Consider the following integral for question 1, 2, 3
\[ \int_{10}^{42} e^{\cos(x/4)} \, dx \]

1. Approximate the integral by using the trapezoid rule with 1, 2, 4, and 8 equal width trapezoids. Use the recursive trapezoid formula. Show your work.

2. Apply Romberg’s algorithm to approximate the integral with 1, 2, 4, and 8 trapezoids to get the best possible approximation from these values. Show your work.

3. Approximate the integral by applying Simpson’s algorithm with 4 and 8 subintervals. Show your work.

4. Derive the Simpson’s \( \frac{3}{8} \) rule in the Newton-Cote formulas:
\[ \int_{x_0}^{x_3} f(x) \, dx = \frac{3}{8} h [f_0 + 3f_1 + 3f_2 + f_3] \]
where \( h = (x_3 - x_0)/3 \).

5. Show that the error term in the basic Simpson’s rule for \( \int_{a}^{b} f(x) \, dx \) is
\[ -\frac{1}{90} h^5 f^{(4)}(\varepsilon) \]
where \( h = (b - a)/2 \) and \( \varepsilon \) in (a, b). (Hint: read page 220 in the text).