# Dynamic User Association and Energy Control in Cellular Networks with Renewable Resources

Yang Yang, Jiashang Liu, Prasun Sinha and Ness B. Shroff

*Abstract*— **In recent years, there has been a growing interest in equipping base-stations with renewable resources and batteries, with an aim to reduce the energy-related operational cost for cellular networks. To fully leverage the cost-saving potential of such base-stations, the design of joint resource-allocation and energy-control algorithms that take both the traffic-level dynamics and energy-level dynamics into account is called for. In this work, we propose a joint user-association and energy-control algorithm, where the induced operational cost can be made arbitrarily close to optimality at the expense of increased battery capacity in each base-station. The key to this algorithm is that it tries to match the traffic-profile with the energy-profile of the network, which leads to an efficient utilization of the renewable resources across the network. Simulation results show that a significant reduction in energy cost can be achieved by using our algorithm, even at a relatively small battery capacity.**

## I. Introduction

The energy-related operational expenditure for cellular networks has increased dramatically over the past few years, driven by the proliferation of wireless devices and the ever-growing traffic demand. Meanwhile, there is a growing interest in equipping base-stations with renewable energy sources and energy storage devices, leading to the concept of "green base-station". In order to exploit the maximum energy-saving potential of such base-stations, it is vital to design joint energy control and wireless resource allocation algorithms that provide an efficient network-wide use of renewable energy and at the same time guarantee high-rate reliable wireless data communication.

However, in the evolution towards green cellular networks, new challenges emerge. The core challenge is that *it is not clear how the control plane of wireless network should interact with energy-related decisions, when traditional wireless network dynamics such as the traffic arrival processes and channel variations are entangled with energy dynamics such as time-varying energy prices, the evolution of battery levels, and stochastic renewable energy arrivals.* In current wireless systems, wireless operations such as transmission-power allocation, antenna allocation, frequency allocation, user-base-station association, and uplink/downlink scheduling are all designed with the goal of achieving desirable throughput/delay performance. To ensure efficient energy consumption and achieve significant cost reduction in purchasing electricity, it is crucial to revisit and redesign network control algorithms that take both user-demand and energy-profile of the network into consideration.

There have been many works that focus on energy harvesting in wireless sensor networks, e.g., [1]–[3]. Since sensor networks usually have a limited life-time and do not induce any energy related operational cost once deployed, the main goal in these works is to develop joint resource allocation and energy control schemes to prolong sensor life and ensure efficient use of the renewable energy under a limited battery size, while fulfilling the QoS constraints. [4]–[6] have focused on designing resource allocation algorithms in cellular networks to reduce the base-station energy consumption. However, they do not consider the use of base-station batteries and renewable resources (i.e., no energy dynamics in the system), and the energy reduction is achieved mainly by exploiting the delay-energy tradeoff. [7] is the closest to our work as it considers the cost minimization problem under stochastic renewable resources, dynamic energy demands, and time-varying energy prices. However, it focuses only on a single battery system.

In this work, we consider a multi-cell network, as depicted in Figure 1, where each base-station in the network has a battery and can potentially harvest energy from renewable resources such as wind or solar. Also, we assume that all the base-stations are connected to the power grid with time-varying energy prices. Each user in the network has the flexibility of associating with one of many base-stations. We focus on the problem of how to dynamically control the battery levels at each base-station and dynamically form associations between users and base-stations taking into account energy harvesting and user traffic dynamics. The objective is to reduce the operational cost of the network, which in turn can translate to a lower-cost for end-customers and a reduced carbon emission footprint of the cellular infrastructure.

The control algorithm proposed in this work naturally decomposes into a user-association component and an energy-control component, and is shown to achieve asymptotically optimal performance as the battery capacity increases. The user-association components requires solving a minimum-weighted maximum-matching problem, which aims to *align the traffic profile with the energy profile* in the network. Once the user-association decision is made, the battery energy-control component runs in a distributed fashion, where each base-station independently decides how much energy is dis-

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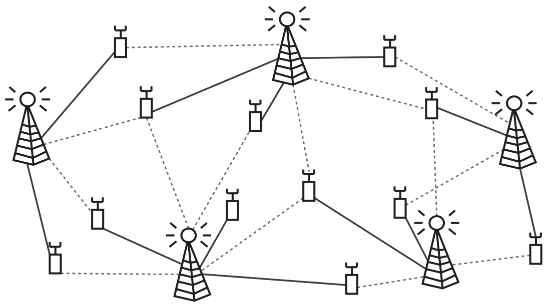


Fig. 1: An example multi-cell network. User $i$ and base-station $j$ is connected with a dashed line (or solid line) if $R(j,i) > 0$. The set of solid lines represents a specific choice of base-station-user association.

charged from/charged to the battery. Simulations with real-life energy price data have shown that a significant amount of cost reduction can be achieved, compared with the fixed base-station-user association scheme, even at a relatively low battery capacity.

Our paper is organized as follows: In Section II, we discuss our network model and energy model. In Section III, we formulate an operational cost minimization problem. Our joint user-association energy-control algorithm is proposed and analyzed in Section IV. Detailed simulation results are presented in Section V. Finally, we conclude our paper in Section VI.

## II. System Model

### A. Network Model

We consider a cellular network with $J$ base-stations and $I$ users, as illustrated in Figure 1, and focus on the problem of downlink dynamic user association. We assume a time slotted system, with each slot representing a time-interval during which the base-station-user association has to be fixed. We also assume that within each time-slot different users are assigned with orthogonal resource blocks, which means that there is no inter-cell interference. For each base-station-user pair $(j,i)$, we denote $R(j,i)$ as the amount of data that can be transmitted from base-station $j$ to user $i$ in a single resource block. In practice, $R(j,i)$ takes on a discrete set of values, with each value corresponding to a specific coding and modulation scheme (MCS). Since the best MCS for each link is determined mainly by the distance related pathloss, which usually changes at a much slower time scale compared with the operation of cellular systems, we assume that $R(j,i)$ is a fixed value across different time-slots and different resource blocks. Also, we set $R(j,i) = 0$ if no direct data-link can be established between the two.

For each user $i$, we denote $w_i(t)$ as the amount of downlink data that needs to be transmitted to it within the $t^{\text{th}}$ association interval. Then, if user $i$ is associated with base-station $j$ in time-slot $t$ and $R(j,i) > 0$, base-station $j$ needs to allocate $w_i(t)/R(j,i)$ resource blocks to user $i$ in that time-slot. For each base-station, we assume that a fixed amount of energy, denoted as $E$, has to be spent on each resource block it uses, where the energy is drawn from either the power grid, or the battery associated with it. In other words, base-station $j$ needs to spend $Ew_i(t)/R(j,i)$ amount of energy to serve user $i$ in time-slot $t$. For notational convenience, we construct a $J$ by $I$ matrix $H$ with

$$H_{ji} = \begin{cases} E/R(j,i) & \text{if } R(j,i) > 0, \\ \infty & \text{otherwise,} \end{cases}$$

where $H_{ji}$ has the metric of *energy per bit* and can be interpreted as the amount of energy that base-station $j$ needs to consume in order to transmit a single unit data to user $i$.

### B. Energy Model

The model of the base-station is illustrated in Figure 2. There exists a battery in each base-station with maximum capacity $B_{\text{max}}$. We let $\lambda_j(t)$ denote the energy harvested from the renewable resources in the $j^{\text{th}}$ base-station during the $t^{\text{th}}$ user-association interval. We assume that the energy price in the power grid may change in different time-slots, but remains the same across all base-stations, which we denote as $P(t)$. This is a reasonable assumption, if the base-stations access the same energy provider for their energy needs. For each base-station $j$ in time-slot $t$, we denote the amount of energy that needs to be consumed as $q_j(t)$, and the amount of energy it purchases from the grid as $g_j(t)$. Then, if $g_j(t) < q_j(t)$, we need to discharge $q_j(t) - g_j(t)$ amount of energy from the battery, while if $g_j(t) > q_j(t)$, the battery will be charged with $g_j(t) - q_j(t)$ amount of energy. Let us denote the battery level in base station $j$ at the beginning of time-slot $t$ as $B_j(t)$ and denote $b_j(t) \triangleq q_j(t) - g_j(t)$, then the battery dynamics can be expressed as

$$B_j(t+1) = \min\left\{B_j(t) - b_j(t) + \lambda_j(t), B_{\text{max}}\right\}.$$

Here we call $b_j(t)$ the energy control decision[1] of base-station $j$ at time-slot $t$. Since the energy drawn from the battery cannot be more than the battery level, and $g_j(t)$ needs to be positive, we must have,

$$b_j(t) \leq B_j(t), \text{ and } b_j(t) \leq q_j(t). \tag{1}$$

Also, due to the finite rate of charging the battery, we assume that at any time-slot, the charged energy from the power grid to the battery cannot be greater than $b_{\max}$, i.e., $b_j(t) > -b_{\text{max}}$ for any base-station $j$ and time-slot $t$.

In order to aid our theoretical analysis, we introduce the notation of *emptiness of battery*, defined as

$$\mathcal{E}_j(t) \triangleq B_{\text{max}} - B_j(t), \tag{2}$$

whose evolution mirrors that of $B_j(t)$ and can be expressed as

$$\mathcal{E}_j(t+1) = \max\{\mathcal{E}_j(t) + b_j(t) - \lambda_j(t), 0\}.$$

### C. Modeling of User-Association

We represent the base-station-user association at any association slot $t$ as a $J$ by $I$ binary matrix $A(t)$, with its $(j,i)^{\text{th}}$

[1]For a given $q_j(t)$, $g_j(t)$ and $b_j(t)$ are complementary: once $b_j(t)$ is determined, $g_j(t) = q_j(t) - b_j(t)$.

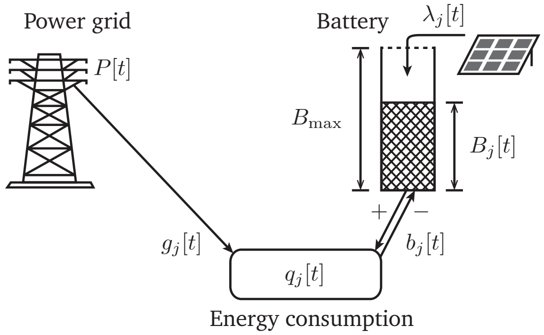



Fig. 2: Base-station energy model (at base-station $j$). A negative $b_j(t)$ indicates that $|b_j(t)|$ amount of energy is charged to the battery in base-station $j$ at time-slot $t$, which happens when the purchased energy $g_j(t)$ is greater than the consumed energy $q_j(t)$.

entry defined as

$$A_{ji}(t) = \begin{cases} 1 & \text{if user } i \text{ is associated with base-station } j, \\ 0 & \text{otherwise,} \end{cases}$$

and for any time-slot $t$, the association matrix $A$ is subject to the following three feasibility constraints:

$\sum_{j=1}^{J} A_{ji} = 1$ for any $1 \leq i \leq I$,
$\sum_{i=1}^{I} A_{ji} \leq m$ for any $1 \leq j \leq J$,
$A_{ji} = 0$ if $R(j,i) = 0$ for any $1 \leq i \leq I, 1 \leq j \leq J$.

The first and the last constraint together implies that each user must be associated with one base-station among those who can establish a direct data-link to the user. The second constraint says that each base-station cannot associate with more than $m$ users. The motivation for setting up the second constraint is to (i) favor the balancing of traffic load in the back-bone, and (ii) avoid the extreme case where too much energy needs to be spent in any base-station at any time-slot. We denote the set of binary matrices that conform to the above three constraints as $\mathcal{A}$, and *assume that it is non-empty*. In other words, each matrix in $\mathcal{A}$ corresponds to a *feasible user-association*.

By combining the definition of $A(t)$ with the discussions in the previous two subsections, we know that $q_j(t)$, the amount of energy that needs to be consumed at base-station $j$ at time-slot $t$, can be expressed as[2]

$$q_j(t) = \sum_{i:A_{ji}(t)=1} A_{ji}(t)H_{ji}w_i(t) = \sum_{i=1}^{I} A_{ji}(t)H_{ji}w_i(t).$$

Since $A_{ji}(t)$ must be one of the feasible user-association in $\mathcal{A}$ and $\mathcal{A}$ is assumed to be non-empty, we know that $q_j(t)$ is always a finite value. Then, for a given the energy control decision $b_j(t)$, we know that the cost of purchasing energy from the power grid in base-station $j$ at time-slot $t$ is

$$P(t)g_j(t) = P(t)\left(q_j(t) - b_j(t)\right)$$

$$= P(t)\left(\sum_{i=1}^{I} A_{ji}(t)H_{ji}w_i(t) - b_j(t)\right). \tag{3}$$

In our analysis, we assume that all the processes in the system are ergodic with bounded values. Denote $P_{\max}$, $w_{\max}$, and $\lambda_{\max}$ as the maximum value for the energy price, the per-slot traffic demand, and the per-slot renewable energy arrival, respectively. Also define $H_{\max} \triangleq \max_{(j,i):H_{ji}<\infty} H_{ji}$.

## III. Problem Formulation

From Equation (3) we know that the cost of buying energy from the grid in any base-station at any time-slot is jointly determined by both the user-association decision at the network, and the energy-control decision at each base-station. Specifically, the user-association decision determines the amount of energy that needs to be consumed in each base-station, while the energy-control decision controls how much energy should be drawn from/charged to the battery, which in turn dictates the amount of energy that needs to be purchased.

The operational cost of the network is the sum of the cost of purchasing energy from the grid for all base-stations. Our main objective, which is captured in the optimization problem below, is to minimize the long-term average operational cost of the network.

(Problem P1)

$$\min_{\mathbf{A},\mathbf{b}} \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{j=1}^{J} \mathbb{E}\left[P(t)\left(\sum_{i=1}^{I} A_{ji}(t)H_{ji}w_i(t) - b_j(t)\right)\right]$$

s.t. For any base-station $j$ and time-slot $t$,

$$A(t) \in \mathcal{A}, \quad B_j(0) = B_{\max}$$

$$b_j(t) \geq -b_{\max} \tag{4}$$

$$b_j(t) \leq \textstyle\sum_{i=1}^{I} A_{ji}(t)H_{ji}w_i(t) \tag{5}$$

$$b_j(t) \leq B_j(t) \tag{6}$$

$$B_j(t+1) = \min\left\{B_j(t) - b_j(t) + \lambda_j(t), B_{\max}\right\} \tag{7}$$

The constraint in Equation (4) limits the maximum charge rate of the battery in each base-station. Equations (5) and (6) are in accord with Equation (1), reflecting the physical limits that the energy drawn from the battery cannot be greater than the battery level, and that the energy should not be drawn beyond what is needed. The last constraint describes the battery evolution, and the first constraint says that each battery starts with being fully charged. The expectation for time-slot $t$ is taken over all potential randomness of the energy prices, traffic arrivals, renewable arrivals, and control actions from time-slot 0 to time-slot $t$.

The optimal solution for Problem P1 is a function of $B_{\max}$, which we denote as $S_1^*(B_{\max})$. Intuitively speaking, a larger battery capacity can benefit the system in two ways and thus should be able to give a better optimal solution: (i) A larger battery can take better advantage of the fluctuation in the

[2] In the second equality, we follow the convention that $\infty \cdot 0 = 0$.

energy price, in the sense that base-stations could stock more energy when the energy price is relatively low. (ii) Increased battery capacity would reduce the chance of battery overflow when there is a large injection of renewable energy.

It is also worth noting that due to the evolution of battery levels in different base-stations, an efficient user-association and energy-control policy should weigh in, among other things, the factor of the status of the batteries, rather than simply building its decision based on the instantaneous traffic load, or the amount of obtained renewable energy within a single time-slot. Moreover, the battery level should be controlled in a fashion that both zero-battery state and full-battery state are avoided: in zero-battery state the base-station has to buy from the grid to serve the users, even if the energy price is high; a full-battery state prevents the base-station from storing renewable resources, resulting in a waste of renewable energy.

Compared with a fixed base-station-user association scheme, the potential benefits of an efficient dynamic user-association policy are twofold: (i) It would help balance *the traffic load pattern* with *the energy profile* in the network. More specifically, by guiding more traffic towards base-stations with larger battery levels, the full-battery and zero-battery states can both be better mitigated, and thus a more efficient use of renewable resources can be achieved. This benefit would be more pronounced when the network has heterogeneous renewable arrival rates and user traffic rates. (ii) It could exploit the *multi-user diversity in terms of user traffic demand*. When a base-station has many users in its close proximity but is only able to serve a fraction of them due to either energy constraints or back-haul data-rate constraint, the dynamic user-association algorithm should prioritize the association of users with large traffic demand, while leaving the users with low traffic demand to be associated with other base-stations, striking for an overall lower energy-expenditure per unit-data-traffic in the network.

Now, let us return our focus to Problem P1. Based on the definition of the level of battery emptiness $\mathcal{E}_j(t)$ in Equation (2), the last two constraints (Equations (6) and (7)) are equivalent to and can be substituted with the following two equations

$$\mathcal{E}_j(t) \leq B_{\max} \quad (8)$$
$$\mathcal{E}_j(t+1) = \max\{\mathcal{E}_j(t) + b_j(t) - \lambda_j(t), 0\}, \quad (9)$$

with the initial states $\mathcal{E}_j(0) = 0$ for any base-station $j$. Then, by contradiction, we can show that the above two equations implies

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} b_j(t) \leq \bar{\lambda}_j, \quad (10)$$

where $\bar{\lambda}_j$ is the arrival rate of the renewable energy in base-station $j$: assume to the contrary that there exists a positive number $\alpha$ with $\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} b_j(t) - \bar{\lambda}_j > \alpha$, then $\mathcal{E}(t)$ will diverge to infinity as $t$ increases, which contradicts with Equation (8).

Next, we switch the last two constraints in Problem P1 with Equation (10) and (9), and obtain a new optimization problem as shown below.

(Problem P2)

$$\min_{\mathbf{A},\mathbf{b}} \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{j=1}^{J} \mathbb{E}\left[ P(t)\Big( \sum_{i=1}^{I} A_{ji}(t) H_{ji} w_i(t) - b_j(t) \Big) \right]$$

s.t. For any base-station $j$ and time-slot $t$,

$$A(t) \in \mathcal{A}, \quad \mathcal{E}_j(0) = 0$$
$$b_j(t) \geq -b_{\max}$$
$$b_j(t) \leq \textstyle\sum_{i=1}^{I} A_{ji}(t) H_{ji} w_i(t)$$
$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} b_j(t) \leq \bar{\lambda}_j \quad (11)$$
$$\mathcal{E}_j(t+1) = \max\{\mathcal{E}_j(t) + b_j(t) - \lambda_j(t), 0\}$$

Since the last two constraints in Problem P1 are equivalent to the combination of Equation (8) and (9), which together imply Equation (10), we know that Problem P2 is a relaxed version of Problem P1. In fact, the last two constraints in Problem P2 only enforce that the evolution of $\mathcal{E}_j(t)$ being rate-stable [8] for any base-station $j$. In other words, in Problem P2, it is feasible for the process $\{\mathcal{E}_j(t)\}_t$ to be unbounded as long as Equation (10) is satisfied[3]. However, in Problem P1, $\mathcal{E}_j(t)$ has to be upper bounded by $B_{\max}$, as is indicated in Equation (8).

Let us denote the optimal solution to Problem P2 as $S_2^*$. Given that Problem P2 is a relaxed version of Problem P1 and is no longer a function of $B_{\max}$, we know that the solution to Problem P2 should be no worse, if not better, than that of Problem P1. More precisely,

$$S_2^* \leq S_1^*(B_{\max}), \text{ for any finite value of } B_{\max}.$$

Interestingly, we find a dynamic user-association and energy-control algorithm under the constraint of Problem P1 whose average operational cost can get arbitrarily close to $S_2^*$ as the increase of battery capacity $B_{\max}$, which we present in the next Section.

## IV. Joint User Association and Energy Control

Let us introduce a free parameter $K > 0$, which is linked with the battery capacity through the equation below

$$B_{\max} = K P_{\max} + m H_{\max} w_{\max},$$

where the definition of $P_{\max}$, $w_{\max}$, and $H_{\max}$ can be found at the end of Section II. Our dynamic joint user-association and energy-control algorithm that conforms to the constraints in Problem P1 naturally breaks into two components as shown below.

*User Association Component:* Solve the following optimization problem, and assign user $i$ to base-station $j$ if

[3] In Problem P2, $\mathcal{E}_j(t)$ can no longer be interpreted as the physical meaning of the emptiness of battery, as it is not necessarily bounded.

$A_{ji} = 1$. We will describe how to solve this problem in Section IV-B.

(Problem P3)

$$\arg\min_{A(t)} \sum_{i=1}^{I}\sum_{j=1}^{J} A_{ji}(t) H_{ji} \min(B_{\max} - B_j(t), KP(t)) w_i(t)$$

$$\text{s.t.} \quad \left.\begin{array}{l} \sum_{j=1}^{J} A_{ji}(t) = 1, \forall i \\ \sum_{i=1}^{I} A_{ji}(t) \le m, \forall j \\ A_{ji}(t) \in \{0,1\}, \forall i,j \\ A_{ji}(t) = 0 \text{ if } R(j,i) = 0, \forall i,j \end{array}\right\} A(t) \in \mathcal{A}$$

*Energy Control Component:* the battery at each base-station is either charged at the maximum rate, or discharged at the exact amount that the base-station needs to consume, depending on both the current battery level and the instantaneous energy price:

$$b_j^*(t) = \begin{cases} -b_{\max} & \text{if } B_j(t) \le B_{\max} - KP(t) \\ \sum_{i=1}^{I} A_{ji}(t) H_{ji} w_i(t) & \text{if } B_j(t) > B_{\max} - KP(t) \end{cases}$$

In the energy control scheme, whether the battery is charged or discharged depends on whether $B_j(t) + KP(t)$ is below or above a fixed threshold $B_{\max}$: when the threshold is exceeded, indicating that either the battery is near full, or the energy price is relatively high, the battery will be discharged; On the other hand, when either the battery is low, or the energy price is relatively low, the scheme will choose to purchase energy from the grid to both serve the current task, and charge the battery at the maximum charge rate. Here $K$ is a parameter that tunes the relative importance of the battery status and the energy price. As we increase $K$, the energy control decision weighs more on the energy price than the battery status, leading to a potentially larger cost reduction, as the battery will be charged only when the price is relatively low. The price to pay, however, is a linear increase in the battery capacity $B_{\max}$. Indeed, as we will show in the performance analysis, $K$ plays a key role in the trade-off between optimality and battery capacity.

The optimization problem in the user-association component essentially tries to *align the traffic profile with the energy profile in the network*. From the objective function in Problem P3, we can observe that each base-station-user pair $(j, i)$ is associated with a product-form metric as shown below

$$\underbrace{\min(B_{\max} - B_j(t), KP(t))}_{\text{energy factor}} \times \underbrace{H_{ji} w_i(t)}_{\text{traffic factor}},$$

with the first term capturing the emptiness of the battery capped by $K$ times the energy price, and the second term representing the amount of energy that needs to be consumed by this association. For a certain base-station-user pair, the metric is low only when the battery-level at the base-station is high and the energy request from the user is low. Then, it is evident that the network can be more energy efficient if the sum of the associated links' metrics is low. Indeed, the simulation results in Section V show that the dynamic user-association component is very crucial in guaranteeing an efficient use of the renewable energy.

### A. Performance Analysis

We define a constant $C$ to be

$$C \triangleq J \min\{b_{\max}, mH_{\max} w_{\max}\}^2 + J\lambda_{\max}^2.$$

The theorem below shows that our proposed policy can achieve a cost that is arbitrarily close to the optimal average cost by increasing the battery capacity $B_{\max}$, with the gap to optimality shrinks in the order of $O(1/B_{\max})$.

*Theorem 1:* The joint user-association and energy control algorithm with battery capacity $B_{\max} = KP_{\max} + mH_{\max} w_{\max}$ achieves

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\Big[P(t) \sum_{j=1}^{J} \Big( \sum_{i=1}^{I} A_{ji}(t) H_{ji} w_i(t) - b_j(t) \Big)\Big] \le S_2^* + C/K, \quad (12)$$

*Idea of proof:* First, we use the stochastic Lyapunov techniques to obtain a near-optimal algorithm for Problem P2 (which, as we discussed in the previous section, is a relaxed version of Problem P1). Then, interestingly, we find that the near-optimal algorithm satisfies the constraints for Problem P1 under the a certain battery capacity.

*Proof:*

Let us define the Lyapunov function as $L(t) = \frac{1}{2}\sum_{j=1}^{J} \mathcal{E}_j(t)^2$, and the one-step conditional Lyapunov drift $\Delta L(t)$ as $\Delta L(t) = \mathbb{E}\Big[L(t+1) - L(t) \big| \vec{\mathcal{E}}(t)\Big]$, where $\vec{\mathcal{E}}(t) = [\mathcal{E}_1(t), \mathcal{E}_2(t), \ldots, \mathcal{E}_J(t)]$, and the expectation is taken over all randomness in traffic arrival, energy arrival, power price, and control action at time-slot $t$.

Before proving the theorem, we need the following three supporting lemmas. The proof of Lemma 1 is very similar to the proof of Theorem 1 in [9] and is thus omitted for brevity. To prove Lemma 2, we only need to sum over the inequality (13) from $t = 0$ to $t = T - 1$, divide both side by $T$, and then let $T$ approach infinity. Lemma 3 follows simply by squaring the evolution equation of $\mathcal{E}_j(t)$ in Equation (9) and taking expectations.

*Lemma 1:* $S_2^*$ can be achieved using a stationary policy that conforms to the feasibility condition in Problem P2. By stationary, we mean that the decision of the policy at any time-slot is not a function of $\mathcal{E}_j(t)$ for any $j$ and $t$. More precisely, there exists a policy that achieves, for any $\vec{\mathcal{E}}(t)$,

$$\mathbb{E}\left[(b_j(t) - \lambda_j(t)) \big| \vec{\mathcal{E}}(t)\right] = 0,$$

$$\mathbb{E}\left[P(t) \sum_{j=1}^{J} \left(\sum_{i=1}^{I} A_{ji}(t) H_{ji} w_i(t) - b_j(t)\right) \big| \vec{\mathcal{E}}(t)\right] = S_2^*.$$

*Lemma 2:* A policy satisfies Equation (12) if

$$\Delta L(t) + K\mathbb{E}\Big[P(t) \sum_{j=1}^{J} \Big( \sum_{i=1}^{I} A_{ji}(t) H_{ji} w_i(t) - b_j(t) \Big) \Big| \vec{\mathcal{E}}(t)\Big] \le C + KS_2^* \quad (13)$$

*Lemma 3:*

$$\Delta L(t) \le C + \mathbb{E}\left[\sum_{j=1}^{J} \mathcal{E}_j(t)(b_j(t) - \lambda_j(t)) \big| \vec{\mathcal{E}}(t)\right]. \quad (14)$$

After adding the same term on both sides of Equation (14), we obtain,

$$\Delta L(t) + K\mathbb{E}\Big[P(t)\sum_{j=1}^{J}\Big(\sum_{i=1}^{I}A_{ji}(t)H_{ji}w_i(t) - b_j(t)\Big)\Big|\vec{\mathcal{E}}(t)\Big]$$
$$\leq C + \mathbb{E}\left[\sum_{j=1}^{J}\mathcal{E}_j(t)(b_j(t)-\lambda_j(t))\middle|\vec{\mathcal{E}}(t)\right]$$
$$+ K\mathbb{E}\Big[P(t)\sum_{j=1}^{J}\Big(\sum_{i=1}^{I}A_{ji}(t)H_{ji}w_i(t) - b_j(t)\Big)\Big|\vec{\mathcal{E}}(t)\Big]. \tag{15}$$

By substitiuting the result of Lemma 1 into Equation (15), we know that there exists a stationary scheme under the constraints of Problem P2 that achieves Equation (13). Then, if we design a policy that *minimize the right hand side of Equation (15) for any values of $\vec{\mathcal{E}}(t)$ while conforming to the constraint in Problem P2*, it must also satisfy Equation (13). According to Lemma 2, such a scheme, which we formally describe below, achieves the performance described in Equation (12).

$$\begin{aligned}&\arg\min_{\mathbf{A},\mathbf{b}}\Big[\sum_{j=1}^{J}\mathcal{E}_j(t)(b_j(t)-\lambda_j(t))\\ &\qquad + KP(t)\sum_{j=1}^{J}\big(\sum_{i=1}^{I}A_{ji}(t)H_{ji}w_i(t)-b_j(t)\big)\Big]\\ =&\arg\min_{\mathbf{A},\mathbf{b}}\Big[\sum_{j=1}^{J}(\mathcal{E}_j(t)-KP(t))b_j(t)\\ &\qquad + KP(t)\sum_{j=1}^{J}\sum_{i=1}^{I}A_{ji}(t)H_{ji}w_i(t)\Big]\end{aligned}$$

subject to the constraints in Problem P2.

It is easy to show that this minimization problem leads to the following solution

$$A^*(t) = \arg\min_{A(t)\in\mathcal{A}}\sum_{i=1}^{I}\sum_{j=1}^{J}A_{ji}(t)H_{ji}\min(\mathcal{E}_j(t), KP(t))w_i(t),$$
$$b_j^*(t) = \begin{cases} -b_{\max} & \text{if } \mathcal{E}_j(t)\geq KP(t)\\ \sum_{i=1}^{I}A_{ji}^*(t)H_{ji}w_i(t) & \text{if } \mathcal{E}_j(t) < KP(t)\end{cases}. \tag{16}$$

It should be noted that the above control policy is designed for Problem P2. However, quite interestingly, it is not hard to see that under this control policy, $\mathcal{E}_j(t)$ is upper bounded by $KP_{\max} + mH_{\max}w_{\max}$ for any base-station $j$ and time-slot $t$. In other words, if we set $B_{\max}$ to be $KP_{\max} + mH_{\max}w_{\max}$, Equation (8) is restored, and thus the above control policy conforms to the constraints in Problem P1. Finally, we can obtain our joint user-association and energy-control policy by switching $\mathcal{E}_j(t)$ back to $B_j(t) = KP_{\max} + mH_{\max}w_{\max} - \mathcal{E}_j(t)$. ■

### B. Revisiting the User Association Algorithm

We now focus on how to solve the optimization problem P3 for any time-slot $t$. Let us represent the multi-cell network as a bipartite graph $G = (\mathcal{J}, \mathcal{I}, \mathcal{L})$, where $\mathcal{J} = \{1, 2, \ldots, J\}$ and $\mathcal{I} = \{1, 2, \ldots, I\}$ denote the set of base-stations and the set of users, respectively. The edge set $\mathcal{L}$ is the set of base-station-user pairs for which a direct data-link can be established. More precisely,

$$\mathcal{L} = \{(j,i)\in\mathcal{J}\times\mathcal{I}|R(j,i)\neq 0 \text{ (i.e., } H_{ji}\leq\infty)\}.$$

Let us define $W$ as a mapping from $\mathcal{L}$ to $\mathbb{R}$ with

$$W(j,i) = H_{ji}\min(B_{\max} - B_j(t), KP(t))w_i(t), \tag{17}$$

and introduce an optimization problem below.

(Problem P4)

$$\arg\min_{\mathcal{M}\subseteq\mathcal{L}}\sum_{(j,i)\in\mathcal{M}}W(j,i)$$
$$\text{s.t. For any } i\in\mathcal{I}, |\{j\in\mathcal{J}|(j,i)\in\mathcal{M}\}| = 1 \tag{18}$$
$$\text{For any } j\in\mathcal{J}, |\{i\in\mathcal{I}|(j,i)\in\mathcal{M}\}| \leq m \tag{19}$$

By comparing the feasibility constraints of Problem P3 and P4, we know that for any feasible $\mathcal{M}$ in Problem P4 ($\mathcal{M}$ that conforms to Equation (18) and (19)), we can construct a $J$ by $I$ binary matrix $A$ by setting $A_{ji}$ to be 1 only if $(j,i)\in\mathcal{M}$, and very easily show that it is feasible in Problem P3, i.e., we must have $A\in\mathcal{A}$. Given that $\mathcal{A}$ is assumed to be non-empty, we know that Problem P4 is feasible. Moreover, by the definition of $W$ in Equation (17), for any $\mathcal{M}^*$ that is one of the solution(s) for Problem P4, the corresponding $A^*$ must be one of the solution(s) for Problem P3. Hence, we only need to focus on the solving of Problem P4.

The constraints in Equations (18) and (19) indicate that each user $i$ needs to incident to exactly one edge in $\mathcal{M}$, and each station $j$ cannot incident to more than $m$ edges in $\mathcal{M}$. When $m = 1$, these constraints implies that any feasible $\mathcal{M}$ is a *maximum matching* in the partite graph $G$, and thus the problem becomes a *minimum-weighted maximum matching* problem, where $W(j,i)$ represents the weight of edge $(j,i)$. In the general case when $m > 1$, we can still convert Problem P4 to a minimum-weighted maximum matching problem by extending each base-station $j$ in $\mathcal{J}$ into a set of $m$ identical virtual base-stations $\{j^{(1)}, j^{(2)}, \ldots, j^{(m)}\}$ where $j^{(v)}$ has the same connectivity to $\mathcal{I}$ as $j$ does and $W(j^{(v)}, i) = W(j,i)$ for any $1\leq v\leq m$. Since Problem P3, being equivalent to P4, can be converted into a minimum-weighted maximum-matching problem, it can be solved in O($|mJ|^3$) complexity using the Hungarian algorithm.

## V. Numerical Example

### A. Simulation Setup

We conduct our simulation on a network topology as shown in Figure 3, with 10 base-stations and 26 users. Each time-slot is set to be 30 seconds, i.e., the user-association and energy-control decision are updated every 30 seconds. The traffic demand $w_i(t)$ is uniformly distributed from 3 to 15 Mbit/time-slot for any user $i$, and the renewable arrival $\lambda_j(t)$ is uniformly distributed from 0 to 5 Wh/time-slot. For any base-station $j$ and user $i$, the amount of data that can be

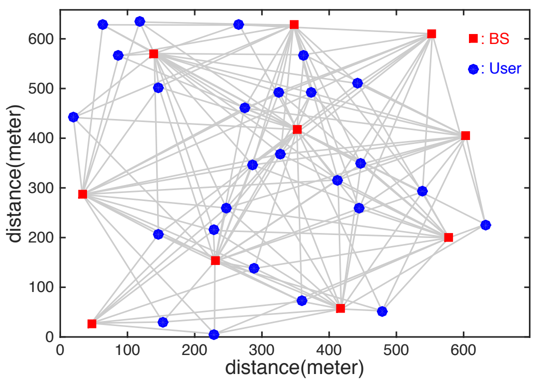



Fig. 3: Topology of the simulation scenario

TABLE I: Simulation Parameter Settings

| Parameters | Value[5] |
|---|---|
| a time-slot | 30 seconds |
| energy price | $(8.1 \sim 331.83) \times 10^{-4}$ cents/Wh |
| $w_i(t)$ | $3 \sim 15$ Mbit/time-slot |
| $E$ | $1/60$ Wh/resource-block |
| SNR[6] | 141 dB |
| $b_{\max}$ | 10 Wh/time-slot |
| $\lambda_j(t)$ | $0 \sim 5$ Wh/time-slot |
| $R_{\max}$ | 200 Kbit/resource-block |
| $R_{\min}$ | 6.67 Kbit/resource-block |
| $m$ | 3 for each base-station |

transmitted per resource block is calculated as[4]

$$R_{ji} = \begin{cases} R_{\max} & \text{if } 100\log\left(1+\text{SNR}d_{ji}^{-4}\right) > R_{\max}, \\ 0 & \text{if } 100\log\left(1+\text{SNR}d_{ji}^{-4}\right) < R_{\min}, \\ 100\log\left(1+\text{SNR}d_{ji}^{-4}\right) \text{ Kbit/resource-block} & \text{o/w.} \end{cases}$$

where $d_{ji}$ is the physical distance between base-station $j$ and user $i$, and the values for $R_{\min}$, $R_{\max}$ and SNR can be found in Table I. We carry out our simulation with the 5-minute market energy price data obtained at [10] for the period of 11/3/2014 to 11/20/2014 (450 hours), where we interpolate the 5-minute energy prices to obtain the 30-second energy prices. The values for the system parameters are summarized in Table I. Figure 4 shows the energy price fluctuation.

## B. Algorithms

We study the performance of three different schemes.

*(1) Fixed Association:* In this scheme, we choose a feasible user-association $A \in \mathcal{A}$ with the smallest $\sum_{i,j} A_{ji}H_{ji}$ and fix this user-association across the entire simulation. Only the energy-control component is activated.

*(2) Dynamic Association:* Both the user-association component and the energy-control component are activated.

*(3) Dynamic Association without traffic information:* From Problem P3, we know that to obtain the user-association decision *at the start of* a certain time-slot, we need to know in advance the amount of traffic arrivals for all the users *within* that time-slot, which can be hard to predict when the association interval is long. Thus, here we apply a sub-

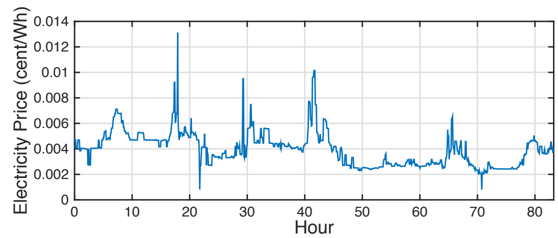



Fig. 4: Real-time Energy Price (11/3/2014-11/5/2014 in [10])

optimal version of the user-association algorithm, where the user-association decision is made assuming that each user has the same amount of traffic demand for the upcoming association interval.

For the baseline scheme, we apply the fixed user-association scheme without energy control, in which case the battery is only charged by renewable resources and energy is purchased from the grid only when the battery is empty.

## C. Performance Evaluation and Analysis

Figure 5a shows the percentage reduction in the energy-induced operational cost for the three schemes compared with the baseline, as a function of the battery capacity. At the battery capacity of 32 KWh[7], the Fixed Association scheme achieves a 23% reduction in cost, which demonstrates the effectiveness of the energy-control algorithm. After we activate the dynamic user-association component on top of the energy-control component, the cost reduction increases dramatically from 23% to 81%. Interestingly, even in the case when the traffic demand information is unknown, the dynamic user-association algorithm can still improve the cost reduction from 23% to 63%, cutting the operational cost by more than half. Further, from the figure we can see that the percentage reduction in cost is close to saturation when the battery is around 32 KWh, and that the improvement is quite significant even at a relatively small battery capacity.

The cost efficiency of the system can be captured by the metric of *cost per bit*. To understand better about where the cost-reduction in these different schemes comes from, it is helpful to break down the metric into two parts

$$\text{cost/bit} = (\text{cost/grid-energy}) \times (\text{grid-energy/bit}),$$

where the first term on the right-hand-side represents the average cost of purchasing energy from the grid, and the second term represents the average amount of energy purchased from grid for every unit of data-transmission. The reduction in operational cost is a combined effect of the reductions in both terms.

Let us focus on the first term. Since the energy price fluctuates over time, the *cost per grid-energy* is directly

[4]This expression of $R_{ji}$ implies that the base-station transmission power is set to be a fixed value for different users, an assumption that is in general not required in our model and analysis.

[5]Wh is a unit of energy equivalent to 1W of power expended for one hour.

[6]This is the normalized SNR when a user is 1-meter away from the associated base-station.

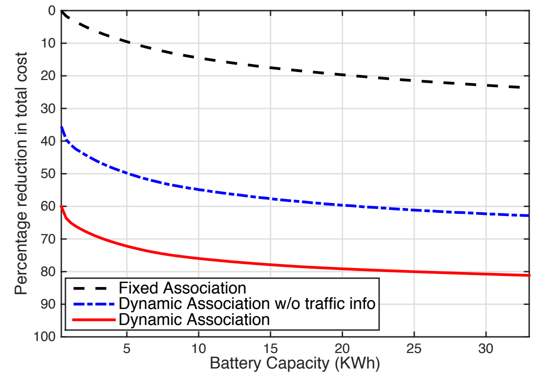
[7]The battery capacity in current green base-stations is around 10-32KWh [11].

determined by the average price at which the energy is purchased, which is in turn dictated by the energy control component. In our energy control algorithm, at any time-slot in any base-station, whether energy is purchased or not depends jointly on the current battery level and the current energy price. According to our discussion in Section IV, as we increase the battery capacity (i.e., increase $K$), the energy price will eventually become the dominating factor, meaning that the energy will be purchased only at the time when energy price is relatively low, which explains the 23% cost-reduction in the Fixed Association scheme.

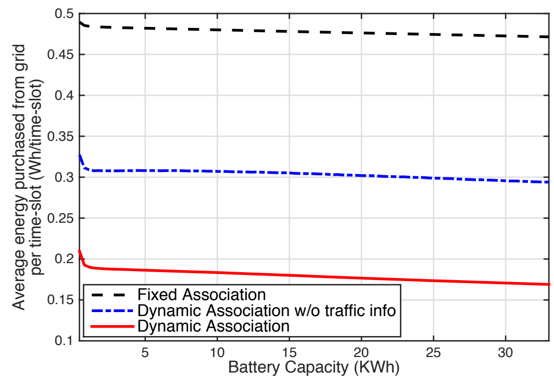
While the reduction in the first term brings down the cost by exploiting the varying energy prices, it is the reduction in the second term, *grid-energy per bit*, that determines how much carbon emission footprint is actually reduced. A smaller *grid-energy per bit* implies a more efficient use of the renewable energy in the network, which can be seen from the equation below

$$\text{grid-energy}/\text{bit} = (\text{energy}/\text{bit}) - (\text{renewable-energy}/\text{bit}).$$

In order to see how much cost-reduction is attributed to the reduction in grid-energy/bit for the dynamic association algorithms, we plot, in Figure 5b, the average energy purchased from the grid as a function of battery capacity. From Figure 5b we can see that the gap between the average purchased energy in the dynamic association scheme and that in the fixed association scheme matches with the gap between the cost-reductions for the two in Figure 5a. This is an evident that the proposed dynamic user-association algorithm indeed favors the alignment of the traffic-profile with the energy-profile in the network and results in an efficient use of renewable resources, which eventually translate to a significant reduction in the energy-related operational cost.




(a) Percentage reduction in operational cost as a function of battery capacity, compared with the baseline scheme.




(b) Average energy purchased from grid per time-slot as a function of battery capacity

Fig. 5: Performance v.s. Battery Capacity.

## VI. Conclusion

In this work, we look at cellular networks with green base-stations – the base-station that has renewable resources and energy storage device – with an objective to design joint energy/user-association control algorithms that minimize the energy-related operational cost of the systems. The proposed algorithm is proven to be able to achieve a cost arbitrarily close to the optimal cost by increasing the battery capacity. Through simulations we show that our energy-aware user-association algorithm, together with our battery control algorithm, significantly improves the utilization of the renewable resources and results in a dramatic reduction in the operational cost.

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