CSE 541 Homework 4
Due: May 14 Friday in class

1. (a) Use the central difference formula with $h = 1.0$ to numerically approximate the derivative of $\cos(\sin(x))$ at $x = 2$;
   (b) Use Richardson’s extrapolation to approximate the derivative of $\cos(\sin(x))$ at $x = 2$ starting with $h = 1.0$ and using four rows in the Richardson extrapolation table (i.e. compute up to $D(3,3)$). Give the complete table.
   (c) Compare with the exact value given by analytically differentiating $\cos(\sin(x))$.

2. (a) Use a single formula to numerically approximate the second derivative of $\cos(\sin(x))$ (without any analytical differentiation) at $x = 2.0$ using $h = 1.0$;
   (b) Use Richardson’s extrapolation to approximate the second derivative of $\cos(\sin(x))$ at $x = 2$ starting with $h = 1.0$ and using two rows in the Richardson extrapolation table (i.e., computing up to $D(1,1)$). Give the complete table.
   (c) Analytically derive the second derivative and compare the value at $x = 2.0$.

3. Derive the approximation formula
   $$ f'(x) \approx \frac{1}{2h} [4f(x + h) - 3f(x) - f(x + 2h)] $$
   and show that its error term is $O(h^2)$. (Use Taylor series expansion for $f(x+h)$ and $f(x+2h)$.)

4. Assume that we are only given $f(x+h)$, $f(x)$, and $f(x-2h)$, and wish to approximate $f''(x)$. Prove that
   $$ f''(x) \approx \frac{1}{3h^2} [2f(x + h) - 3f(x) + f(x - 2h)] $$
   (Use the Taylor series expansion for $f(x+h)$ and $f(x-2h)$.)