

# Syntax Analysis

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Chapter 1, Section 1.2.2

Chapter 4, Section 4.1, 4.2, 4.3, 4.4, 4.5

CUP Manual

# Inside the Compiler: **Front End**

## Lexical analyzer (aka scanner)

- Provides a stream of token to the syntax analyzer (aka parser), which then creates a **parse tree**
- Usually the parser calls the scanner: **getNextToken()**

## Syntax analyzer (aka parser)

- Based on a **context-free grammar** which specifies precisely the syntactic structure of well-formed programs
  - Token names are terminal symbols of this grammar
- Error checking, reporting, and recovery are important concerns; we will not discuss them

# Overview

Context-free grammars

Ambiguous grammars

Top-down parsing

- Essential first step: elimination of left recursion
- Predictive parsing for LL(1) grammars

Bottom-up parsing

- Example: shift-reduce parsing for LR(1) grammars

# Context-Free Grammars

Productions:  $x \rightarrow y$

- $x$  is a single non-terminal: the **left side**
- $y$  has zero or more terminals and non-terminals: the **right side** of the production
- E.g.  $expr \rightarrow expr + const$

Alternative notation: Backus-Naur Form (BNF)

- E.g.  $\langle expr \rangle ::= \langle expr \rangle + \langle const \rangle$

Example: simple arithmetic expressions

$$E \rightarrow E + T \mid E - T \mid T$$
$$T \rightarrow T * F \mid T / F \mid F$$
$$F \rightarrow ( E ) \mid id$$

# Derivations and Parse Trees

Start with the starting non-terminal, apply productions until a string of terminals is derived

- **Leftmost** derivation: the leftmost non-terminal at each step is chosen for expansion
- **Rightmost** derivation: the rightmost non-terminal

Each derivation can be represented by a **parse tree**

- Leaves are terminals or non-terminals
- After a full derivation: leaves are terminals (or  $\epsilon$ )

Parser: builds the parse tree for a given string of terminals

Example: using the grammar from the previous slide, show the parse tree for  **$a + b * ( c + d ) * e$**

# Ambiguity

**Ambiguous grammar:** more than one parse tree for some sentence

- Choice 1: make the grammar unambiguous
- Choice 2: leave the grammar ambiguous, but define some disambiguation rules to use during parsing

Example: the dangling-else problem

```
stmt →  if expr then stmt
      |  if expr then stmt else stmt
      |  other
```

Two parse trees for **if a then if b then x=1 else x=2**

- Choice 1: complex non-ambiguous version in Fig 4.10 in the Dragon Book (**else** is matched with the closest unmatched **then**); do not need to study it
- Choice 2: a “hint” to the parser (used in our projects)

# Elimination of Ambiguity

$expr \rightarrow expr + expr \mid expr * expr \mid ( expr ) \mid id$

Why is this grammar ambiguous?

Earlier grammar: equivalent non-ambiguous grammar with the “normal” precedence and associativity

- $*$  has higher precedence than  $+$
- both are left-associative

Recall the parse tree for  $a + b * ( c + d ) * e$

# Top-Down Parsing

Goal: find the leftmost derivation for a given string

General solution: **recursive-descent parsing**

- To use this: need to eliminate any **left recursion** from the grammar
- In the general case, parsing may require **backtracking**

**Predictive recursive-descent parsing**

- **LL( $k$ ) grammars**: only need to look at the next  $k$  symbols to decide which production to apply (no backtracking)
  - Important case in practice: LL(1) grammars



# Prerequisite: Elimination of Left Recursion

**Left-recursive** grammar: possible  $A \Rightarrow \dots \Rightarrow A\alpha$

Simple case (here  $\alpha$  and  $\beta$  are arbitrary sequences of terminals and non-terminals)

- Original grammar:  $A \rightarrow A\alpha \mid \beta$
- New grammar:  $A \rightarrow \beta A'$  and  $A' \rightarrow \alpha A' \mid \epsilon$

More complex case

- Original:  $A \rightarrow A\alpha_1 \mid \dots \mid A\alpha_m \mid \beta_1 \mid \dots \mid \beta_n$
- New:  $A \rightarrow \beta_1 A' \mid \dots \mid \beta_n A'$  and  $A' \rightarrow \alpha_1 A' \mid \dots \mid \alpha_m A' \mid \epsilon$

Still not enough

- E.g.  $S$  is left-recursive in  $S \rightarrow A\mathbf{a} \mid \mathbf{b}$  and  $A \rightarrow A\mathbf{c} \mid S\mathbf{d} \mid \epsilon$
- More details in Section 4.3.3 of Dragon book; we will not discuss in this course

# Example with Left Recursion

Original grammar

$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * F \mid T / F \mid F$$

$$F \rightarrow ( E ) \mid \text{id}$$

Modified grammar

$$E \rightarrow T E'$$

$$E' \rightarrow + T E' \mid - T E' \mid \varepsilon$$

$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid / F T' \mid \varepsilon$$

$$F \rightarrow ( E ) \mid \text{id}$$

# Recursive-Descent Parsing

One procedure for each non-terminal

Parsing starts with a call to the procedure for the starting non-terminal

- Success: if at the end of this call, the entire input string has been processed (no leftover symbols)

```
void A() /* procedure for a non-terminal A */  
  choose some production  $A \rightarrow X_1 X_2 \dots X_k$   
  for (i = 1 to k)  
    if ( $X_i$  is non-terminal) call  $X_i()$   
    else if ( $X_i$  is equal to the current input symbol)  
      move to the next input symbol  
    otherwise report parse error
```

# A Few Issues

Choosing which production  $A \rightarrow X_1 X_2 \dots X_k$  to use

- There could be many possible productions for  $A$
- If one of the choices does not work, backtrack the algorithm and try another choice
- Expensive and undesirable in practice

Top-down parsing for programming languages:  
**predictive** recursive-descent (no backtracking)

# LL(1) Grammars

Suitable for predictive recursive-descent parsing

- LL = “scan the input left-to-right; produce a leftmost derivation”; 1 = “use 1 symbol to decide”
- A left-recursive grammar cannot be LL(1)
- An ambiguous grammar cannot be LL(1)

For any  $A \rightarrow \alpha \mid \beta$

- $\text{FIRST}(\alpha)$  and  $\text{FIRST}(\beta)$  must be disjoint sets
  - $\text{FIRST}(\alpha)$  = terminals that could be the first symbol of something derived from  $\alpha$
- If current input symbol is in  $\text{FIRST}(\alpha)$ : use  $A \rightarrow \alpha$
- If current input symbol is in  $\text{FIRST}(\beta)$ : use  $A \rightarrow \beta$
- Otherwise report parsing error
- Only look at current input symbol to make a decision

# Some Examples of Sets FIRST

Grammar with eliminated left recursion

$$E \rightarrow T E'$$

$$E' \rightarrow + T E' \mid - T E' \mid \varepsilon$$

$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid / F T' \mid \varepsilon$$

$$F \rightarrow ( E ) \mid \text{id}$$

$$\text{FIRST}(F) = \text{FIRST}(T) = \text{FIRST}(E) = \{ (, \text{id} \}$$

$$\text{FIRST}(E') = \{ +, -, \varepsilon \} \text{ and } \text{FIRST}(T') = \{ *, /, \varepsilon \}$$

$$\text{FIRST}( ( E ) ) = \{ ( \} \text{ and } \text{FIRST}( \text{id} ) = \{ \text{id} \}$$

*Use for LL(1) parsing:*

e.g. for  $F \rightarrow ( E ) \mid \text{id}$

Parser code for  $F$

```
if (currToken==LPAREN) ...  
else if (currToken==ID) ...  
else error()
```

# Sets FIRST

For any string  $\alpha$  of terminals and non-terminals:

**FIRST( $\alpha$ )** contains all terminals that could be the first symbol of some string derived from  $\alpha$

- $\alpha \xRightarrow{*} a\beta$  where  $a$  is a terminal, means  $a \in \text{FIRST}(\alpha)$
- $\alpha \xRightarrow{*} \epsilon$  means  $\epsilon \in \text{FIRST}(\alpha)$  – some complications ...

The simple cases:

- If  $\alpha$  is just a single terminal  $a$ , **FIRST( $\alpha$ )** = {  $a$  }
- If  $\alpha$  is a terminal  $a$  followed by anything, **FIRST( $\alpha$ )** = {  $a$  }
- If  $\alpha$  is the empty string  $\epsilon$ , **FIRST( $\alpha$ )** = {  $\epsilon$  }

The more complex cases: next slide

- If  $\alpha$  is just a single non-terminal
- If  $\alpha$  is a non-terminal followed by something

## Sets FIRST (cont)

FIRST( $X$ ) for a non-terminal  $X$  : consider each production  $X \rightarrow Y_1 Y_2 \dots Y_n$

- Any terminal in FIRST( $Y_1$ ) is also in FIRST( $X$ )
- If  $\epsilon \in \text{FIRST}(Y_1)$ , any terminal in FIRST( $Y_2$ ) is in FIRST( $X$ )
  - And if  $\epsilon \in \text{FIRST}(Y_2)$ , any terminal in FIRST( $Y_3$ ) is in FIRST( $X$ ), etc.
  - If  $\epsilon \in \text{FIRST}(Y_i)$  for all  $i$ , FIRST( $X$ ) also contains  $\epsilon$
- If  $X \rightarrow \epsilon$  is a production, FIRST( $X$ ) contains  $\epsilon$

FIRST( $X_1 X_2 \dots X_n$ )

- Any terminal in FIRST( $X_1$ )
- If FIRST( $X_1$ ) contains  $\epsilon$ , any terminal in FIRST( $X_2$ ), etc.
- If all FIRST( $X_i$ ) contain  $\epsilon$ , FIRST( $X_1 X_2 \dots X_n$ ) contains  $\epsilon$



## Special Case: $\epsilon \in \text{FIRST}(\dots)$

Example: consider  $E' \rightarrow +TE' \mid -TE' \mid \epsilon$

- $\text{FIRST}(+TE') = \{ + \}$ ,  $\text{FIRST}(-TE') = \{ - \}$ ,  $\text{FIRST}(\epsilon) = \{ \epsilon \}$
- When do we choose production  $E' \rightarrow \epsilon$ ?
- What is the actual code for the parser?

We will not discuss in this course, but there is a systematic approach to handle this; leads to

```
if (currToken==PLUS) {nextToken(); T(); Eprime();}
else if (currToken==MINUS) { ... }
else if (currToken==RPAREN ||
        currToken==END_INPUT) { } // do nothing
else error()
```

# LL(1) Parser

- Define a predictive parsing table
  - A row for a non-terminal  $A$ , a column for a terminal  $a$
  - Cell  $[A, a]$  is the production that should be applied when we are inside  $A$ 's parsing procedure and we see  $a$
  - If the grammar is LL(1) – only one choice per cell

	id	+	-	*	/	(	)	\$
$E$	$E \rightarrow TE'$					$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$	$E' \rightarrow -TE'$				$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$					$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$	$T' \rightarrow /FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow \text{id}$					$F \rightarrow (E)$		

# Example: $a + b * (c + d) * e$

$E$	$a + b * (c + d) * e \$$	$E \rightarrow T E'$
$T$	$a + b * (c + d) * e \$$	$T \rightarrow F T'$
$F$	$a + b * (c + d) * e \$$	$F \rightarrow \text{id}$
$T'$	$+ b * (c + d) * e \$$	$T' \rightarrow \varepsilon$
$E'$	$+ b * (c + d) * e \$$	$E' \rightarrow + T E'$
$T$	$b * (c + d) * e \$$	$T \rightarrow F T'$
$F$	$b * (c + d) * e \$$	$F \rightarrow \text{id}$
$T'$	$* (c + d) * e \$$	$T' \rightarrow * F T'$
$F$	$(c + d) * e \$$	$F \rightarrow ( E )$
$E$	$c + d) * e \$$	$E \rightarrow T E'$
$T$	$c + d) * e \$$	$T \rightarrow F T'$
$F$	$c + d) * e \$$	$F \rightarrow \text{id}$

# Instead of Procedure Calls: Explicit Stack

Top of stack: terminal or nonterminal  $X$  ; current input symbol: terminal  $a$

1. Push  $S$  on top of stack

2. While stack is not empty

– If ( $X == a$ )

    Pop stack and move to the next input symbol

– Else if ( $X ==$  some other terminal) Error

– Else if (table cell [ $X, a$ ] is empty) Error

– Else: table cell [ $X, a$ ] contains  $X \rightarrow Y_1 Y_2 \dots Y_n$

    Pop stack

    Push  $Y_n$ , Push  $Y_{n-1}$ , ..., Push  $Y_1$

Exercise at home: apply to the example from the previous slide

# Different Approach: Bottom-Up Parsing

In general, more powerful than top-down parsing

- E.g.,  $LL(k)$  grammars are not as general as  $LR(k)$

Basic idea: start at the leaves and work up

- The parse tree “grows” upwards

**Shift-reduce parsing:** general style of bottom-up parsing

- Used for parsing  $LR(k)$  grammars
- Used by **automatic parser generators**: given a grammar, it generates a shift-reduce parser for it (e.g., yacc, CUP)
  - yacc = “Yet Another Compiler Compiler”
  - CUP = “Constructor of Useful Parsers”

# Reductions

Expressions again (now it is OK to be left-recursive)

$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * F \mid T / F \mid F$$

$$F \rightarrow ( E ) \mid \text{id}$$

At a **reduction step**, a substring matching the right side a production is replaced with the left side

– E.g.,  $E + T$  is reduced to  $E$  because of  $E \rightarrow E + T$

Parsing is a sequence of reduction steps

$$(1) \text{id} * \text{id} \quad (2) F * \text{id} \quad (3) T * \text{id}$$

$$(4) T * F \quad (5) T \quad (6) E$$

This is a **derivation in reverse**:  $E \Rightarrow T \Rightarrow T * F \Rightarrow T * \text{id} \Rightarrow F * \text{id} \Rightarrow \text{id} * \text{id}$

# Overview of Shift-Reduce Parsing

Left-to-right scan of the input

Perform a sequence of reduction steps which correspond (in reverse) to a **rightmost** derivation

- If the grammar is not ambiguous: there exists a unique rightmost derivation  $S = \gamma_0 \Rightarrow \gamma_1 \Rightarrow \dots \Rightarrow \gamma_n = w$
- Each step also updates the tree (adds a parent node)

At each reduction step, find a “handle”

- If  $\gamma_k \Rightarrow \gamma_{k+1}$  is  $\alpha A v \Rightarrow \alpha \beta v$ , then  $\beta$  is a handle of  $\gamma_{k+1}$ 
  - Note that  $v$  is a string of terminals
- Non-ambiguous grammar: only one handle of  $\gamma_{k+1}$

# Overview of Shift-Reduce Parsing (cont)

A **stack** holds grammar symbols; an **input buffer** holds the rest of the string to be parsed

- Initially: the stack is empty, the buffer contains the entire input string
- Successful completion: the stack contains the starting non-terminal, the buffer is empty

Repeat until success or error

- **Shift** zero or more input symbols from the buffer to the stack, until the top of the stack forms a handle
- **Reduce** the handle



# Example of Shift-Reduce Parsing

Stack	Input	Action
empty	$\text{id}_1 * \text{id}_2 \$$	Shift
$\text{id}_1$	$* \text{id}_2 \$$	Reduce by $F \rightarrow \text{id}$
$F$	$* \text{id}_2 \$$	Reduce by $T \rightarrow F$
$T$	$* \text{id}_2 \$$	Shift
$T *$	$\text{id}_2 \$$	Shift
$T * \text{id}_2$	$\$$	Reduce by $F \rightarrow \text{id}$
$T * F$	$\$$	Reduce by $T \rightarrow T * F$
$T$	$\$$	Reduce by $E \rightarrow T$
$E$	$\$$	Accept

# LR Parsers and Grammars

**LR( $k$ ) parser**: knowing the content of the stack and the next  $k$  input symbols is enough to decide

- LR=“scan left-to-right; produce a rightmost derivation”
- LR( $k$ ) grammar: we can define an LR( $k$ ) parser for it

**Non-LR grammar**: conflicts during parsing

- Shift/reduce conflict: shift or reduce?
- Reduce/reduce conflict: several possible reductions
- Typical example: **any ambiguous grammar**

**SLR parsers** (“simple-LR”, Section 4.6), **LALR parsers** (“lookahead-LR”, Section 4.7), **canonical-LR** (most general; Section 4.7); details will not be discussed

# CUP Parser Generator

Input: grammar specification

- Has embedded Java code to be executed during parsing

Output: a parser written in Java

Often uses a scanner produced by JFlex

Key components of the specification:

- Terminals and non-terminals
- Precedence and associativity
- Productions: terminals, non-terminals, actions
- Project 2: change the parser from Project 1
  - And related changes to scanner and AST
  - **while** loops; **for** loops; operators  $<$ ,  $<=$ ,  $>$ ,  $>=$ ,  $==$ ,  $!=$