate-power function as $\mu(P) = \ln(1+P)$ (bits/sec), and also set inaccurate estimation parameters $\beta_1 = \beta_2 = 0.2$. The battery size M is equal to 1V/120mAh. In this figure, the red dot curve is the cumulative harvested energy $R(t)$, and the green dot curve is $R(t) - M$. The blue curve represents the energy allocations of shortest-path solution and the black dot-dash curve represents that of the online algorithm. The shortest path is calculated using the linear time algorithm in [16], whose complexity is $O(T)$. From the figure, it can be observed that the energy allocations of the online algorithm are close to those of the shortest-path solution. As a result, the throughput achieved by the online algorithm is 2.4507 bits/sec for the sunny day, which is 99.27% of the optimal (2.4685 bits/sec). For the cloudy day, the throughput by the online algorithm is 2.0518 bits/sec, which is also close to the optimal throughput, 2.0790 bits/sec.

Now we consider the energy allocation problem in a network. Since solving Problem **D** is highly complex and time-consuming, in this paper, we consider a simple six-node network, as shown in Fig. 6. From Fig. 7, it can be seen that the throughput performance achieved by our heuristic scheme, NetOnline, is approximately 97% of the optimal throughput. An interesting observation is that the performance by NetOnline is higher than that of the optimal scheme at the beginning.

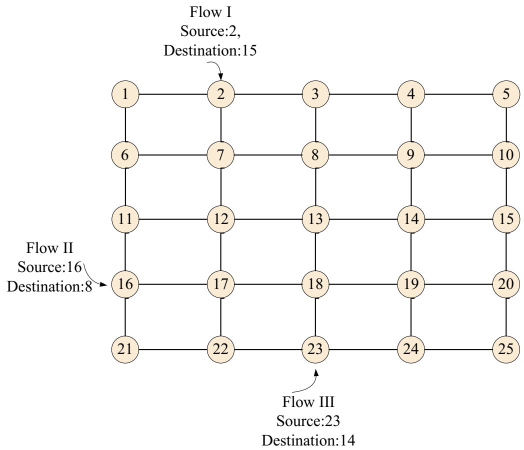

Fig. 8. A large network with three flows

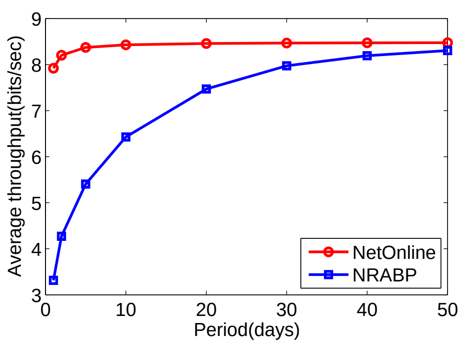

Fig. 9. The performance by NetOnline and a state-of-the-art infinite-horizon based scheme

The reason for this phenomenon is that NetOnline excludes the time coupling, while the optimal scheme allocates energy considering the entire time period. Thus, the optimal scheme may have less throughput at the beginning, but outperforms the heuristic scheme in the end.

We also consider a larger network with twenty five nodes and three flows, as shown in Fig. 8, and compare the performance of NetOnline with the state-of-the-art infinite-horizon based scheme *NRABP* in [12]. The results shown in Fig. 9, demonstrate that the infinite-horizon based technique is highly inefficient even over fairly long time periods. This observation verifies the importance of our work.

VIII. Conclusion

In this paper, we have studied the problem of maximizing the throughput in a finite-horizon period for a sensor network with energy replenishment. The problem can be formulated as a convex optimization problem, but turns out to be highly complex. To address this complexity, we employ a three-step approach. First, we have shown that a shortest-path based energy allocation scheme maximizes the throughput of a single node with renewal energy. Then an online algorithm based on the shortest-path solution is proposed and proven to guarantee a fraction of the optimal performance. Finally, we proposed a low-complexity heuristic distributed scheme NetOnline, where each node follows the energy allocation scheme by the single-node results. We prove that this heuristic scheme is optimal under homogeneous replenishment profiles. Further, in more general settings, we show via simulations, using data collected from a testbed sensor network, that NetOnline significantly outperforms the state-of-the-art infinite-horizon scheme and also achieve throughput performance close to optimal.

Appendix A
Proof of Theorem 1

Proof of Theorem 1: Let $\vec{o}$ and $\vec{O}$ denote the optimal power allocation, and its cumulative sequence, respectively, satisfying that $o(t) = O(t+1) - O(t)$. Assume that the shortest path $\vec{s}$ differs from the optimal path $\vec{o}$. We are going to show that this assumption leads to a contradiction.

Let x denote the smallest time, such that $S(x) \neq O(x)$. Clearly, we have $S(t) = O(t)$, for all $t < x$. In this paper, we consider the case when $S(x) < O(x)$. The other case $S(x) > O(x)$ can be shown similarly. We further divide the case into two sub-cases.

Case 1: $O(t)$ is strictly concave at x, i.e., $O(x) > O(x-1)/2 + O(x+1)/2$. We can define a new sequence $O'(t)$, such that

$$O'(t) = \begin{cases} O(t) - \epsilon/2, & t = x, \\ O(t), & \text{otherwise}, \end{cases}$$

where $\epsilon \triangleq O(x) - \max\{S(x), O(x-1)/2 + O(x+1)/2\} > 0$.

Note that $O'(x)$ is within the domain D from $R(x) - M \leq S(x) \leq O'(x) < O(x) \leq R(x)$. Since $O'(t)$ is non-decreasing, thus it is feasible. Also it follows that:

$$o'(x) = O'(x) - O'(x-1) = o(x) - \epsilon/2,$$
$$o'(x+1) = O'(x+1) - O'(x) = o(x+1) + \epsilon/2.$$

Then, we can obtain that

$$\begin{aligned} &o'(x) - o'(x+1) = o(x) - o(x+1) - \epsilon \\ &= \max\{S(x), O(x-1)/2 + O(x+1)/2\} \\ &\quad + O(x) - O(x-1) - O(x+1) \\ &\geq O(x) - (O(x-1) + O(x+1))/2 > 0, \end{aligned}$$

where the last inequality comes from the concavity of $O(t)$ at x. Hence, we have

$$o(x) > o'(x) > o'(x+1) > o(x+1),$$

and $o(t) = o'(t)$ for all $t \neq x, x+1$. Note that $\mu(o(x)) - \mu(o'(x)) < d\mu_+(o'(x))\frac{\epsilon}{2}$, and also $\mu(o'(x+1)) - \mu(o(x+1)) > d\mu_-(o'(x+1))\frac{\epsilon}{2}$, where $d\mu_\pm(\cdot)$ is the right and left derivative of μ, which always exist due to the concavity of $\mu(\cdot)$. Therefore, from the decreasing property of the derivative of the concave function $\mu(\cdot)$ and $o'(x) > o'(x+1)$, we can obtain that

$$\begin{aligned} &\sum_{t=1}^{T} \mu(o'(t)) - \sum_{t=1}^{T} \mu(o(t)) \\ &> \{d\mu_-(o'(x+1)) - d\mu_+(o'(x))\}\epsilon/2 > 0, \end{aligned} \quad (23)$$

which implies that $O(t)$ is not optimal.

Case 2: $O(t)$ is convex at x, i.e., $O(x) \leq (O(x-1)+O(x+1))/2$. Recall that $S(x-1) = O(x-1)$ and $S(x) < O(x)$. . Then from Lemma 3, $S(t)$ is concave at x, and thus, we have that $S(x+1) \leq 2S(x) - S(x-1) < 2O(x) - O(x-1) \leq O(x+1) < R(x+1)$. If $O(t)$ is strictly concave at $x+1$, we can follow the same line of analysis as in Case 1, resulting in a contradiction to the optimality of $O(t)$. Otherwise, it can be easily shown that $S(x+2) < O(x+2)$, since $S(t)$ is concave. Hence, it follows that $O(t)$ is convex at all $t \geq x$, and we have that $S(T) < O(T)$, which contradicts our assumption that $S(T) = O(T)$.

Similar contradiction can be obtained when assuming that $S(x) > O(x)$. Therefore, we can conclude that $S(t) = O(t)$, for all $t \in (1, \cdots T)$.□

APPENDIX B
PROOF OF PROPOSITION 2

Proof of Proposition 2: We will first prove a lemma that will be used in the proof.

Lemma 5*:* If the replenishment rates are $\underline{r}(t)$ and $\overline{r}(t)$ as defined by Eqn. (13), the corresponding shortest-path solutions satisfy $\underline{e}(t) = (1-\beta_1)\hat{e}(t)$ and $\overline{e}(t) = (1+\beta_2)\hat{e}(t)$.

Proof of Lemma 5: First, consider the case of $\underline{r}(t) = (1-\beta_1)\hat{r}(t)$. It follows that the corresponding cumulative harvested energy $\underline{R}(t) = \sum_{i=1}^{t} \underline{r}(i) = \sum_{i=1}^{t}(1-\beta_1)\hat{r}(i) = (1-\beta_1)\hat{R}(t)$, where $\hat{R}(t)$ is the cumulative harvested energy associated with $\hat{r}(t)$. Let $\underline{e}(t) = (1-\beta_1)\hat{e}(t)$ and $\underline{E}(t) = \sum_{i=1}^{t} \underline{e}(i) = (1-\beta_1)\hat{E}(t)$. We will show that the corresponding shortest-path scheme is $\vec{\underline{E}} = (\underline{E}(1), \underline{E}(2), \cdots, \underline{E}(T))$.

Assume that $\underline{S}(t)$ is the shortest path associated with $\underline{R}(t)$, and it is different from $\underline{E}(t)$. Let x denote the smallest time, such that $\underline{S}(x) \neq \underline{E}(x)$. We consider the case when $\underline{S}(x) > \underline{E}(x)$. The other case where $\underline{S}(x) < \underline{E}(x)$ can be shown similarly.

Since we have $\underline{S}(x) > \underline{E}(x) \geq \underline{R}(t) - M$, it follows that $\underline{S}(t)$ is convex at point x according to Lemma 2. On the other hand, since we have $\underline{R}(t) \geq \underline{S}(x) > \underline{E}(x)$, it follows that $\hat{R}(x) > \hat{E}(x)$. Therefore, we know that $\hat{E}(t)$ is concave at point x according to Lemma 2. Note that $\underline{E}(t) = (1-\beta_1)\hat{E}(t)$, which preserves the concavity of $\hat{E}(x)$. Therefore, we have $\underline{S}(x+1) > \underline{E}(x+1)$. Hence, we can follow the same line of analysis and get that $\underline{S}(T) > \underline{E}(T)$, which contradicts $\underline{S}(T) = \underline{E}(T)$.

Similar contradiction can be obtained when assuming that $\underline{S}(x) < \underline{E}(x)$. Therefore, we can conclude that $\underline{S}(t) = \underline{E}(t)$, for all $t \in (1, \cdots T)$ for the case of $\underline{r}(t) = (1-\beta_1)\hat{r}(t)$, which results in the conclusion that the shortest-path solution corresponding to $\underline{R}(t)$ is $\underline{e}(t) = (1-\beta_1)\hat{e}(t)$.

Similarly, we can prove that the shortest-path solution corresponding to $\overline{R}(t)$ is $\overline{e}(t) = (1+\beta_2)\hat{e}(t)$. □

Let $\hat{U} \triangleq \sum_{t=1}^{T} \mu(\hat{e}(t))$, $\overline{U} \triangleq \sum_{t=1}^{T} \mu(\overline{e}(t))$, and $\underline{U} \triangleq \sum_{t=1}^{T} \mu(\underline{e}(t))$ denote the throughput by the shortest-path solution associated with $\hat{r}(t)$, $\overline{r}(t)$, and $\underline{r}(t)$, respectively.

From the strictly concave and nondecreasing properties of $\mu(\cdot)$ and the fact that $\mu(0) = 0$, we have

$$\begin{aligned}(1-\beta_1)\mu(\hat{e}(t)) &< \mu((1-\beta_1)\hat{e}(t)) < \mu(\hat{e}(t)) \\ &< \mu((1+\beta_2)\hat{e}(t)) < (1+\beta_2)\mu(\hat{e}(t)). \end{aligned} \tag{24}$$

By summing over all t, we obtain that

$$(1-\beta_1)\hat{U} < \underline{U} < \hat{U} < \overline{U} < (1+\beta_2)\hat{U}. \tag{25}$$

Now, from Step 2 of the online algorithm, it follows that $e(t) > \underline{e}(t)$, and thus we have that $U > \underline{U}$. Similarly, we can obtain that $U^* \leq \overline{U}$, where U^* is the throughput achievable when the optimal resource allocation is used. Therefore, we have that

$$U/U^* \geq \underline{U}/\overline{U} > \frac{(1-\beta_1)}{(1+\beta_2)}, \tag{26}$$

which proves Proposition 2. □

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