## Brief Overview

Also see document from Education Board of ACM SIGPLAN ("Motivation" on web page)

## Main Questions in PL

Q1: Is this a valid program?
Compile-time and run-time checking (in 6341: attribute grammars and type systems)
Q2: What is this program supposed to do?
Precise language semantics (in 6341: operational semantics)
Q3: How do we execute this program correctly and efficiently?

Implementation of compilers and interpreters (in 6341: projects to build an interpreter; attribute grammars for code generation in a compiler; static analysis for performance optimization)

## Why Study Foundations of PL?

Understand your tools better
Compilers, interpreters, virtual machines, code checking tools, debuggers, assemblers, linkers
Write your own languages, compilers, analyzers, ... Happens more often than you'd think [example: Forma]
To fix bugs \& make programs fast, often you need to understand what's happening "under the hood"

Most importantly: PLs are the foundations of software; we need to be clear on what they mean and how to support their users with useful tools

## Example: Inside a Compiler



## Attribute Grammars

## Pagan Ch. 2.1, 2.2, 2.3, 3.2

Slonneger and Kurtz Ch 3.1, 3.2 [online; under Resources on course web page]

Dragon Book Ch. 5.1, 5.2

## Outline

Review context-free grammars [expected background] Introduce attribute grammars
Use scenario: simple type checking
Two flavors of attribute grammars: (1) pure and (2) with limited side effects

Use scenario: more complex type checking
Use scenario: generation of assembly code

## Formal Languages

Theoretical basis for the design and implementation of programming languages
Alphabet: finite set $\mathbf{T}$ of symbols
String: finite sequence of symbols
Empty string $\varepsilon$ (i.e., sequence of length 0 )
$\mathbf{T}^{*}$ - set of all strings over $\mathbf{T}$ (incl. $\varepsilon$ )
$\mathrm{T}^{+}$- set of all non-empty strings over $\mathbf{T}$
Language: set of strings $\mathrm{L} \subseteq \mathrm{T}^{*}$

## Grammars

$\mathbf{G}=(\mathbf{N}, \mathrm{T}, \mathrm{S}, \mathrm{P})$
Finite set of non-terminal symbols $\mathbf{N}$
Finite set of terminal symbols $\mathbf{T}$ (this is our alphabet)
Starting non-terminal symbol $\mathbf{S} \in \mathbf{N}$
Finite set of productions $\mathbf{P}$
Goal: define a language $\mathbf{L} \subseteq \mathbf{T}^{*}$
Production: $\mathbf{x} \rightarrow \mathbf{y}$
$\mathbf{x}$ : non-empty sequence of terminals and non-terminals
$\mathbf{y}$ : possibly-empty sequence of terminals/non-terminals
Applying a production: uxv $\Rightarrow$ uyv

## Languages and Grammars

Derivation of a string

$$
\mathbf{w}_{1} \Rightarrow \mathbf{w}_{2} \Rightarrow \ldots \Rightarrow \mathbf{w}_{\mathrm{n}} ; \text { denoted } \mathbf{w}_{1} \stackrel{*}{\Rightarrow} \mathbf{w}_{\mathbf{n}}
$$

Language generated by a grammar

$$
L(G)=\left\{w \in T^{*} \mid S \stackrel{*}{\Rightarrow} w\right\}
$$

Traditional classification of languages and grammars Regular $\subset$ Context-free $\subset$ Context-sensitive $\subset$ Unrestricted

## Use in Compilers and Interpreters

stream of characters
w,h,i,l,e,(,a,1,5,>,b,b, ),d,o,...

## Lexical Analyzer (uses a regular grammar)

stream of tokens id[bb],rightparen,keyword[do], ...
keyword[while],leftparen,id[a15],op[>],

## Parser (uses a context-free grammar)

parse
each token is a leaf in the parse tree tree
... more components

## Context-Free Languages

Strict superset of regular languages
Example: $L=\left\{a^{n} b^{n} \mid n>0\right\}$ is context-free but not regular
Generated by a context-free grammar
Each production: A $\rightarrow \mathbf{w}$
$\mathbf{A}$ is a non-terminal, $\mathbf{w}$ is a (possibly empty) sequence of terminals and non-terminals
BNF: alternative notation for context-free grammars
Backus-Naur form: John Backus and Peter Naur, for ALGOL60 (both have received the ACM Turing Award)

## BNF Example (related to the language for the project)

<program> ::= <stmtList>
<stmtList> ::= <stmt> <stmtList>
| <stmt>
<stmt> ::= <varDecl> = <expr> ;
| ident = <expr> ;
<varDecl> ::= int ident
<expr> ::= intconst
| ident
| <expr> + <expr>
If there are several productions
<X> ::= ...
for convenience we write them as a single production
<X> ::= ... | ... | ...
We say "the $i^{\text {th }}$ production alternative"

## String Derivation

## Example of a string from the language

[next slide shows the leftmost derivation sequence (always expands the leftmost non-terminal)] [try this at home: the rightmost derivation sequence (always expands the rightmost non-terminal)] int $x=1 ; y=x+2 ;$

## Example of a string not from the language

$\mathbf{x + 1}=\mathbf{y} ;$

```
<program> ::= <stmtList>
<stmtList> ::= <stmt> <stmtList> | <stmt>
<stmt> ::= <varDecl> = <expr> ; | ident = <expr> ;
<varDecl> ::= int ident
<expr> ::= intconst | ident | <expr> + <expr>
int x = 1 ; y = x + 2 ;
<program> => <stmtList> =>
<stmt> <stmtList> =>
<varDecl> = <expr> ; <stmtList> =>
int ident }\mp@subsup{}{\textrm{x}}{}=<<<expr> ; <stmtList> =>
int ident }\mp@subsup{}{\textrm{x}}{}=\mp@subsup{\mathrm{ intconst }}{1}{}; <stmtList> =>
int ident }\mp@subsup{}{\textrm{x}}{}=\mp@subsup{\mathrm{ intconst }}{1}{}; <stmt> =>
int ident }\mp@subsup{}{\textrm{x}}{}=\mp@subsup{\mathrm{ intconst }}{1}{};\mp@subsup{\mathrm{ ident }}{\textrm{y}}{=}=<expr> ; =>
int ident }\mp@subsup{}{\textrm{x}}{}=\mp@subsup{\mathrm{ intconst }}{1}{};\mp@subsup{\mathrm{ ident }}{\textrm{y}}{}=<\mathrm{ <expr> + <expr> ; =>
int ident }\mp@subsup{}{\textrm{x}}{}=\mp@subsup{\mathrm{ intconst }}{1}{}\mathrm{ ; ident }\mp@subsup{}{\textrm{y}}{\prime}=\mp@subsup{\mathrm{ ident }}{\textrm{x}}{}+<<expr> ; =>
int ident }\mp@subsup{}{x}{}=\mp@subsup{\mathrm{ intconst }}{1}{};\mp@subsup{\mathrm{ ident }}{\textrm{y}}{}=\mp@subsup{\mathrm{ ident }}{\textrm{x}}{}+\mp@subsup{\mathrm{ intconst }}{2}{}

\section*{Parse Tree}

Also called derivation tree or concrete syntax tree Leaf nodes: terminals
Inner nodes: non-terminals
Root: starting non-terminal of the grammar Leaf nodes, from left to right, define the string

Each non-leaf node \(X\) has children that correspond to some production <X> ::= ...

Children are ordered as they appear in the production

\section*{Parse Tree Examples}

Example 1: int \(\mathbf{x}=\mathbf{1}\); int \(\mathbf{y}=\mathbf{x}+\mathbf{2}\);
Example 2: int \(\mathbf{y}=\mathbf{x}+\mathbf{w}\); int \(\mathbf{x}=\mathbf{y}\);
Example 3: \(\mathbf{w}=\mathbf{x}+\mathbf{y}+\mathbf{z}\);
```

int $y=x+w ;$ int $x=y ;$

```

\section*{<program> \\ I \\ <stmtList>}

```

<program> ::= <stmtList>
<stmtList> ::= <stmt> <stmtList> | <stmt>
<stmt> ::= <varDecl> = <expr> ; | ident = <expr> ;
<varDecl> ::= int ident
<expr> ::= intconst | ident | <expr> + <expr>

```
\[
w=x+y+z ;
\]


\section*{Ambiguous Grammar}

For some string, there are multiple parse trees

\section*{An ambiguous grammar}

Gives more freedom to the compiler writer: e.g., for code optimizations (several possible translations) Allows under-specification of irrelevant details [we will see this later when we discuss operational semantics and abstract interpretation] Must be disambiguated when we build a real parser

\section*{To remove ambiguity}

Change the grammar, or
Keep it ambiguous, but tell the parser how to resolve ambiguity so that we have only one possible parse tree [this is the approach used in the programming projects]

\section*{Classic Examples of Ambiguity}
<expr> ::= <expr> + <expr> | ident

Two different parse trees for \(\mathbf{x + y + z}\)
<expr> ::= <expr> + <expr> | <expr> * <expr> | ident

Two different parse trees for \(\mathbf{x + y}\) * \(\mathbf{z}\)
We will illustrate the importance of ambiguity in one specific scenario: associativity and precedence of binary operators in programming languages

\section*{Binary Operators in Math}

Commutativity: \(a\) op \(b=b\) op \(a\)
Example: + is commutative and - is not commutative Associativity [same op]: \((a\) op \(b)\) op \(c=a \mathbf{o p}(b\) op \(c)\)
+ is associative; we can write \(a+b+c\) since the location of parentheses does not matter
- is not associative; for convenience, we write \(a-b-c\)
to mean \((a-b)-c\) [i.e., left-to-right reading of \(a-b-c\) ]
Precedence [two different ops]: \(a \mathbf{o p}_{1} b \mathbf{o p}_{2} c\)
Does it mean \(\left(a \mathbf{o p}_{1} b\right) \mathbf{o p}_{2} c\) or is it \(a \mathbf{o p}_{1}\left(b \mathbf{o p}_{2} c\right)\) ?
If the precedence is defined, we can omit parentheses
+ vs \(\times\) : higher precedence for \(\times\), so \(a+b \times c\) is \(a+(b \times c)\)

\section*{Operator Associativity in PL}

Associativity [same op]: how should \(a\) op \(b\) op \(c\) be evaluated when we execute the program?

Left-associative operator: \(a\) op \(b\) op \(c\) should be evaluated as "first compute the value of the left subexpression \(a\) op \(b\); then compute the result op \(c^{\prime \prime}\) [i.e., treat it as ( \(a\) op \(b\) ) op \(c\) ]
Right-associative operator: \(a\) op \(b\) op \(c\) should be evaluated as "first compute the value of the right subexpression \(b\) op \(c\); then compute \(a\) op the result" [i.e., treat it as a op (b op c)]

\section*{Why Does Ambiguity Matter?}
<expr> ::= <expr> + <expr> | ident
\(\mathbf{w}=\mathbf{x}+\mathbf{y}+\mathbf{z} ;\)

Parse tree version 1 leads to assembly code version 1
ADD R1, \(\mathbf{x}, \mathrm{y}\)
ADD R2, R1, z
STORE w, R2

Parse tree version 2 leads to assembly code version 2
ADD R1, y, z
ADD R2, x, R1
STORE w, R2

Left or right associativity? Same as asking "how should \(\mathbf{x + y + z}\) be parsed"? It does matter ...
// non-associative math operations int \(p=1-2-3\);
// floating-point computations
double \(x=(0.1+0.2)+0.3\);
double \(y=0.1+(0.2+0.3)\);
\({ }_{23}\) System.out.print \(\ln (x==y)\); // what will be printed here?

SUB R1, 1, 2
SUB R2, R1, 3 vs
STORE \(p\), R2

SUB R1, 2, 3
SUB R2, 1, R1 STORE \(p\), R2

\section*{Why Does Ambiguity Matter?}
<expr> ::= <expr> + <expr> | <expr>* <expr> | ident
\(\mathbf{w}=\mathbf{x}+\mathbf{y}\) * \(\mathbf{z ;}\)
Assembly code version 1 Assembly code version 2
ADD R1, x, y
MUL R2, R1, \(z\)
STORE w, R2
MUL R1, \(\mathbf{y}, \mathbf{z}\)
ADD R2, \(x\), R1
STORE w, R2
Precedence: how should \(\mathbf{x}+\mathbf{y}^{*} \mathbf{z}\) be parsed? The shape of the parse tree matters ...

Exercise: how many different parse trees are possible for \(\mathbf{x} * \mathbf{y}+\mathbf{z}^{*} \mathbf{w}\)

\section*{In C++}
\begin{tabular}{|c|c|c|c|}
\hline Level & Operators & Description & Associativity \\
\hline 15 & \[
\begin{gathered}
0 \\
\text { [ } \\
\text {-> } \\
++ \\
++
\end{gathered}
\] & \begin{tabular}{l}
Function Call \\
Array Subscript \\
Member Selectors \\
Postfix Increment/Decrement
\end{tabular} & Left to Right \\
\hline 14 & \[
\begin{gathered}
\hline++ \\
+- \\
!\sim \\
(\text { type) } \\
* \\
\& \\
\text { \& } \\
\text { sizeof }
\end{gathered}
\] & \begin{tabular}{l}
Prefix Increment / Decrement \\
Unary plus / minus \\
Logical negation / bitwise complement \\
Casting \\
Dereferencing \\
Address of \\
Find size in bytes
\end{tabular} & Right to Left \\
\hline 13 & \[
\begin{aligned}
& * \\
& \prime \\
& \% \\
& \%
\end{aligned}
\] & Multiplication Division Modulo & Left to Right \\
\hline 12 & +- & Addition / Subtraction & Left to Right \\
\hline 11 & < & \begin{tabular}{l}
Bitwise Right Shift \\
Bitwise Left Shift
\end{tabular} & Left to Right \\
\hline 10 & \[
\begin{aligned}
& \text { \ll= } \\
& \ggg=
\end{aligned}
\] & Relational Less Than / Less than Equal To Relational Greater / Greater than Equal To & Left to Right \\
\hline 9 & \[
\begin{aligned}
& = \\
& \text { != }
\end{aligned}
\] & Equality Inequality & Left to Right \\
\hline 8 & \& & Bitwise AND & Left to Right \\
\hline 7 & \(\wedge\) & Bitwise XOR & Left to Right \\
\hline 6 & | & Bitwise OR & Left to Right \\
\hline 5 & \& \& & Logical AND & Left to Right \\
\hline 4 & II & Logical OR & Left to Right \\
\hline 3 & ?: & Conditional Operator & Right to Left \\
\hline 2 & \[
\begin{gathered}
= \\
+=-= \\
*=1=\%= \\
\&=A=1= \\
\ll=\gg=
\end{gathered}
\] & Assignment Operators & Right to Left \\
\hline 1 & , & Comma Operator & Left to Right \\
\hline
\end{tabular}

\section*{Elimination of Ambiguity}
<expr> ::= <expr> + <expr> | <expr> * <expr> | ident
Simple solution: change the language to force the programmer to write all parentheses
<expr> ::= ( <expr> + <expr> ) | ( <expr> * <expr> ) | ident
Exercise: convince yourself that this grammar is not ambiguous

Problem: too much work for the programmer - e.g., cannot just write \(\mathbf{x + y + z}\) but must write ( \((\mathbf{x}+\mathbf{y})+\mathbf{z})\)

Better solution: let's not force the programmer to write all these ( and ), but rather change the grammar accordingly

\section*{Elimination of Ambiguity}
<expr> ::= <expr> + <expr> | <expr> * <expr> | ident | const
Note: added const to make the grammar more interesting

Goal: Create an equivalent non-ambiguous grammar with the appropriate precedence and associativity:
* has higher precedence than +
both are left-associative
Solution: two new non-terminals
<expr> ::= <expr> + <term> | <term>
<term> ::= <term>* <factor> | <factor>
<factor> ::= ident | const
Exercise: construct parse trees for \(\mathbf{x +} \mathbf{y + z}\) and \(\mathbf{x}+\mathbf{y}\) * \(\mathbf{z}\) and imagine what the generated assembly code may look like

\section*{Adding Parentheses}

Goal: extend the language to allow for parenthesized subexpressions - e.g., \(x\) * \((y+z)\)

Solution:
<expr> ::= <expr> + <term> | <term>
<term> ::= <term> * <factor> | <factor>
<factor> ::= ident | const | ( <expr> )
Exercise: construct the parse tree for \(\mathbf{x}\) * \(\mathbf{y + z})\) and imagine what the generated assembly code may look like

Exercise 2: look at the grammar definition in the CUP file for Project 1, convince yourself it is ambiguous, and see the extra "hints" to the parser about precedence and associativity (to resolve ambiguity)

\section*{Abstract Syntax Trees (AST)}

A simplified version of a concrete syntax tree, without loss of information [we will use ASTs in the programming projects] <funcDef> ::= ident (<formalDeclList> ) \{ <stmtList> \}

parse tree


\section*{Use of Context-Free Grammars}

Syntax of a programming language
Java: Chapter 19 of the language specification (JLS)
defines a grammar [under Resources on the web page]
Terminals: identifiers, keywords, literals, separators, operators
Starting non-terminal: CompilationUnit
Implementation of a parser in a compiler
e.g. the JLS grammar (Ch. 19) is used by the parser inside the javac compiler

\section*{Limitations of Context-Free Grammars}

\section*{Cannot represent semantics}

Example: "every variable used in a statement should be declared earlier in the code" or "the use of a variable should conform to its type declaration" (type checking) Need to allow only programs that satisfy certain context-sensitive conditions
An example of a context: "an earlier declaration of \(\mathbf{x}\) must exist, and it must declare an int type"
Cannot generate things other than parse trees
Example: what if we wanted to generate assembly code for the given program?

\section*{Attribute Grammars}

Generalization of context-free grammars
Used for semantic checking and other compile-time analyses
e.g. type checking in a compiler

Used for translation
e.g. parse tree \(\rightarrow\) assembly code

Implicitly represents a traversal of the parse tree and the computation of information during traversal

\section*{Structure of an Attribute Grammar}
1. Underlying context-free grammar
2. For a terminal or non-terminal: some attributes
3. For each attribute: type of its possible values e.g., integer or string or map(string \(\rightarrow\) list(integer))
4. Set of evaluation rules for each production
5. Set of boolean conditions for attribute values

\section*{Example}
\(L=\left\{a^{n} b^{n} c^{n} \mid n>0\right\}\); not a context-free language BNF
\[
\begin{array}{ll}
<\text { start> }::=<A><B><C> & <A>::=a \mid a<A> \\
\langle B\rangle::=b \mid b<B> & \langle C>::=c| c<C>
\end{array}
\]

\section*{Attributes}

Na : associated with <A>
Nb : associated with <B>
Nc: associated with <C>
Type of possible values for \(\mathrm{Na}, \mathrm{Nb}, \mathrm{Nc}\) : integer values

\section*{Example}

Evaluation rules (similar for \(\langle\mathrm{B}\rangle,<\mathrm{C}\rangle\) )
\[
\begin{aligned}
&<A>::=\mathrm{a} \\
&<A>. N a:=1 \\
& \mid \quad \mathrm{a}<A>_{2} \\
&<A>. N a:=1+\langle A\rangle_{2} . N a
\end{aligned}
\]

\section*{Conditions}
<start> ::= <A><B><C>
Cond: [ \(\langle A\rangle . N a=\langle B\rangle . N b=\langle C\rangle . N c]\)
a string belongs to the language defined by this attribute grammar if and only if the parse tree satisfies the condition

\section*{Parse Tree}


\section*{Parse Tree for an Attribute Grammar}

Valid tree for the underlying BNF
Each node has (attribute,value) pairs
One pair for each attribute associated with the node
Some nodes have boolean conditions
If there is a corresponding Cond: ... rule
Valid parse tree
Attribute values are consistent with the evaluation rules
All boolean conditions are true

\section*{Modified Example}

Same evaluation rules as before e.g.
\[
<A>::=\mathbf{a}
\]
\[
\begin{aligned}
& <A>. N a:=1 \\
& \left\lvert\, \begin{array}{l}
\mathrm{a}<A>_{2} \\
<A>. N a:=1+<A>_{2} \cdot N a
\end{array}\right.
\end{aligned}
\]

Different conditions
<start> ::= <A><B><C>
Cond: [ \(\langle A\rangle . N a=3]\)
Cond: [ \(\langle A\rangle . N a><B>. N b]\)
Cond: [ <B>.Nb > <C>.Nc]

How many valid parse trees exist for this attribute grammar?

\section*{Comments}

If non-terminal \(X\) has an attribute \(A\), each occurrence of \(X\) in the parse tree must have a value for \(A\). The evaluation rules should define exactly one value for \(A\) for a particular \(X\) node.

Attributes are not like program variables; cannot have: \(<Z>\). \(A:=1+<Z>\). \(A\)
In rules/conditions, can only refer to attributes of non-terminals and terminals in the current production alternative

Cannot look at "grandparent"/"grandchild" parse tree nodes, or even further away up/down the tree

Synthesized vs. Inherited Attributes
Each non-terminal X: disjoint sets of synthesized attributes and inherited attributes

An attribute \(A\) for \(X\) cannot be both
For each synthesized attribute A: each production alternative in X ::= ... should have exactly one evaluation rule for X.A

For each inherited attribute A: each occurrence of X in ... ::= ... X ... X ... X ... should have exactly one evaluation rule for X.A

Synthesized attributes convey information about the subtree rooted at the node

Inherited attributes convey context conditions
E.g., information about variable declarations that have appeared earlier in the program
The starting non-terminal does not have inherited attributes
For convenience: assume each terminal symbol has one attribute lexval with a pre-defined value The lexical analyzer sets these values (e.g., some int value for a token representing an integer constant)

\section*{Example (revisited)}
\[
\begin{aligned}
<\text { start> } & ::=\langle A><B><C> \\
& <B>. \operatorname{expectedNb}:=<A>. N a \\
& <C>. \operatorname{expectedN}:=<A>. N a
\end{aligned}
\]
\[
<A>::=\mathbf{a}
\]
\[
<A>. N a:=1
\]
\[
\mid a<A>_{2}
\]
\[
\langle A\rangle . \mathrm{Na}:=1+\langle A\rangle_{2} \cdot \mathrm{Na}
\]
\[
<B>::=\mathbf{b}
\]

Cond: [ \(\langle B\rangle\).expectedNb \(=1\) ] | \(b<B>_{2}\)
\(\langle B\rangle_{2}\). expected \(N b:=<B>\).expected \(N b-1\)
Na is synthesized, expected \(\mathrm{Nb} / \mathrm{Nc}\) are inherited

\section*{Example: Binary Numbers}

Context-free grammar
\[
\begin{aligned}
& <B>::=<D> \\
& <B>::=<D><B> \\
& <D>::=0 \\
& <D>::=1
\end{aligned}
\]

Goal: compute the value of the binary number Needed, for example, in compilers during code translation

\section*{BNF Parse Tree for Input 1010}


\section*{Example: Binary Numbers}
<B> ::= <D>
\[
\begin{aligned}
& <B>. p o s:=1 \\
& <B>. v a l:=<D>. v a l \\
& <D>. \text { pow }:=0
\end{aligned}
\]
\(\langle B\rangle_{1}::=\langle D\rangle\langle B\rangle_{2}\)
\[
\begin{aligned}
& \langle B\rangle_{1} \cdot \text { pos }:=\langle B\rangle_{2} \cdot \text { pos }+1 \\
& \langle B\rangle_{1} \cdot v a l:=\langle B\rangle_{2} \cdot v a l+\langle D>\cdot v a l \\
& \left\langle D>\text {.pow }:=\langle B\rangle_{2} \cdot\right. \text { pos }
\end{aligned}
\]
<D> ::= 0
\[
<D>. v a l:=0
\]
<D> ::= 1
\[
<D>. v a l:=2^{<D>. p o w}
\]

\section*{Evaluated Parse Tree}


\section*{Complex Evaluation Rules}
\(\langle X\rangle\).A := ... could be rather complex - e.g. with helper functions, conditional expressions, etc.
Example:
\(\langle X\rangle . A\) := if (<Y>.B = <Z>.C) then f1(<Y>.D) else f2(<Z>.E)
Must be if-then-else; cannot be if-then. Why?
Helper functions such as f1 and f2 can use basic algorithms and data structures/operations

Can only use attributes of non-terminals and terminals that appear in this production alternative

\section*{Attribute Evaluation: Dependence Graph}
\(\langle X\rangle . A:=\langle Y\rangle . B+\langle Z\rangle . C\)
Since the value of \(\langle X\rangle\).A depends on \(\langle Y>\). \(B\) : Y.B \(\rightarrow\) X.A dependence edge

Since the value of \(\langle X>\).A depends on \(\langle Z\rangle\). \(C\) :
Z.C \(\rightarrow\) X.A dependence edge
\(\langle X\rangle_{1} \cdot A:=\langle X\rangle_{2} \cdot A \quad\) two different \(X\) nodes in the parse tree Since the value of \(\langle X\rangle_{1}\).A depends on \(\langle X\rangle_{2}\).A: \(\mathrm{X}_{2}\). \(\mathrm{A} \rightarrow \mathrm{X}_{1}\).A dependence edge

\section*{Algorithm for Attribute Evaluation}

Given a parse tree with attributes attached to tree nodes, how do we compute the attribute values?

Step 1: find evaluation order of attributes
a) Build dependence graph where a node is a pair (parse tree node, attribute)
b) Complain about cycles in the graph: cannot evaluate
c) Topologically sort the graph

Step 2: evaluate the attributes in sorted order

\section*{Example: Binary Numbers}
<B> ::= <D>
\[
\begin{aligned}
& <B>. p o s:=1 \\
& <B>. v a l:=<D>. v a l \\
& <D>. \text { pow }:=0
\end{aligned}
\]
\(\langle B\rangle_{1}::=\langle D\rangle\langle B\rangle_{2}\)
\[
\begin{aligned}
& \langle B\rangle_{1} \cdot \text { pos }:=\langle B\rangle_{2} \cdot \text { pos }+1 \\
& \langle B\rangle_{1} \cdot v a l:=\langle B\rangle_{2} \cdot v a l+\langle D>\cdot v a l \\
& \left\langle D>\text {.pow }:=\langle B\rangle_{2} \cdot\right. \text { pos }
\end{aligned}
\]
<D> ::= 0
\[
<D>. v a l:=0
\]
<D> ::= 1
\[
<D>. v a l:=2^{<D>. p o w}
\]

\section*{Dependence Graph for Binary Numbers}


\section*{Sort the Graph}

Topological sort: \(\mathbf{x}\) is "smaller" than \(\mathbf{y}\) iff \(\mathbf{x} \rightarrow \mathbf{y}\)

D4.pow, B4.pos, D4.val, B4.val,


\section*{Cycles}

The notion of "topological sort" only makes sense for directed acyclic graphs

Cycles in the dependence graph means we have recursive dependencies

In general, there are approaches to solve meaningful recursive systems of equations

But, in this course we will disallow cycles
No cyclic dependencies in exams and homeworks

\section*{Use Scenario 1: Simple Type Checking}
<program> ::= <stmtList>
<stmtList> ::= <stmt> <stmtList> | <stmt>
<stmt> ::= <varDecl> = <expr> ;
| ident = <expr> ;
<varDecl> ::= int ident | float ident
<expr> ::= intconst | floatconst | ident
\[
\begin{aligned}
& \mid<\text { expr }>+ \text { <expr }>\quad \begin{array}{l}
\text { [grammar is ambiguous; assume the parser } \\
\text { resolved this somehow] }
\end{array} \\
& \mid(\text { <expr }>)
\end{aligned}
\]

\section*{Type Checking: Simple Examples}

Example 1:
int \(y=x+w ;\) int \(x=y ; \quad\) vs int \(y=5+3\); int \(x=y ;\)
Discussed in the next few slides
Example 2: for practice
float \(x=5.0\); float \(y=x+1.0\); int \(z=x+y ;\)
Will this type check in Java?
Should it type check in our language? It's up to us. We will choose "No".
int \(y=x+w ;\) int \(x=y ;\)
<program>
<stmtList>tbl: \(\}\)


Note: not showing some of the "uninteresting" terminal symbols, to simplify the picture (they are still in the tree)

\section*{Our Type Checking Goals}

Goal 1: Any variable in an <expr> must have a corresponding declaration in an earlier <stmt> Example: do not allow int \(\mathbf{x}=1\); int \(\mathbf{y}=\mathbf{x}+\mathbf{w}\); Example: do not allow int \(\mathbf{x}=\mathbf{x + 1}\);
Note: in the programming project will also check that no variable is declared more than once; in class we will not discuss this check, but you should think how the solution should be changed to perform such checking

Goal 2: Both operands of + must be of the same type Example: do not allow int \(\mathbf{x}=1\); float \(\mathbf{y}=\mathbf{x}+3.14\);

\section*{Attributes for Type Checking Solution} Inherited attribute tbl (short for "symbol table"). The attribute is a map from strings to INT/FLOAT. Each <stmtList>, <stmt>, and <expr> has its tbl.

Synthesized attribute type for <expr>: INT/FLOAT When the <expr> is an ident (just a variable name), need to look inside <expr>.tbl to figure out if the variable was already declared and with what type

\section*{Type Checking: Expressions}
<expr> ::= intconst
| floatconst <expr>.type := FLOAT
| ident
<expr>.type := INT

Cond: [ ident.lexval has a type in <expr>.tbl ]
<expr>.type := <expr>.tbl.lookupld(ident.lexval)
\[
\mid\left(\text { <expr> }{ }_{2}\right)
\]
<expr> \({ }_{2} . t b l:=\) expr>.tbl.clone() Copies the entire table <expr>.type := <expr> \({ }_{2}\).type

\section*{Type Checking: Expressions}
<expr> ::= <expr> \({ }_{2}+\) <expr \(_{3}\)
\[
\begin{aligned}
& \text { <expr> }{ }_{2} . t b l:=\text { eexpr>.tbl.clone() } \\
& \text { <expr> }{ }_{3} . t b l:=\text { eexpr>.tbl.clone() } \\
& \text { Cond: [ <expr> } \left.{ }_{2} . \text { type }=\text { <expr> }_{3} \cdot \text {.type }\right] \\
& \text { <expr>.type }:=\text { <expr> }{ }_{2} \text {.type }
\end{aligned}
\]

Note: this would disallow code such as int \(\mathbf{x}=1\); float \(\mathbf{y}=\mathbf{x + 3 . 1 4 ;}\)
int \(y=5+3 ;\) int \(x=y ;\)
<program>
<stmtList> tbl: \{ \}

int \(y=x+w ;\) int \(x=y ;\)
<program>
<stmtList> tbl: \{ \}


\section*{Attributes for Type Checking Solution} Inherited attribute tbl (short for "symbol table"). The attribute is a map from strings to INT/FLOAT. Each <stmtList>, <stmt>, and <expr> has its tbl.

Synthesized attribute type for <expr>: INT/FLOAT When the <expr> is an ident (just a variable name), need to look inside <expr>.tbl to figure out if the variable was already declared and with what type

Synthesized attribute decl for <varDecl> and <stmt>: a set containing zero or one pair (string,INT/FLOAT)
int \(y=x+w ;\) int \(x=y ;\)
<program>
<stmtList> tbl: \{ \}
<stmt> tbl: \(\}\) decl: \(\{(y, I N T)\}\)
<varDecl> decl: \(\{(y, I N T)\}\) <expr> tbl: \(\}\) type:
int ident \({ }_{y}\)

<stmtList> tbl: \(\{y \rightarrow\) INT 1
<stmt> tbl: \(\{y \rightarrow\) INT \(\}\)


Note: not showing decl in this part of the tree (but it is there)

\section*{Type Checking: Symbol Tables}
<program> ::= <stmtList>
<stmtList>.tbl := newTable() emptytable
<stmtList> ::= <stmt> <stmtList> \({ }_{2}\)
<stmt>.tbl := <stmtList>.tbl.clone()
<stmtList> \({ }_{2}\).tbl := <stmtList>.tbl.clone(<stmt>.decl)
Creates a copy of <stmtList>.tbl and adds to it <stmt>.decl
| <stmt>
<stmt>.tbl := <stmtList>.tbl.clone()

\section*{Type Checking: Symbol Tables}
<varDecl> ::= int ident
<varDecl>.decl := newSet(ident.lexval,INT) Set with one element:
a pair (string,INT)
| float ident similary here
<stmt> ::= <varDecl> = <expr> ;

> <stmt>.decl := <varDecl>.decl.clone()
> <expr>.tbl \(:=\) <stmt>.tbl.clone()
> | ident \(=\) <expr> ;
<stmt>.decl := newSet() empty set
<expr>.tbl := <stmt>.tbl.clone()

\section*{Type Checking: Assignments}

Goal 1: Any variable in an <expr> must have a corresponding declaration in an earlier <stmt>

Example: do not allow int \(\mathbf{x}=1\); int \(\mathbf{y}=\mathbf{x}+\mathbf{w}\);
Example: do not allow int \(x=x+1\);

Goal 2: Both operands of + must be of the same type Example: do not allow int \(\mathrm{x}=1\); float \(\mathrm{y}=\mathrm{x}+3.14\);

Goal 3: Both sides of an assignment must be of the same type

Example: do not allow int \(\mathbf{x}=\mathbf{1}\); float \(\mathbf{y}=\mathbf{x}\);

\section*{Type Checking: Assignments}
<stmt> ::= <varDecl> = <expr> ;
Cond: [ <expr>.type = type in <varDecl>.decl]
| ident = <expr> ;
Cond: [ ident.lexval has a type in <stmt>.tbl ]
Cond: [ <expr>.type = <stmt>.tbl.lookupld(ident.lexval) ]

\section*{Example}

Consider again Example 1: int \(\mathbf{y}=\mathbf{x}+\mathbf{w}\); int \(\mathbf{x}=\mathbf{y}\); vs int \(\mathbf{y}=\mathbf{5} \mathbf{+ 3}\); int \(\mathbf{x}=\mathbf{y}\); Already saw parse tree and attributes tbl, type, and decl Where in the tree do the type checks occur?
```

<expr> ::= indent
<expr> ::= <expr> }\mp@subsup{2}{2}{+}<\mp@subsup{e\operatorname{expr>}}{3}{}\quad\mathrm{ Cond: [<expr>}\mp@subsup{2}{2}{\prime}.type = <expr> >3.type ]
<stmt> ::= <varDecl> = <expr> ;
<stmt> ::= ident = <expr> ;
Cond: [ ident.lexval has a type in <expr>.tbl]
Cond: [ <expr>.type = type in <varDecl>.decl ]
Cond: [ ident.lexval has a type in <stmt>.tbl ]
Cond:[<expr>.type=<stmt>.tbl.lookupId(ident.lexval)]

```

\section*{Efficiency Of Type Checking}

Inherited attribute tbl: each <stmtList>, <stmt>, and <expr> has its own table, which is inefficient

Consider a list of \(n\) variable declarations. What is the total size of all tbl attributes?

Let's just have one single "global" table
Advantage: more efficient use of space; no need for clone() operations
Disadvantage: need to be very careful in which order attributes are evaluated and how this affects the table

Modified solution: at each <stmtList>, <stmt>, and <expr>, tbl is a pointer to a single global table
```

Inefficient
int p = ..; int q = ...; float r = ...; int s = ...;
<program>
<stmtList> tbl: { }
<stmt> tbl: {} <stmtList> tbl: {p-> INT }
<expr> tbl: {}
<stmt> tbl: {p-> INT }
<expr> tbl: {p > INT }
<stmtList> tbl: {p> INT, q > INT }
<stmt> tbl:

```

```

            q}->\mathrm{ INT, 
                <stmt> tbl:
    ```

```

                    <expr> tbl:
    
## Efficient



## Typechecking: Expressions

<expr> ::= intconst
| floatconst <expr>.type := FLOAT | ident
<expr>.type := INT

Cond: [ ident.lexval has a type in <expr>.tbl ]
<expr>.type := <expr>.tbl.lookupld(ident.lexval)

$$
\mid\left(<\operatorname{expr}_{2}\right)
$$

<expr> ${ }_{2} . t b l:=<$ expr>.tbl copies the pointer; both point to the same global table
<expr>.type := <expr> ${ }_{2}$.type

## Typechecking: Expressions

$$
\left.\begin{array}{l}
\text { <expr> ::= <expr> } 2_{2}+\text { <expr> } \\
\\
\text { <expr> } \\
2
\end{array}\right) \text { tbl }:=\text { <expr>.tbl was <expr>.tbl.clone() } \quad \text { was <expr>.tbl.clone() }
$$

## Typechecking: Symbol Tables

<program> ::= <stmtList>
<stmtList>.tbl := newTable() empty table
<stmtList> ::= <stmt> <stmtList> ${ }_{2}$
<stmt>.tbl := <stmtList>.tbl
\{ <stmtList> ${ }_{2}$.tbl := <stmtList>.tbl;
<stmtList> ${ }_{2}$.tbl.insertld(<stmt>.decl) "side effect" of the evaluation \}
| <stmt>
<stmt>.tbl := <stmtList>.tbl

## Typechecking: Symbol Tables

<varDecl> ::= int ident
<varDecl>.decl := newSet(ident.lexval,INT) Set with one element:
a pair (string, INT)
| float ident similarly here
<stmt> ::= <varDecl> = <expr> ;
<stmt>.decl := <varDecl>.decl
<expr>.tbl := <stmt>.tbl
| ident = <expr> ;
<stmt>.decl := newSet() empty set
<expr>.tbl := <stmt>.tbl

## Example

## Consider

int $\mathbf{y}=5 \mathbf{+ 3}$; int $\mathbf{x}=\mathbf{y}$;
All tbl are now pointers to the same table
A It matters when the checks are performed relative to the insertld side effects

Specifically, for <stmtList> ::= <stmt> <stmtList> ${ }_{2}$ : checks inside <stmt> should happen before the side effect of <stmtList> ${ }_{2}$.tbl.insertld(<stmt>.decl) but after insertld side effects for <stmtList> nodes that are higher in the parse tree

## Attribute Grammars with Side Effects

More generally, can we have "global" data structures, i.e., data shared among tree nodes?
Pure attribute grammars: nothing is shared; each node has its own local data, computed once and unchangeable (for example, the first version of type checking)

Advantage: easy to decide the order of evaluation of attributes as we don't have to worry about order of updates to shared data
Attribute grammars with side effects: some shared data and limited side effects on it (Oragon book, See
5.1 and 5.2: also known as "syntax-directed definitions")

Advantage: efficiency

## Side Effects and Order of Evaluation

 Pure attribute grammars: any topological sort order is a valid evaluation orderAttribute grammars with side effects: we need to define additional restrictions on the evaluation
(e.g., as we did for insertld for the type checking attribute grammar)

Think of these restrictions as additional artificial edges in the dependence graph (Oragon book, Sec 5.2.5)

## Use Scenario 2: More Type Checking

In general, type checking is a form of semantic checking that a compiler will perform after parsing, on the parse tree (or, more likely, on the AST)

An attribute grammar specifies both the goals of typechecking and (implicitly) the actual algorithm

A generalization of our earlier example: given program with declarations, check types of identifiers (integers, floats, functions)

For type checking inside a nested block, use the innermost variable declarations
Will not discuss the complete grammar, just key ideas

## Context-Free Grammar

<program> ::= <funcDefList>
<funcDefList> ::= <funcDef> <functDefList> | <funcDef>
<funcDef> ::= <varDecl> ( <formalDeclList> ) \{ <stmtList> \}
<varDecl> ::= int ident | float ident
<stmt> ::= ... | \{ <stmtList> \} | while, if, return statements (not shown)
<expr> ::= ... | ident ( <exprList> ) function call
Example:
int $f$ (int $x$, int $y)\{$ int $z=x+y ;$ return $z ;\}$
int g(int $x)\{$ int $z=5 ;$ int $t=x+z ;$ return $t ;\}\}$
int main (int w) $\{$ return $f(w+1, w+2)+g(8) ;\}$

## Type Checking Goals

Goal 1: Any variable in an <expr> must have an earlier corresponding declaration, including (1) vars from surrounding blocks and (2) function parameters

Goal 2: Any function name in a call must have a corresponding definition somewhere in the program; types of actual parameters at the call should match types of formal parameters at the definition; similarly for the return type

Idea: use a tree of symbol tables
float f (int $x$, int $y$ ) \{
int w = x+y;
w = w+1;
$\{$ int $p=w+1 ;$ int $q=p+x ; \ldots\}$
$\{$ float $w=f(x, y) ;$ float $p=w ; ~ . .$.
<program>
<funcDefList> tbl:
<funcDef> tbl:
<varDecl> <formalDeclList> tbl:


## Type Checking: Expressions

<expr> ::= ... | ident
Cond: [ ident.lexval has a type in <expr>.tbl ]
<expr>.type := <expr>.tbl.lookupld(ident.lexval)

Note: lookupld checks the table, its parent table, the grandparent table, etc. until a match is found

## Type Checking: Symbol Tables

<stmt> ::= ... | \{ <stmtList> \} nested block <stmtList>.tbl := <stmt>.tbl.newChildTable()
int f (int $x$, float $y$ ) $\{$ int $z$... \{ float $w . .\}.\{$ int $v . .\}$.
Root table $\mathrm{T}_{1}:(\mathrm{f},(\mathrm{INT}, \mathrm{FLOAT}) \rightarrow$ INT $)$
$T_{2}$, child of $T_{1}$, table for formals: ( $x, I N T$ ) and ( $y, F L O A T$ )
$T_{3}$, child of $T_{2}$, table for locals: (z,INT)
$T_{4}$, child of $T_{3}$, table for first nested block: ( $w, F L O A T$ )
$T_{5}$, child of $T_{3}$, table for second nested block: ( $\mathrm{v}, \mathrm{INT}$ )

## Type Checking: Function Calls

<expr> ::= ... | ident ( <exprList> )
Cond: [ ident.lexval has a function type in <expr>.tbl]
Cond: [formal types in <expr>.tbl.lookupld(ident.lexval) match actual types in <exprList>]
<expr>.type := return type from <expr>.tbl.lookupld(...)

## Use Scenario 3: Code Generation

Given: parse tree for a simple program (after type checking)
Goal: translate to assembly code
The evaluation rules of the attribute grammar generate the assembly code

Note: in a real compiler, the parse tree (or AST) will be translated to a machine-independent simplified representation (e.g., threeaddress code) which is then optimized and translated to machinespecific assembly code. Details in CSE 5343 "Compilers".

## Code Generation for Expressions

<expr> ::= intconst | ident | (<expr> )
| <expr> + <expr> | <expr> * <expr>

## Output language

Assembly language for a machine with an infinite number of registers $R_{1}, R_{2}, \ldots$ and instruction set as follows

LOAD $R_{i}, x$ : copy the value of variable $x$ into $R_{i}$
LOAD $R_{i}$, const: set the value of $R_{i}$ to an integer constant STORE $x, R_{i}$ : write $R_{i}$ to variable $x$
ADD $\mathbf{R}_{\mathbf{i}}, \mathbf{R}_{\mathbf{j}}, \mathbf{R}_{\mathbf{k}}$ : add $\mathrm{R}_{\mathbf{j}}$ and $\mathrm{R}_{\mathrm{k}}$ and store in $\mathrm{R}_{\mathbf{i}}\left(R_{i}\right.$ could be same as $R_{j}$ or $\left.R_{k}\right)$ MUL $\mathbf{R}_{\mathbf{i}}, \mathbf{R}_{j}, \mathbf{R}_{\mathbf{k}}$ : multiply $\mathrm{R}_{\mathrm{j}}$ and $\mathrm{R}_{\mathrm{k}}$ and store in $\mathrm{R}_{\mathrm{i}}$

## Code Generation Strategy

Synthesized attribute code contains a sequence of instructions: concatenation of subsequences from its children, plus new instructions

```
<expr> ::= <expr> \({ }_{2}+\) <expr> \(_{3}\)
    <expr>.code :=
        <expr> \({ }_{2}\).code
<expr> \({ }_{3}\).code
    "ADD" \(R_{[f o r ~<e x p r>],}, R_{[f o r ~<e x p r>2]}, R_{\left.[f o r ~<e x p r]_{3}\right]}\)
```


## Simple Code Generation

<expr> ::= intconst
<expr>.reg := newReg() // create a new register name
<expr>.code := newInstr(LOAD, <expr>.reg, intconst.lexval)
| ident
<expr>.reg := newReg()
<expr>.code := newInstr(LOAD, <expr>.reg, ident.lexval)
| <expr> ${ }_{2}+$ <expr> $>_{3} / /$ similarly for *
<expr>.reg := newReg()
<expr>.code := concat(<expr> ${ }_{2}$. code, <expr> ${ }_{3}$.code, newInstr(ADD, <expr>.reg, <expr> ${ }_{2} \cdot r e g$, eexpr> $\left.{ }_{3} \cdot r e g\right)$ )
| (<expr> ${ }_{2}$ )
<expr>.reg := <expr> ${ }_{2}$.reg
<expr>.code := <expr> ${ }_{2}$.code

## Observations

newReg(): creates a unique register name. This is a side effect, but the order of these side effects does not matter

We are assuming an infinite number of "abstract" registers, but in reality there is a limit; in compilers, a register allocation pass re-maps the abstract registers to a finite number of real registers

## Example

$(x+99)^{*} z+v^{*} w$
LOAD R1, $x$
LOAD R2, 99
ADD R3, R1, R2
LOAD R4, z
MUL R5, R3, R4
LOAD R6, v
LOAD R7, w
MUL R8, R6, R7
ADD R9, R5, R8

## Code Generation for Statements

<stmtList> ::= <stmt> <stmtList> | <stmt>
<stmt> ::= ident = <expr> ; | if (<cond>) <stmt> else <stmt>
| while (<cond>) <stmt> | \{<stmtList> \}

## Output language

Labels for some instructions; jump instructions BR and BZ
BR $\mathrm{L}_{\mathrm{i}}$ : branch unconditionally to instruction with label $\mathrm{L}_{\mathrm{i}}$
$B Z R_{i}, L_{k}$ : branch to instruction with label $L_{k}$ but only if the value in register $R_{i}$ is zero ( $B Z=$ branch on zero); in many machine languages, zero is a way to represent "false"
$L_{i}:$ : label $L_{i}$; target of $B R / B Z$

## Code Generation for Statements

<stmtList> ::= <stmt> <stmtList> ${ }_{2}$
<stmtList>.code := concat(<stmt>.code,<stmtList> ${ }_{2}$.code)
| <stmt>
<stmtList>.code := <stmt>.code
<stmt> ::= ident = <expr> ;
<stmt>.code := concat(<expr>.code, newInstr(STORE, ident.lexval, <expr>.reg))
| \{ <stmtList> \}
<stmt>.code := <stmtList>.code

## Code Generation for Statements

<stmt> ::= if (<cond>) <stmt> ${ }_{2}$ else <stmt> ${ }_{3}$
<stmt>.elseLabel := newLabel()
<stmt>.exitLabel := newLabel()
<stmt>.code := concat(
<cond>.code, // leaves 0 in <cond>.reg if condition is "false"
newInstr(BZ, <cond>.reg, <stmt>.elseLabel),
<stmt> ${ }_{2}$.code,
newInstr(BR, <stmt>.exitLabel),
<stmt>.elseLabel,
<stmt> ${ }_{3}$.code,
<stmt>.exitLabel)

## Example

if (...) $x=y+5$; else $x=8$;
code for ... // leaves 0 in $R 8$ if condition is "false"
BZ R8, L33 // jump to "else" if condition is "false"
LOAD R1, y
LOAD R2, 5
ADD R3, R1, R2
STORE x, R3
BR L34
L33: // else label
LOAD R4, 8
STORE x, R4
L34: // exit label

## Code Generation for Statements

<stmt> ::= while (<cond>) <stmt> ${ }_{2}$
<stmt>.startLabel := newLabel()
<stmt>.exitLabel := newLabel()
<stmt>.code := concat(
<stmt>.startLabel,
<cond>.code, // leaves 0 in <cond>.reg if condition is "false"
newInstr(BZ, <cond>.reg, <stmt>.exitLabel),
<stmt> ${ }_{2}$.code,
newInstr(BR, <stmt>.startLabel),
<stmt>.exitLabel)

## Example

while (...) $x=x+1$;
L15: // start label
code for ... // leaves 0 in $R 8$ if condition is "false"
BZ R8, L16 // jump to "exit" if condition is "false"
LOAD R1, x
LOAD R2, 1
ADD R3, R1, R2
STORE x, R3
BR L15
L16: // exit label

## Summary: Attribute Grammars

 Useful for expressing arbitrary cycle-free traversals over context-free parse treesSynthesized and inherited attributes
Conditions to reject invalid parse trees Evaluation order depends on attribute dependencies Uses: type checking and code generation Basic data structures (sets, maps, etc.) can be used The evaluation rules can call helper functions If functions have global effects ("side effects"), need to define when these effects happen

