Compiler Optimizations

Chapter 8, Section 8.5

Chapter 9, Section 9.1.7

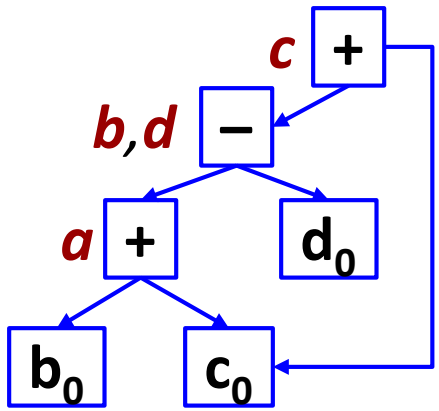
Local vs. Global Optimizations

- Local: inside a single basic block
  - Simple forms of common subexpression elimination, dead code elimination, strength reduction, etc.
  - May sometimes require the results of a dataflow analysis (e.g., Live Variables analysis)
- Global: **intra**procedurally, across basic blocks
  - Code motion, more complex common subexpression elimination, etc.
- **Inter**procedural optimizations: across procedure boundaries
  - Tricky; not used often
  - Sometimes we do procedure inlining and use intraprocedural optimizations

Part 1: Local Optimizations

- Based on a DAG representation of a basic block
- DAG nodes are
  - The initial values of variables (before the basic block)
  - Nodes labeled with operations – e.g. +, –, etc.
- DAG construction
  - Traverse the sequence of instructions in the basic block
  - Maintain a mapping: variable v → DAG node that corresponds to the last write to this variable
  - Construct new nodes only when necessary

a = b + c
b = a – d
c = b + c
d = a – d



Common Subexpression Elimination

- In the previous example, we identified the common subexpression **a – d**
  - This can be used to optimize the code – eventually we will reassemble the basic block from this DAG
- Reverse topological sort traversal; if a node has multiple variables, write to one of them and then copy to the others; ignore vars that are not live
- Is b live at the end of the basic block?

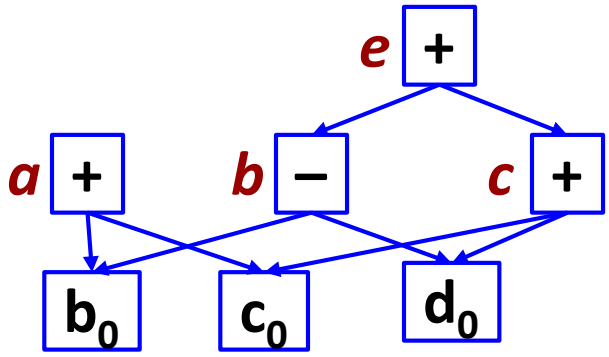
a = b + c
d = a – d
b = d
c = d + c
if b is live

a = b + c
d = a – d
c = d + c
if b is not live

Common Subexpression Elimination

- This approach does not always find all redundancies

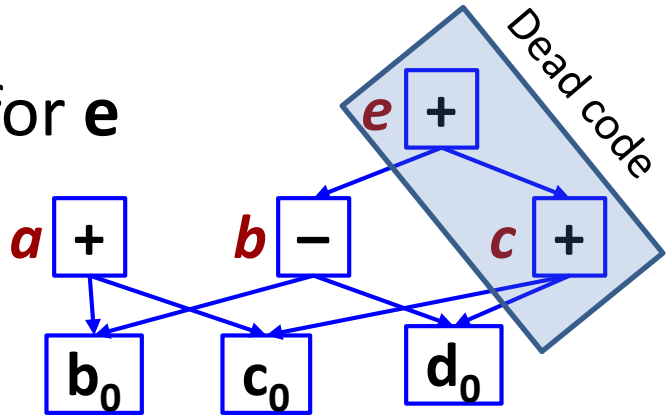
a = b + c
b = b – d
c = c + d
e = b + c



There are no common subexpressions here, but in fact **a** and **e** both have the value $\mathbf{b_0}+\mathbf{c_0}$

Dead Code Elimination

- Assumes the output of Live Variables analysis (aka liveness analysis)
- Consider the DAG together with the map of DAG node that have the last write of a variable
- If a DAG node has zero incoming edges, and has zero writes of **live** variables, it can be eliminated
  - And this may enable other removals
  - Example (from before): suppose that **e** and **c** are not live at the exit of the basic block
    - First we can remove the node for **e**
    - Then, the node for **c**



Algebraic Identities

- Arithmetic

  $x + 0 = 0 + x = x$ $\qquad$ $x * 1 = 1 * x = x$

  $x - 0 = x$ $\qquad$ $x / 1 = x$

- Strength reduction: replace a more expensive operator with a cheaper one

  $2 * x = x + x$ $\qquad$ $x / 2 = 0.5 * x$

- Constant folding: evaluate expressions at compile time and use the result

  x = 2*3.14 is replaced with x = 6.28 (e.g., due to symbolic constants with **#define** in C, **final** in Java, etc.)

- Commutativity
  - E.g. a = x*y and b = y*x have a common subexpression

Local vs. Global Optimizations

- Local: inside a single basic block
  - Simple forms of common subexpression elimination, dead code elimination, strength reduction, etc.
  - May sometimes require the results of a dataflow analysis (e.g., Live Variables analysis)
- **Global: intraprocedurally, across basic blocks**
  - **Code motion, more complex common subexpression elimination, etc.**
- Interprocedural optimizations: across procedure boundaries
  - Tricky; not used often
  - Sometimes we do procedure inlining and use intraprocedural optimizations

Part 2: Loop-Invariant Computations

- Motivation: again, avoid redundancy

```
a = …
b = …
c = …
start loop
…
d = a + b
e = c + d
…
end loop
```

All instructions whose right-hand side operands have reaching definitions that are **only** from outside the loop

But, this also applies transitively …

Need an algorithm to compute the set ***Inv*** of all loop-invariant instructions

Complete Definition and Algorithm

- Add an instruction to ***Inv*** if each operand on the right-hand side satisfies one of the following
  - It is a constant: e.g., -5 or 3.1415
  - All its reaching definitions are from outside of the loop
- After this initialization, repeat until no further changes to ***Inv*** are possible
  - Find an instruction that is not in ***Inv*** but each operand on the right-hand side satisfies one of the following
    - It is a constant
    - All its reaching definitions from outside of the loop
    - It has exactly one reaching definition, and that definition is already in ***Inv***
  - Add this instruction to ***Inv***
  - Note: use the ud-chains to find all reaching defs

The Last Condition

- A right-hand side operand has exactly one reaching def, and that def is already in ***Inv***
- Why not two reaching defs, both already in *Inv*?

  if (…) a = 5; else a = 6;
  b = a+1;

  – Even though each definition of **a** is in ***Inv***, the value of **b** is not guaranteed to be the same for each iteration
- But: not every loop-invariant computation (i.e., member of ***Inv***) can be moved out of the loop
  – **for (…) { a = 1; … if (…) break; … b = a+1; … }**
  – **a = 1; b = a+1; for (…) { … if (…) break; … }**

Conditions for the Actual Code Motion

- Given: a loop-invariant instruction from ***Inv***
- Condition 1: the basic block that contains the instruction must dominate all exits of the loop
  - i.e., all nodes that are sources of loop-exit edges
  - This means that it is impossible to exit the loop before the instruction is executed
    - And thus, it is safe to move this instruction before the loop header
- Condition 2: to move instruction **a = …**, this must be the only assignment to **a** in the loop
  - **for (…) { a = 1; … if (…) { … a = 2; … } … b = a+1; … }**
  - **a = 1; for (…) { … if (…) { … a = 2; … } … b = a+1; … }**

Conditions for the Actual Code Motion

- Condition 3: to move instruction **a = …**, every use of **a** in the loop must be reached **only** by this definition of **a**
  - Condition 2 ensures that there does not exist another definition of **a** in the loop; Condition 3 guarantees that even if there are definitions of **a** outside of the loop, they do not reach any use of **a** in the loop
  - **a = 1; for (…) { … b = a+1; … a = 2; … }**
  - **a = 1; a = 2; for (…) { … b = a+1; … }**
- Given the subset of instructions from ***Inv*** that satisfy the three conditions from above, we can now modify the three-address code

Complications with Arrays

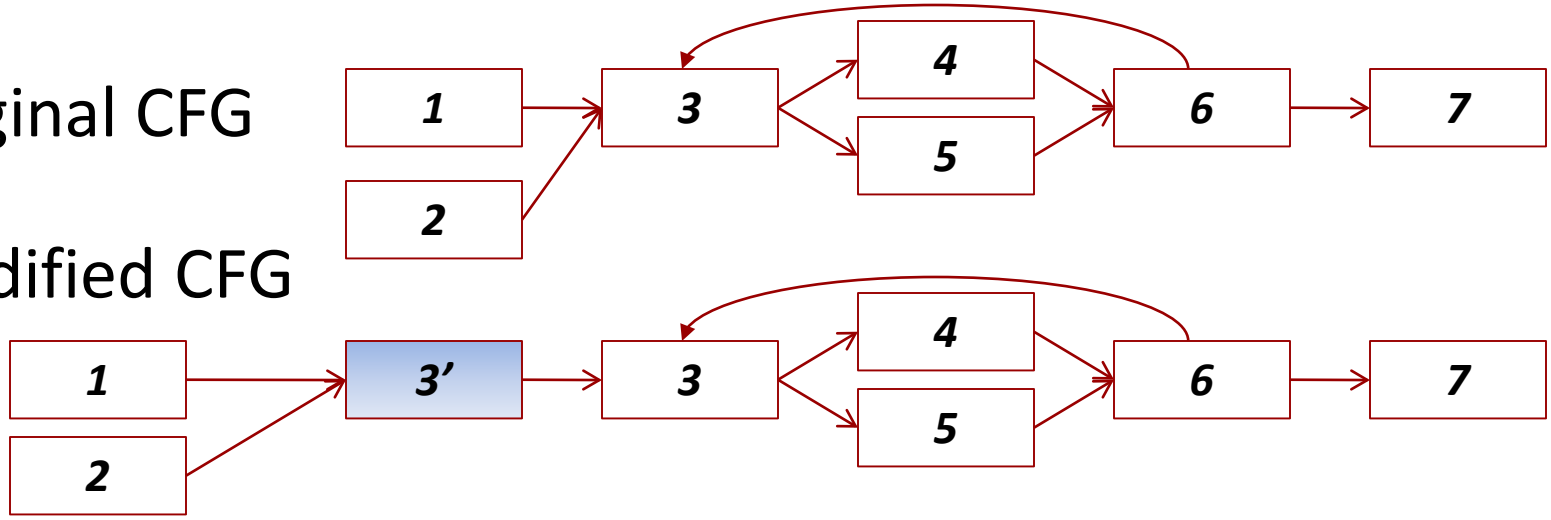
- Reaching definitions and ud-chains for arrays:
  - Any **x[y] = z** is treated as a "definition" of the entire array **x** (really, a possible def for each array element)
  - Any **x = y[z]** is treated as a "use" of the entire array **y** (really, a possible use for each array element)
  - This is very approximate ... (better approaches exist)
- Various issues

  a[x] = y
  a[z] = w
  u=a[v]

  - Both definitions of **a** are reaching for the use of **a**
    - Unless we can prove that **z** is always equal to **x**

Code Transformation

- First, create a preheader for the loop
  - Original CFG
  - Modified CFG



- Next, consider all instructions in ***Inv***, in the order in which they were added to ***Inv***
  - Each instruction that satisfies the three conditions is added at the end of the preheader, and removed from its basic block

Problem for While/For Loops

$S \rightarrow$ **while (** E **)** S_1

$S.startLabel$ = *newLabel*()

$S.exitLabel$ = *newLabel*()

$S.code$ =

$S.startLabel$ ||

$E.code$ ||

"**if (!** " $E.addr$ "**) goto** " $S.exitLabel$ ||

$S_1.code$ ||

"**goto** " $S.startLabel$ ||

$S.exitLabel$

Example

```
while (i <= n) {
  x = 3*4;
  res *= x;
  i += 1;
}
...
```

```
_l1:
 _t1 = i <= n;
 if (!_t1) goto _l2;
 x = 3*4;
 _t2 = res * x; res = _t2;
 _t3 = i + 1; i = _t3;
 goto _l1;
 _l2: …
```

Condition 1: the basic block that contains the instruction must dominate all exits of the loop – violated!

The Fix

```
_l1:
  _t1 = i <= n;
  if (!_t1) goto _l2;
  x = 3*4;
  _t2 = res * x; res = _t2;
  _t3 = i + 1; i = _t3;
  goto _l1;
  _l2: …
```

```
  _t1 = i <= n;
  if (!_t1) goto _l2;
  _l1:
  x = 3*4;
  _t2 = res * x; res = _t2;
  _t3 = i + 1; i = _t3;
  _t1 = i <= n;
  if (!_t1) goto _l2;
  goto _l1;
  _l2: …
```

Conceptually …

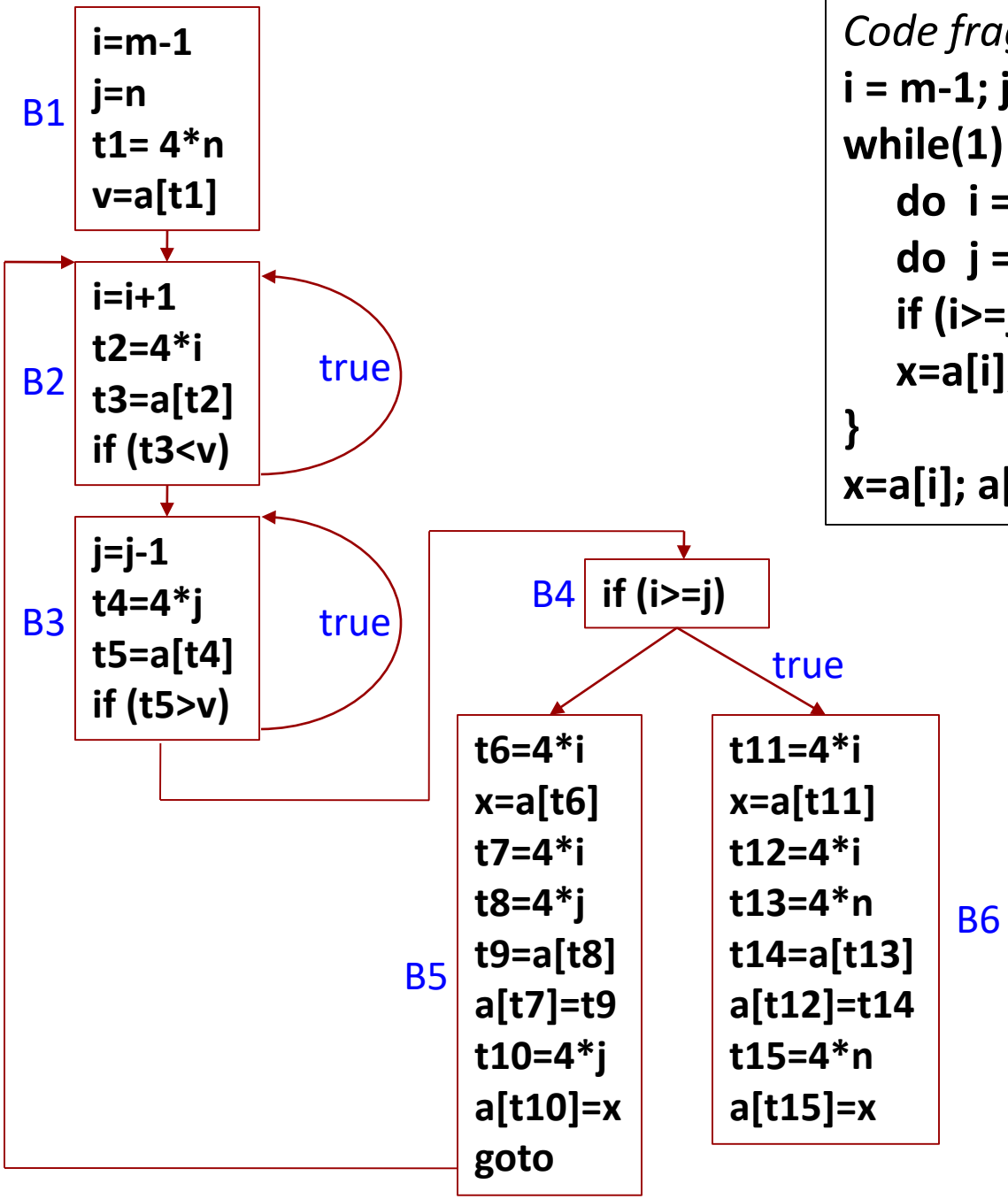
```
while (i <= n) {
  x = 3*4;
  res *= x;
  i += 1;
}
…
```

```
if (i <=n)
  do {
   x = 3*4;
   res *= x;
   i += 1;
  } while (i<=n)
…
```

```
if (i <=n)
  x = 3*4;
  do {
   res *= x;
   i += 1;
  } while (i<=n)
…
```

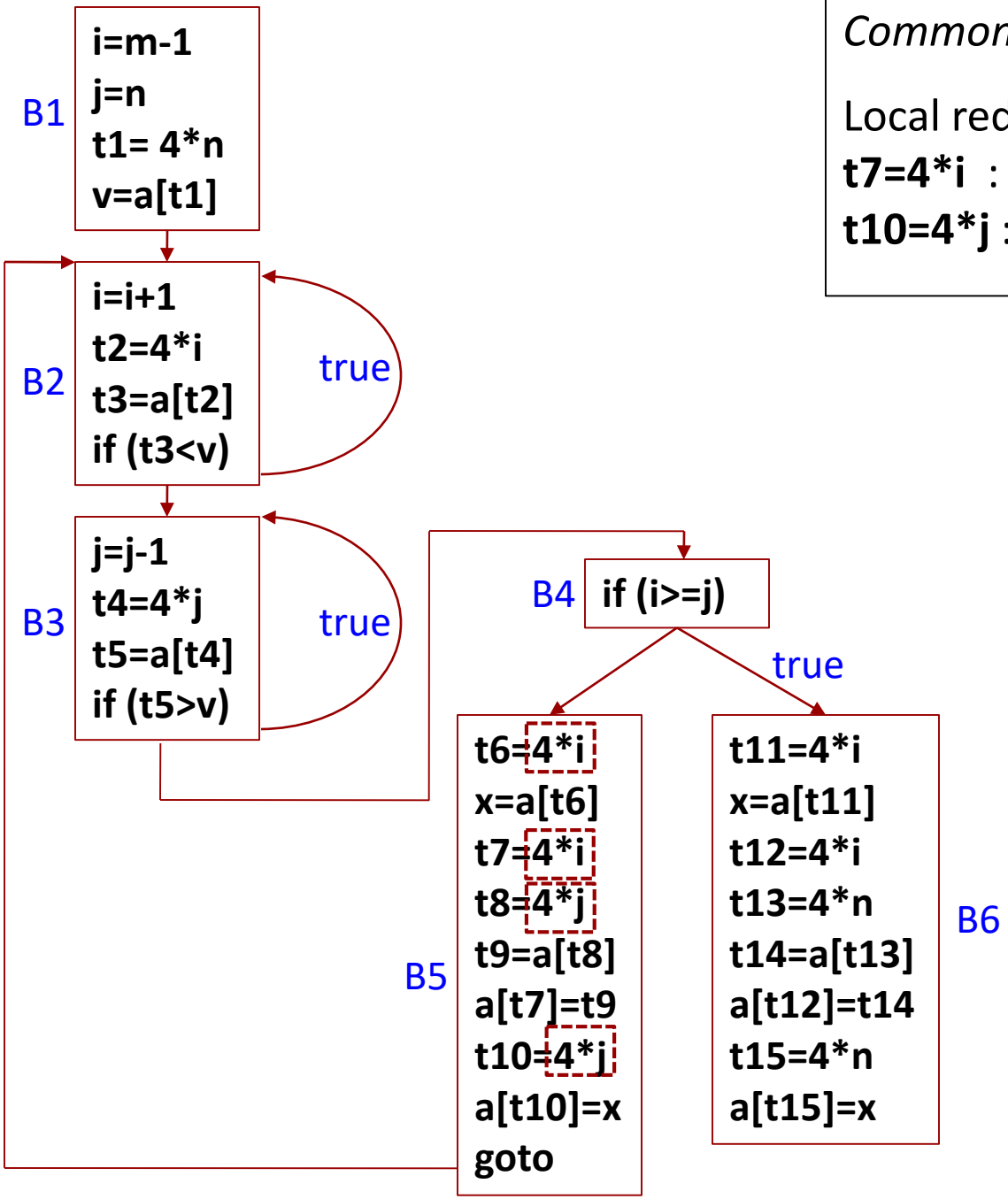
Part 3: Other Global Optimizations

- We have already have seen one simple form of code motion for loop-invariant computations
- Common subexpression elimination
- Copy propagation
- Dead code elimination
- Elimination of induction variables
  - Variables that essentially count the number of iterations around a loop
- Partial redundancy elimination
  - Powerful generalization of code motion and common subexpression elimination (Section 9.5)



Code fragment from quicksort:

```
i = m-1; j = n; v = a[n];
while(1) {
    do  i = i+1;  while  (a[i] < v);
    do  j = j–1;  while  (a[j] > v);
    if (i>=j) break;
    x=a[i]; a[i] = a[j]; a[j] = x;
}
x=a[i]; a[i] = a[n]; a[n] = x;
```

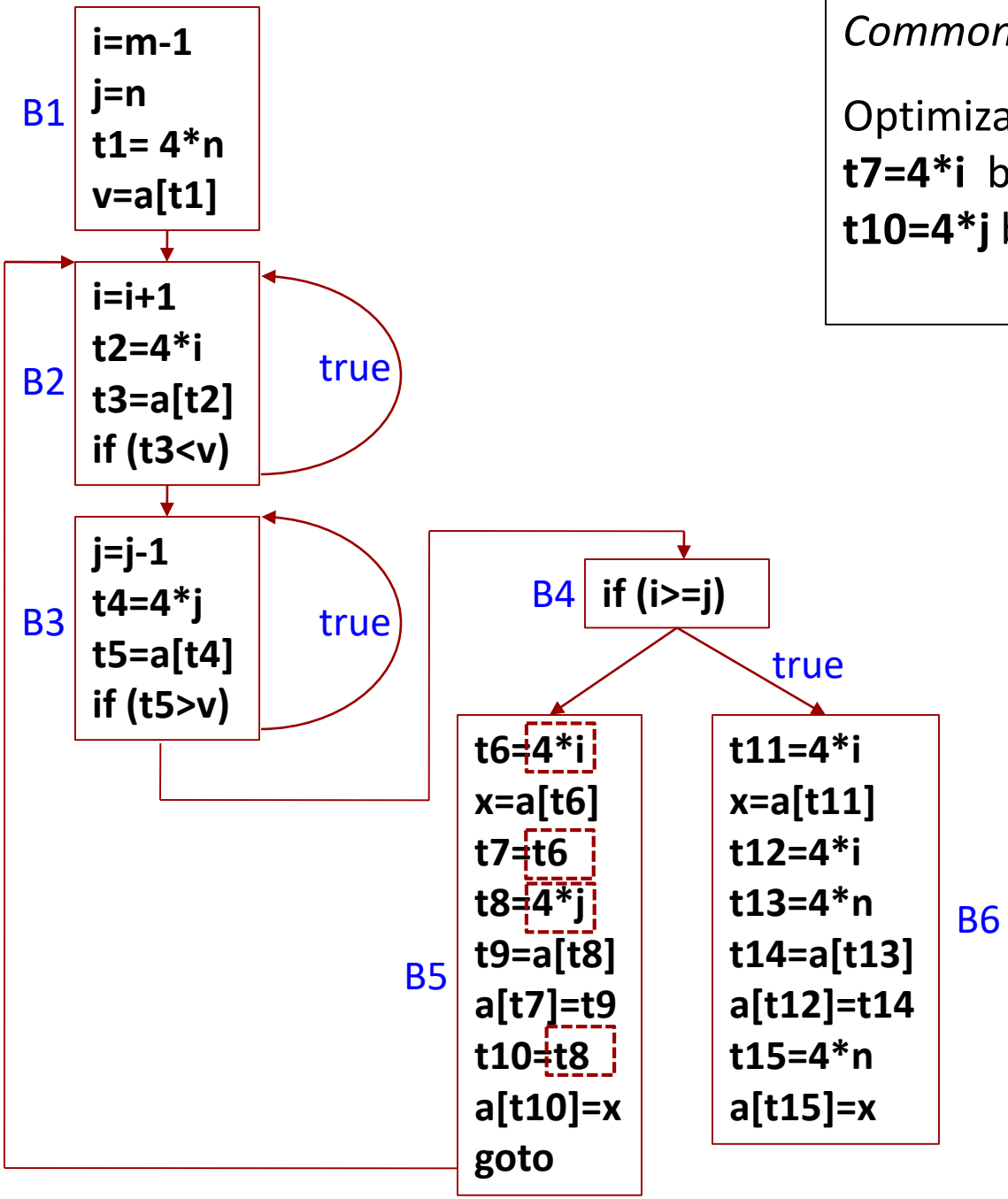


Common subexpression elimination

Local redundancy in B5:

t7=4*i : value already available in **t6**

t10=4*j : value already available in **t8**

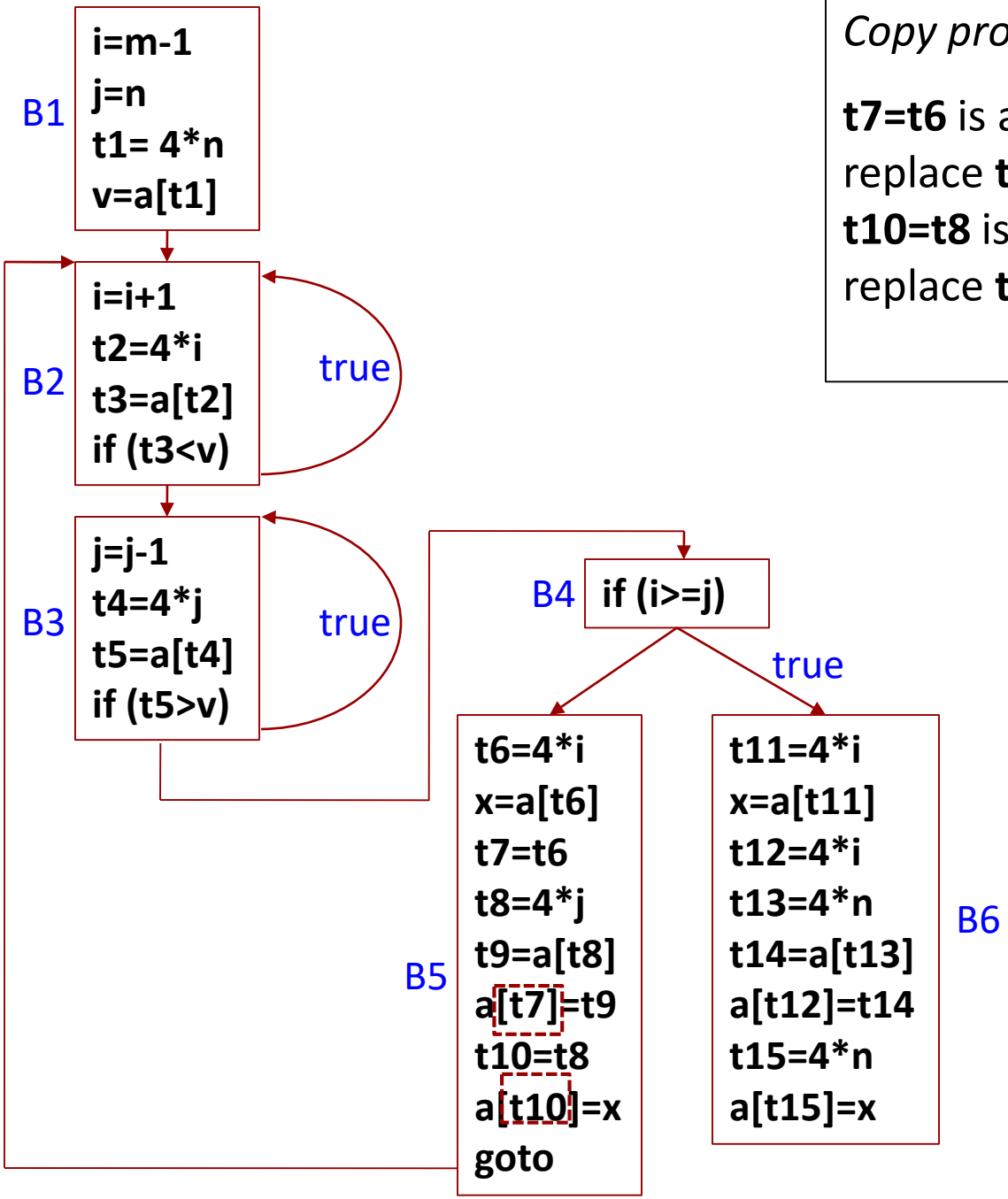


Common subexpression elimination

Optimization:

t7=4*i becomes **t7=t6**

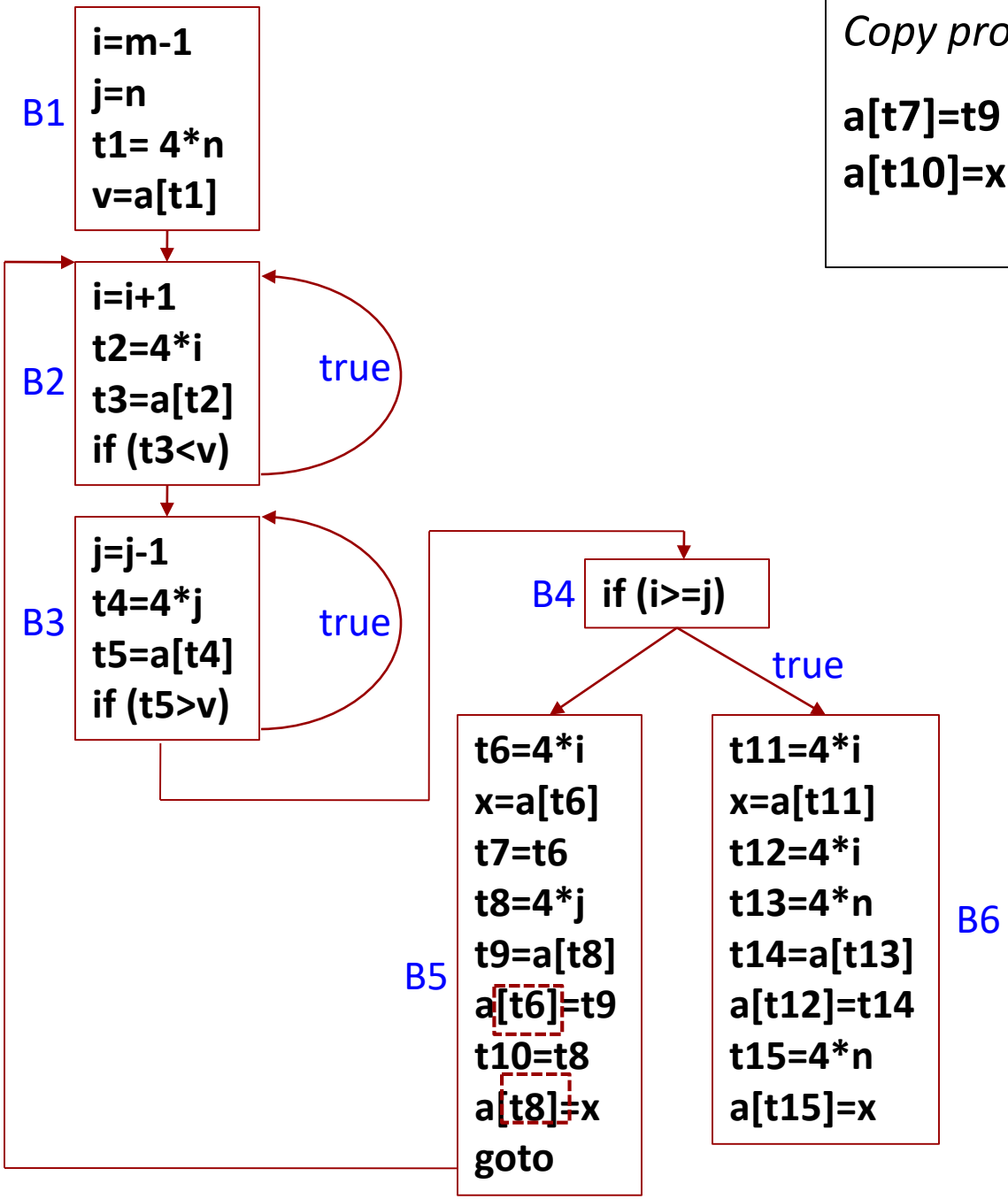
t10=4*j becomes **t10=t8**



Copy propagation (details later)

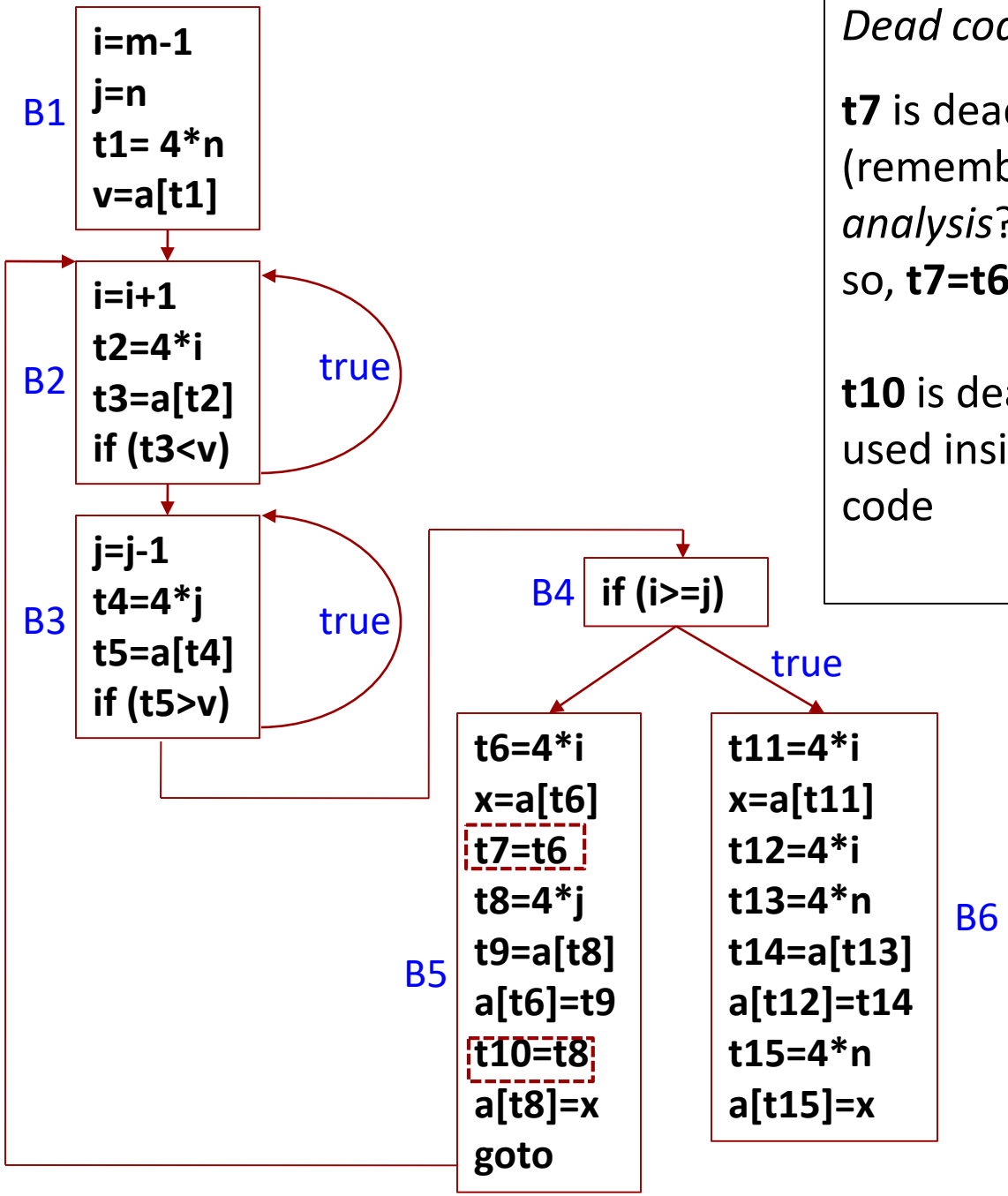
t7=t6 is a copy instruction; can we replace **t7** with **t6**?

t10=t8 is a copy instruction; can we replace **t10** with **t8**?



Copy propagation (details later)

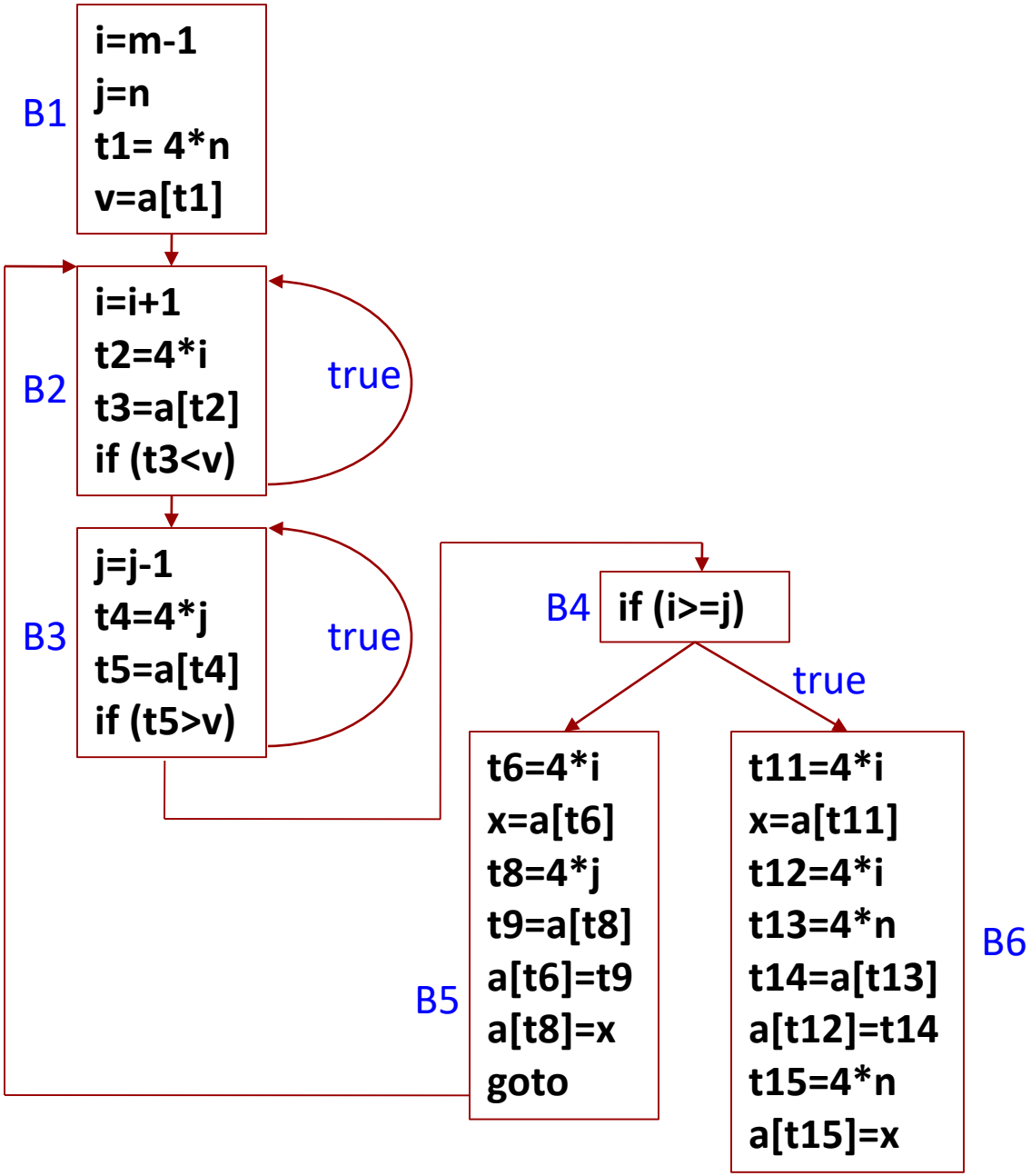
a[t7]=t9 becomes **a[t6]=t9**
a[t10]=x becomes **a[t8]=t9**

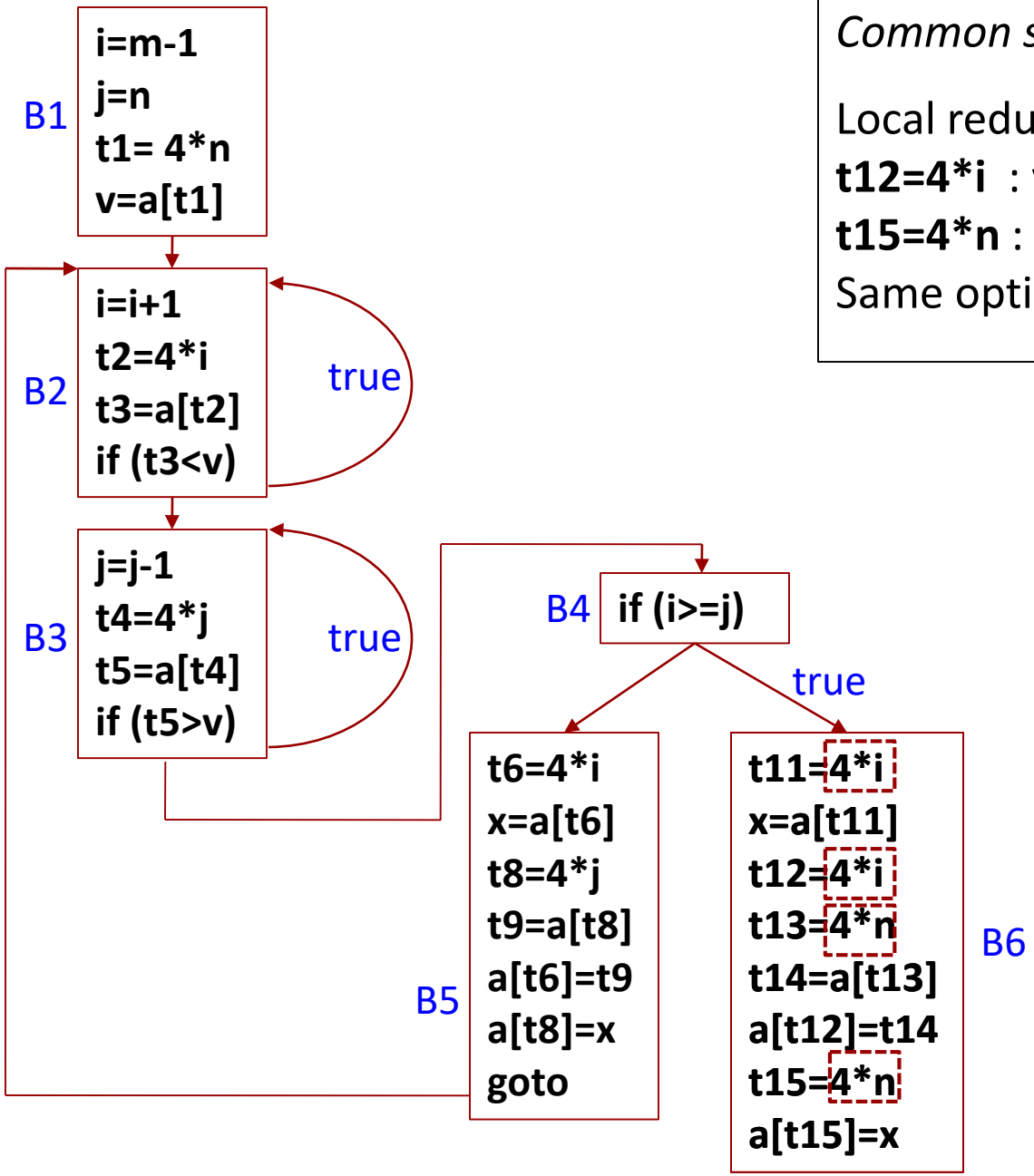


Dead code elimination

t7 is dead at the end of B5 (remember *Live Variables data-flow analysis*?) and is not used inside B5; so, **t7=t6** is dead code

t10 is dead at the end of B5 and is not used inside B5; so, **t10=t8** is dead code


B1
i=m-1
j=n
t1= 4*n
v=a[t1]
B2
i=i+1
t2=4*i
t3=a[t2]
if (t3<v)
true
B3
j=j-1
t4=4*j
t5=a[t4]
if (t5>v)
true
B4
if (i>=j)
true
B5
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto
B6
t11=4*i
x=a[t11]
t12=4*i
t13=4*n
t14=a[t13]
a[t12]=t14
t15=4*n
a[t15]=x



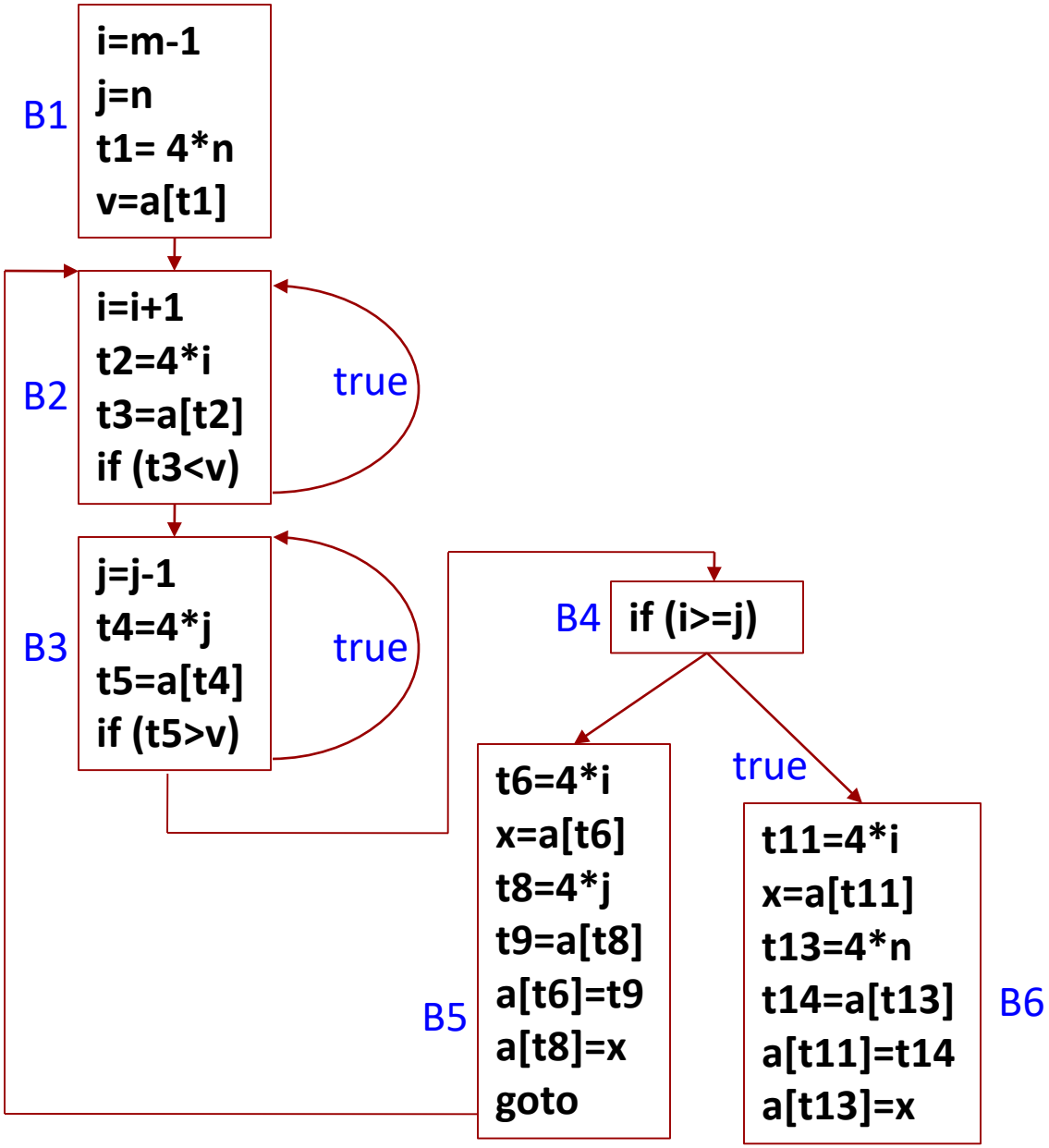
Common subexpression elimination

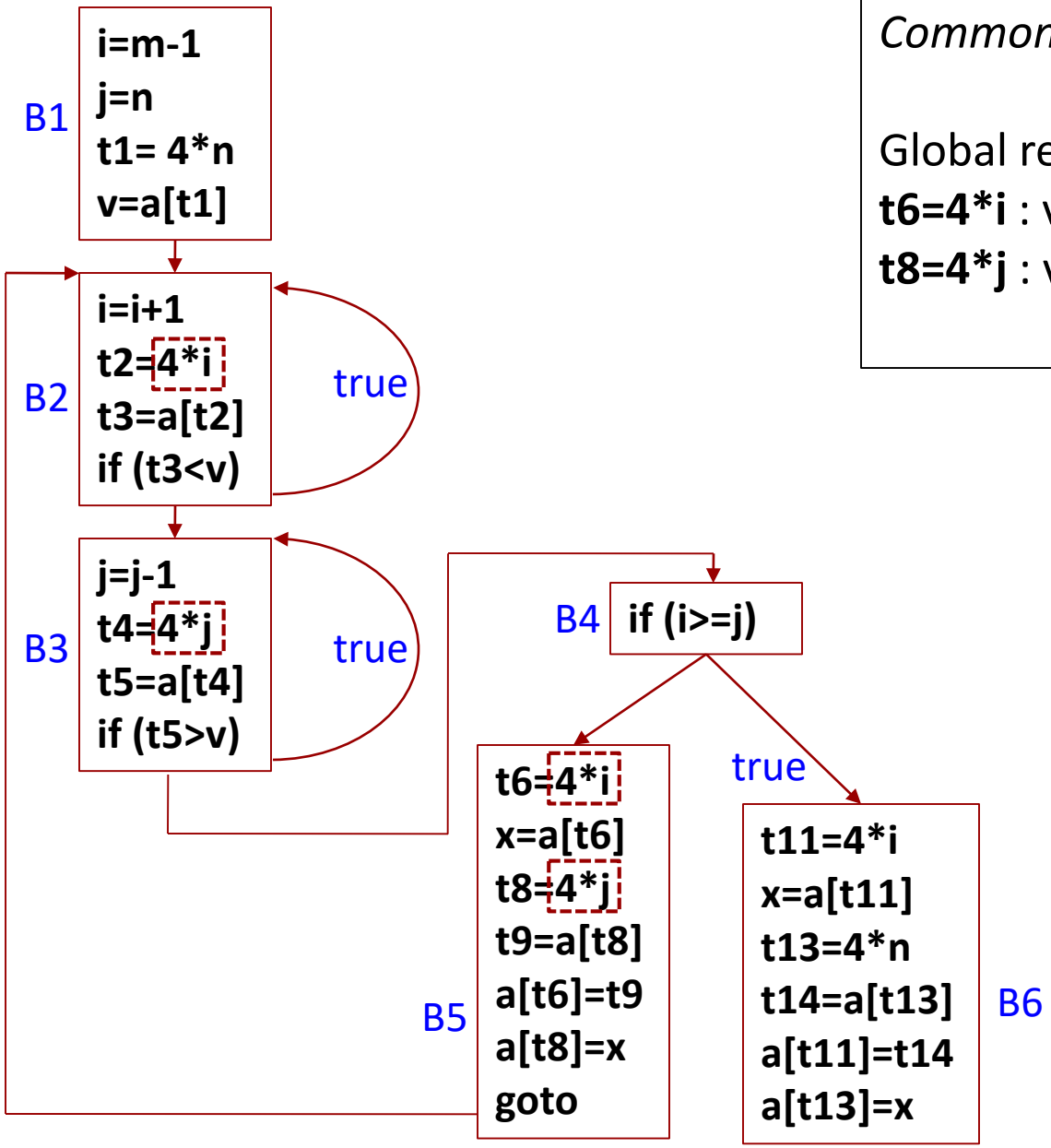
Local redundancy in B6:

t12=4*i : value already available in **t11**

t15=4*n : value already available in **t13**

Same optimizations as was done for B5


B1
i=m-1
j=n
t1= 4*n
v=a[t1]
B2
i=i+1
t2=4*i
t3=a[t2]
if (t3<v)
true
B3
j=j-1
t4=4*j
t5=a[t4]
if (t5>v)
true
B4
if (i>=j)
true
B5
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto
B6
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=t14
a[t13]=x

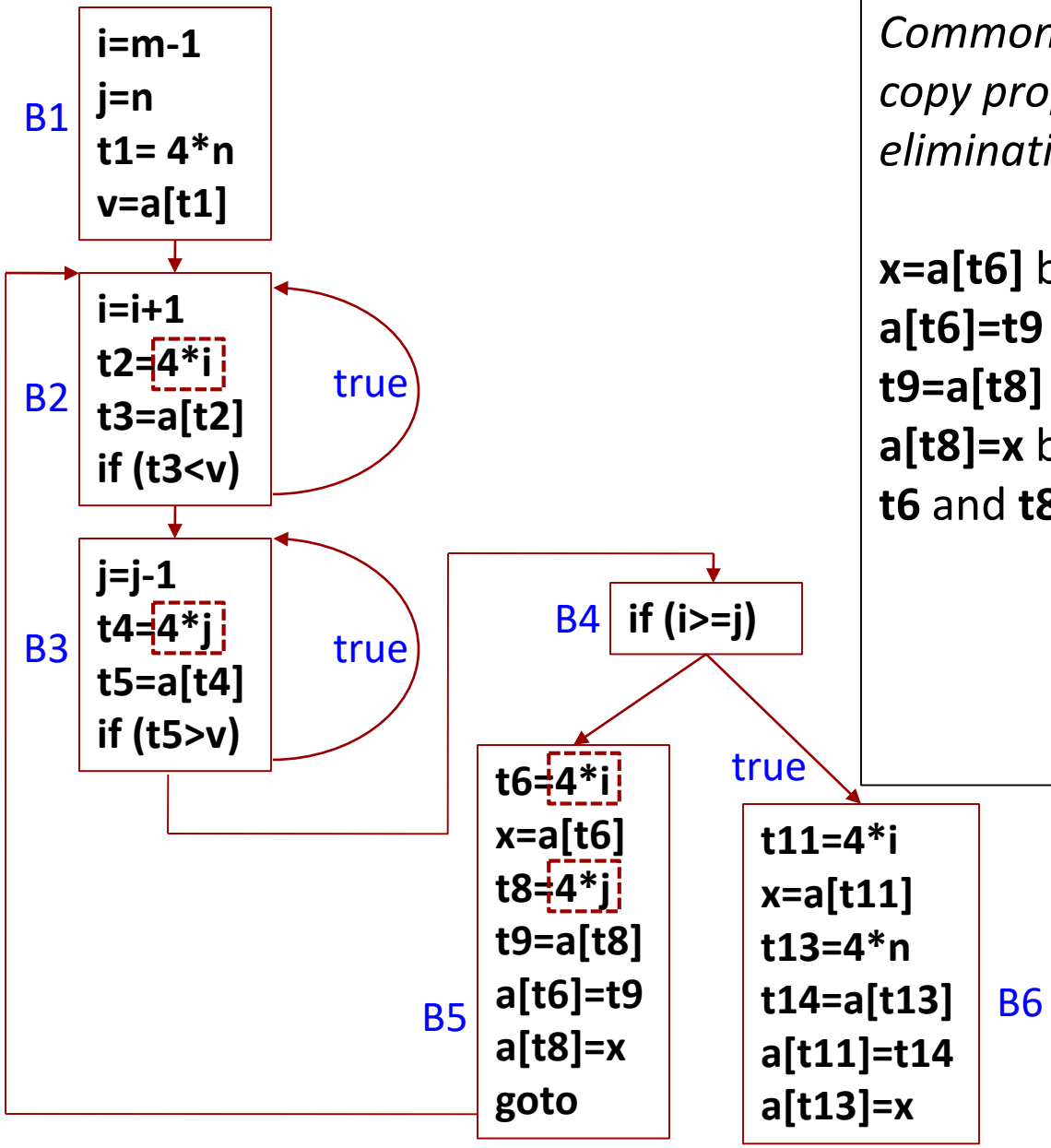


Common subexpression elimination

Global redundancy in B5:

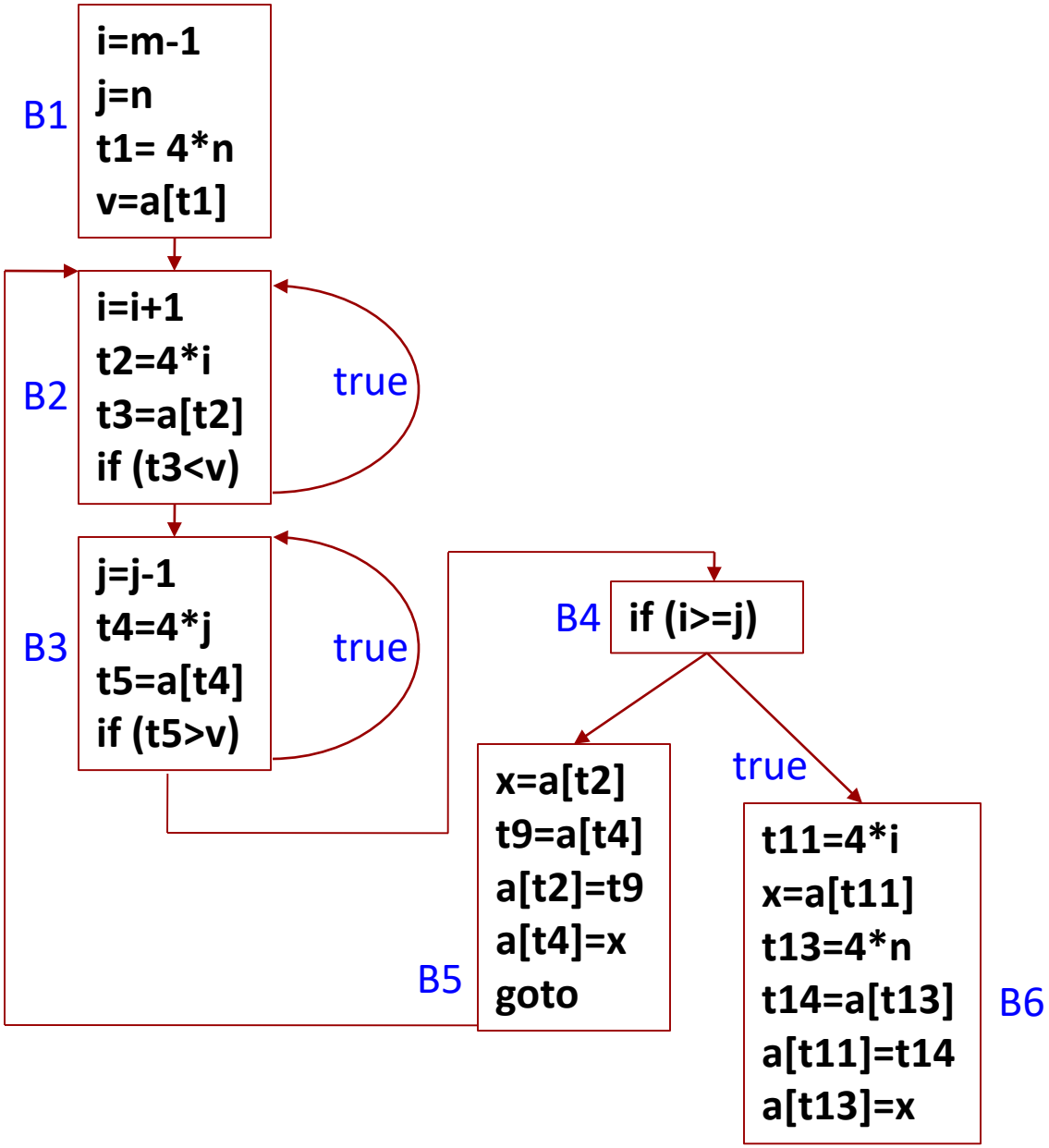
t6=4*i : value already available in **t2**

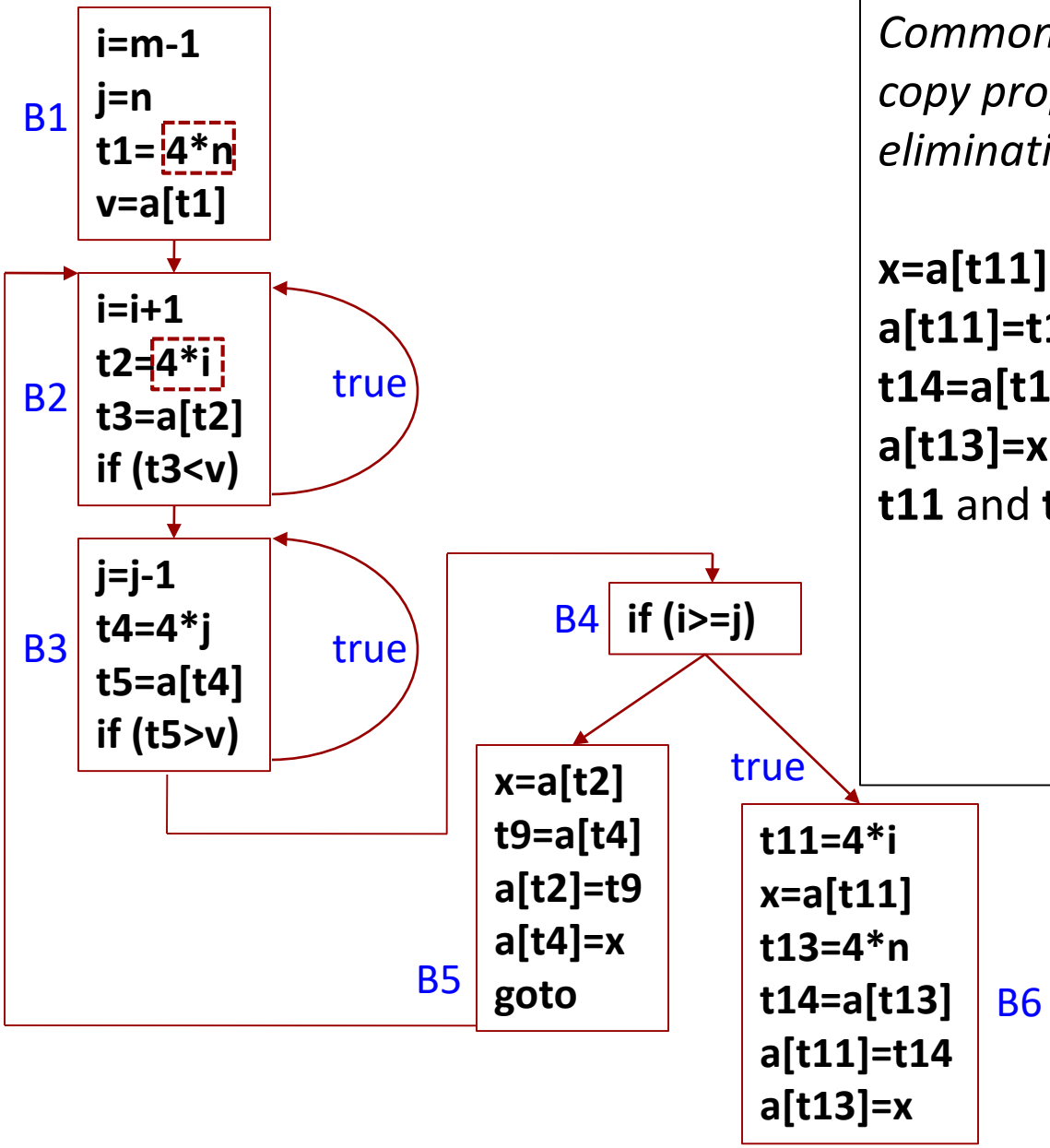
t8=4*j : value already available in **t4**



Common subexpression elimination, copy propagation, dead code elimination:

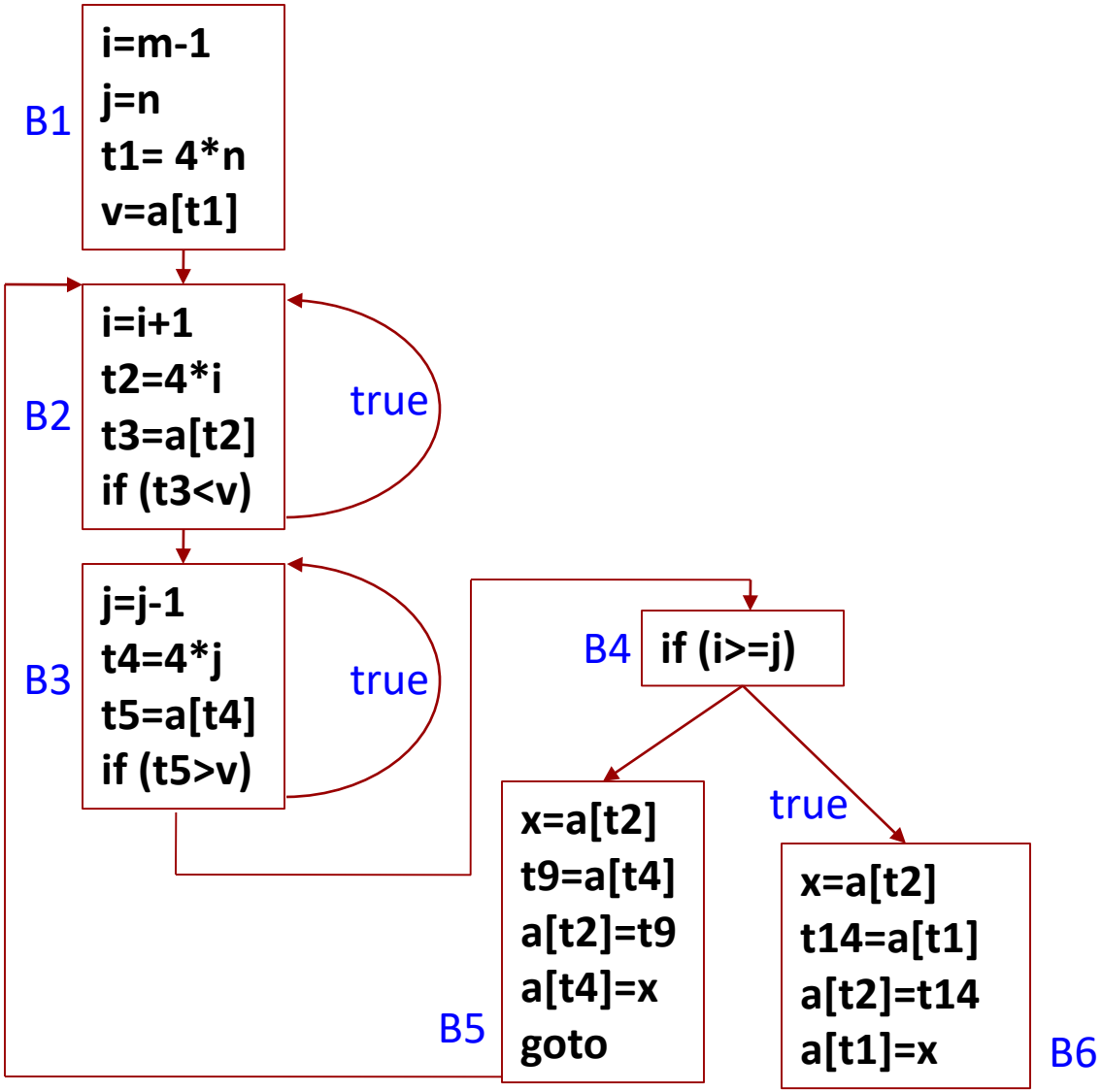
x=a[t6] becomes **x=a[t2]**
a[t6]=t9 becomes **a[t2]=t9**
t9=a[t8] becomes **t9=a[t4]**
a[t8]=x becomes **a[t4]=x**
t6 and **t8** are eliminated

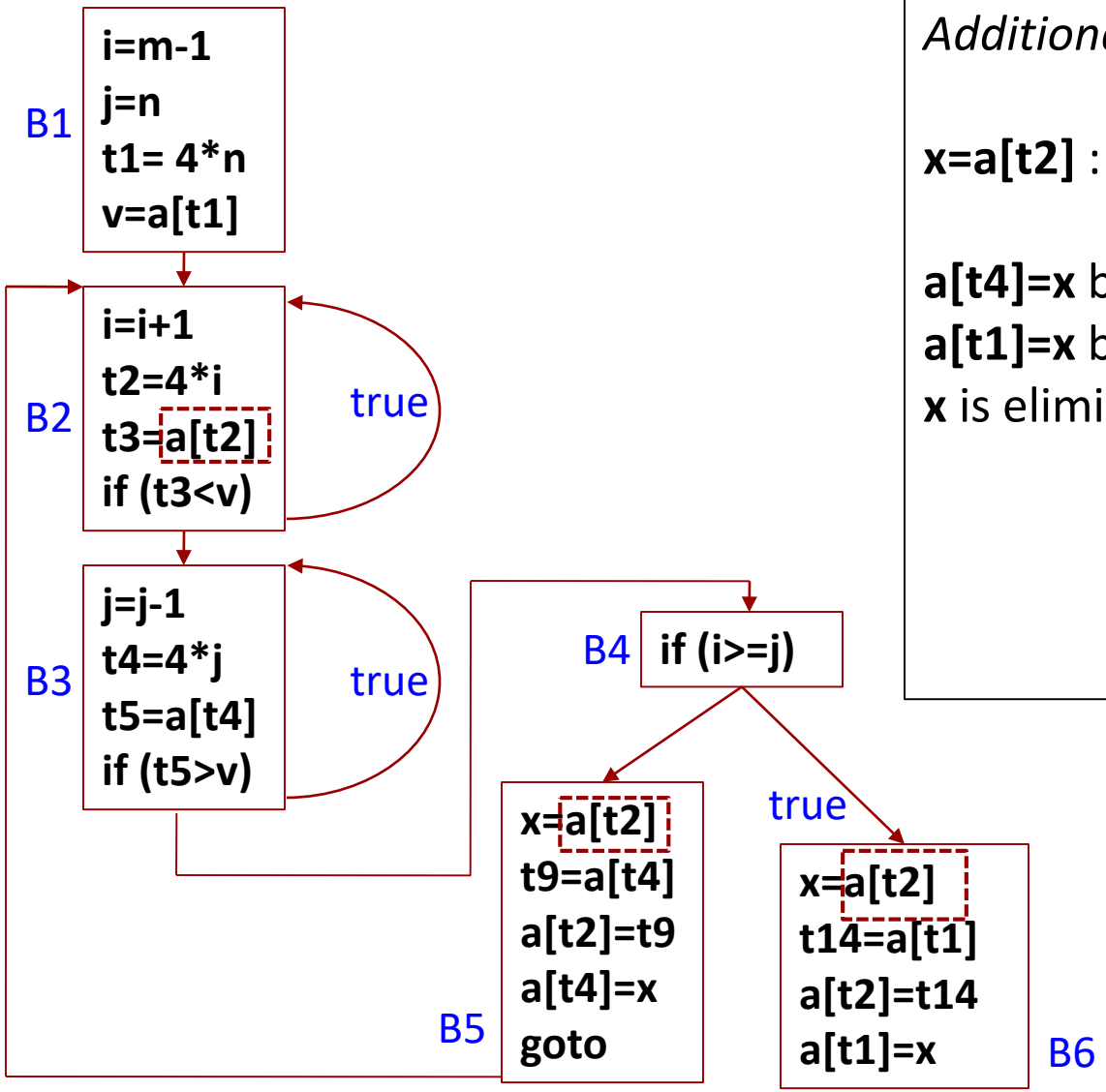

B1
i=m-1
j=n
t1= 4*n
v=a[t1]
B2
i=i+1
t2=4*i
t3=a[t2]
if (t3<v)
true
B3
j=j-1
t4=4*j
t5=a[t4]
if (t5>v)
true
B4
if (i>=j)
true
B5
x=a[t2]
t9=a[t4]
a[t2]=t9
a[t4]=x
goto
B6
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=t14
a[t13]=x



Common subexpression elimination, copy propagation, dead code elimination:

x=a[t11] becomes **x=a[t2]**
a[t11]=t14 becomes **a[t2]=t14**
t14=a[t13] becomes **t14=a[t1]**
a[t13]=x becomes **a[t1]=x**
t11 and **t13** are eliminated

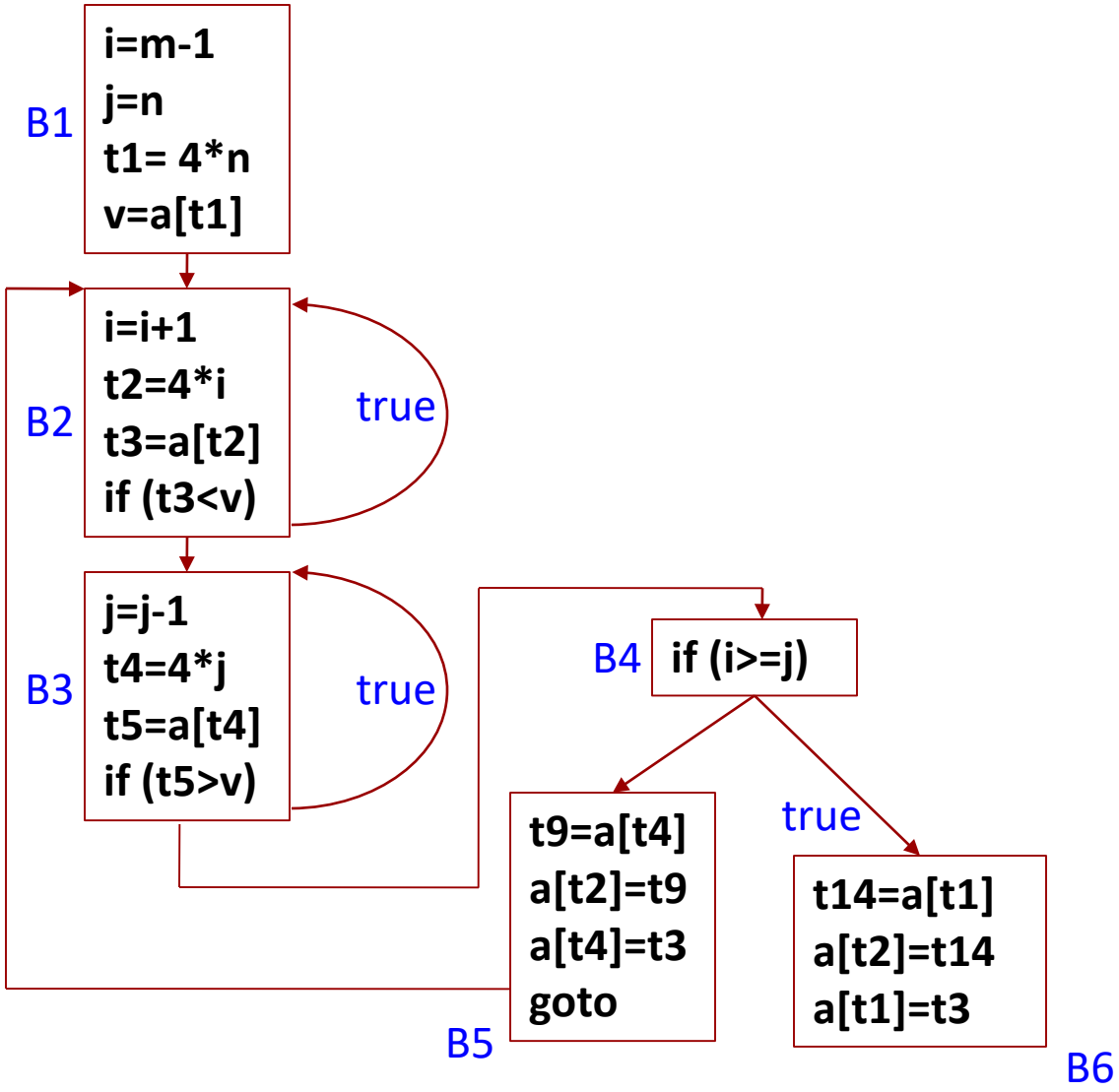

B1
i=m-1
j=n
t1= 4*n
v=a[t1]
B2
i=i+1
t2=4*i
t3=a[t2]
if (t3<v)
true
B3
j=j-1
t4=4*j
t5=a[t4]
if (t5>v)
true
B4
if (i>=j)
true
B5
x=a[t2]
t9=a[t4]
a[t2]=t9
a[t4]=x
goto
x=a[t2]
t14=a[t1]
a[t2]=t14
a[t1]=x
B6

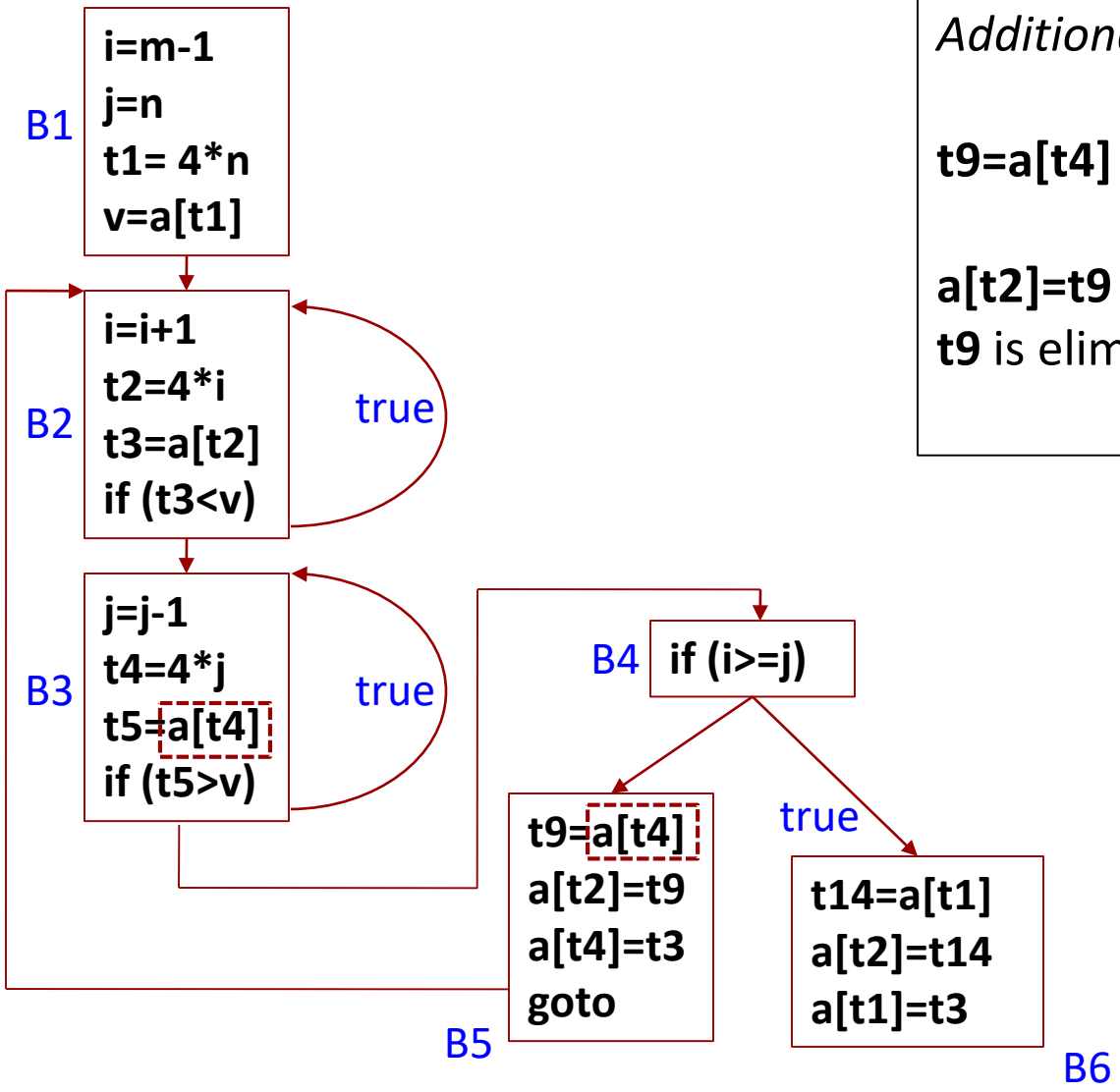


Additional common subexpressions?

x=a[t2] : value already available in **t3**

a[t4]=x becomes **a[t4]=t3**
a[t1]=x becomes **a[t1]=t3**
x is eliminated

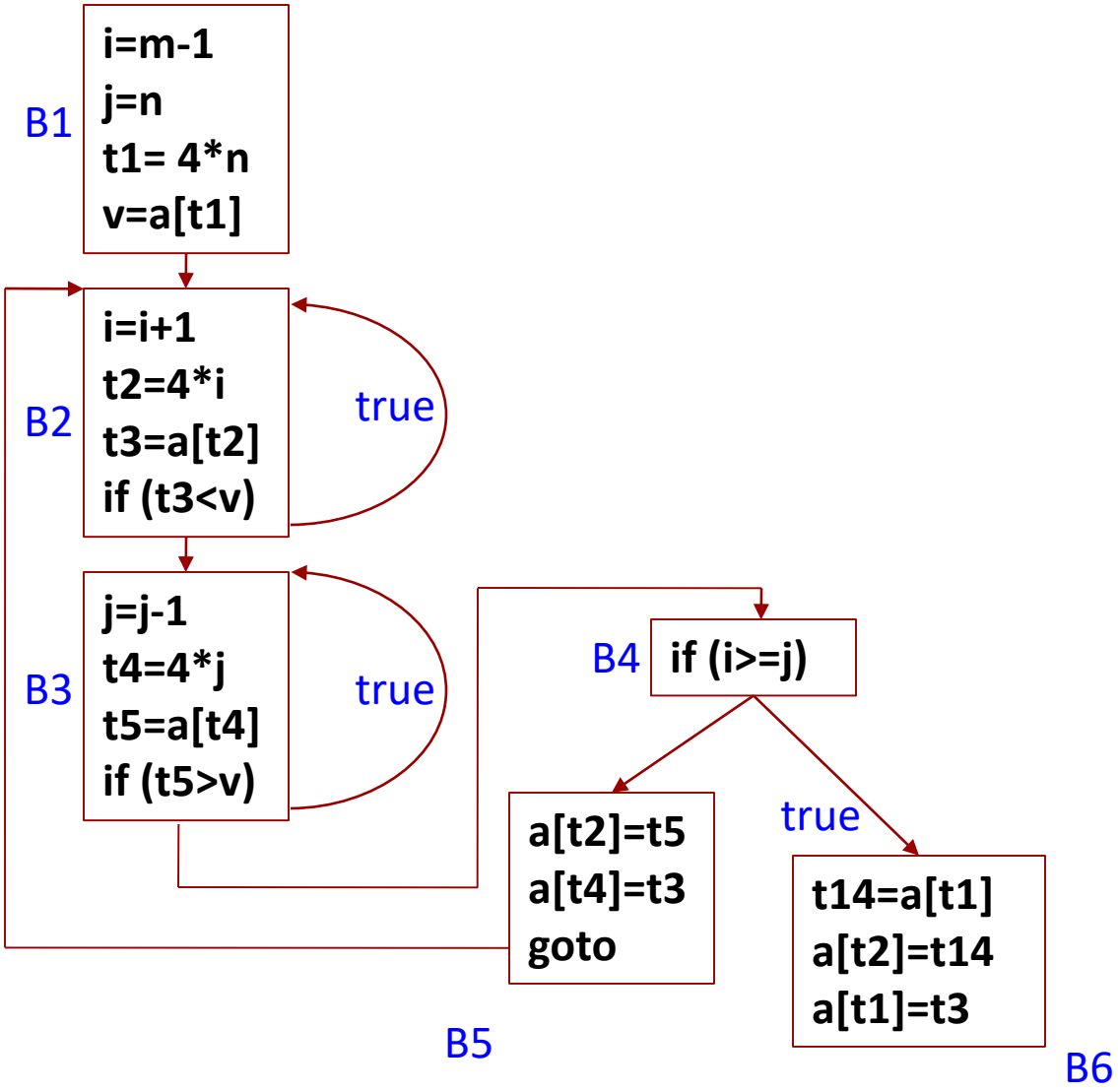

B1
i=m-1
j=n
t1= 4*n
v=a[t1]
B2
i=i+1
t2=4*i
t3=a[t2]
if (t3<v)
true
B3
j=j-1
t4=4*j
t5=a[t4]
if (t5>v)
true
B4
if (i>=j)
true
B5
t9=a[t4]
a[t2]=t9
a[t4]=t3
goto
B6
t14=a[t1]
a[t2]=t14
a[t1]=t3

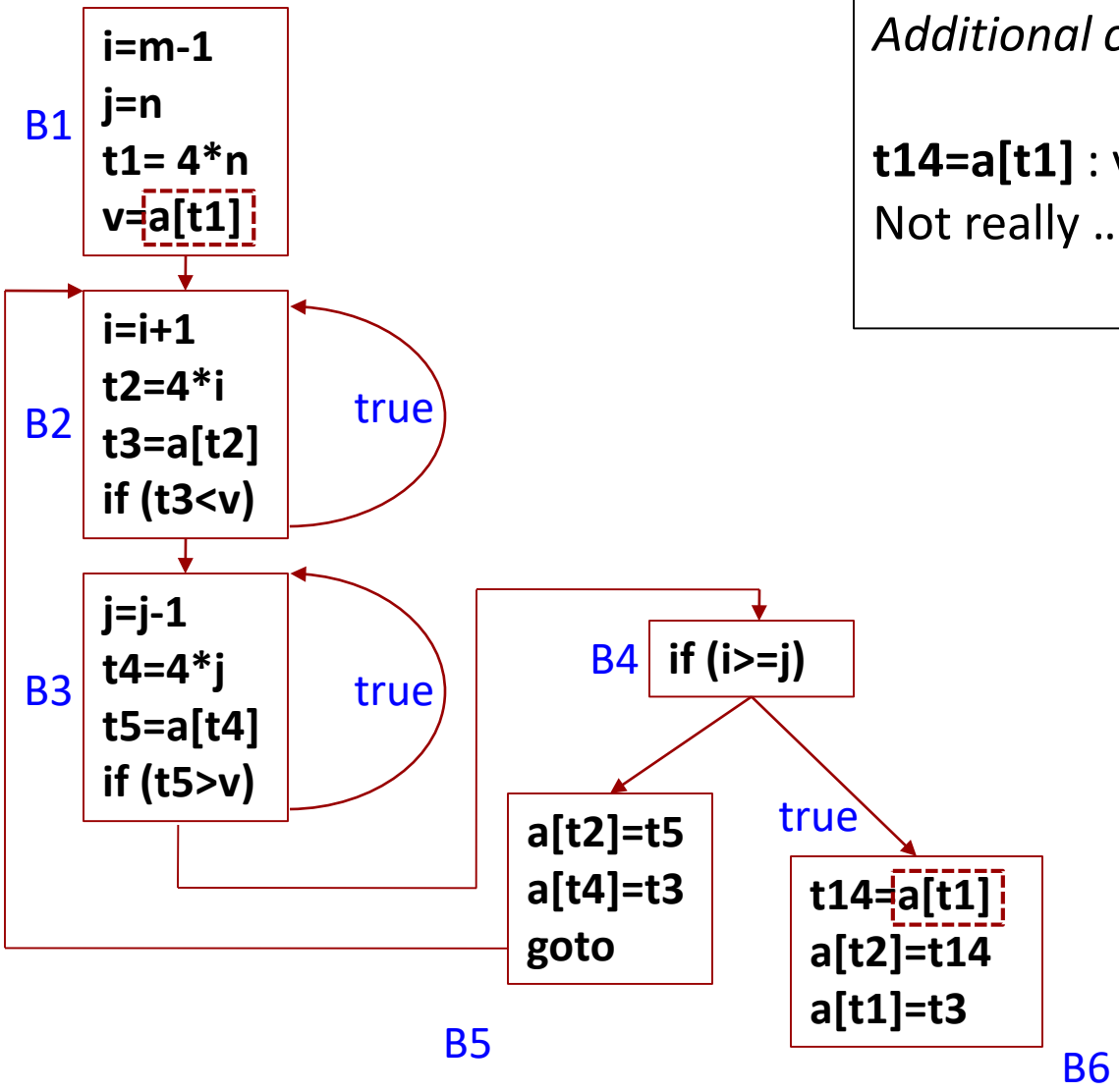


Additional common subexpressions?

t9=a[t4] : value already available in **t5**

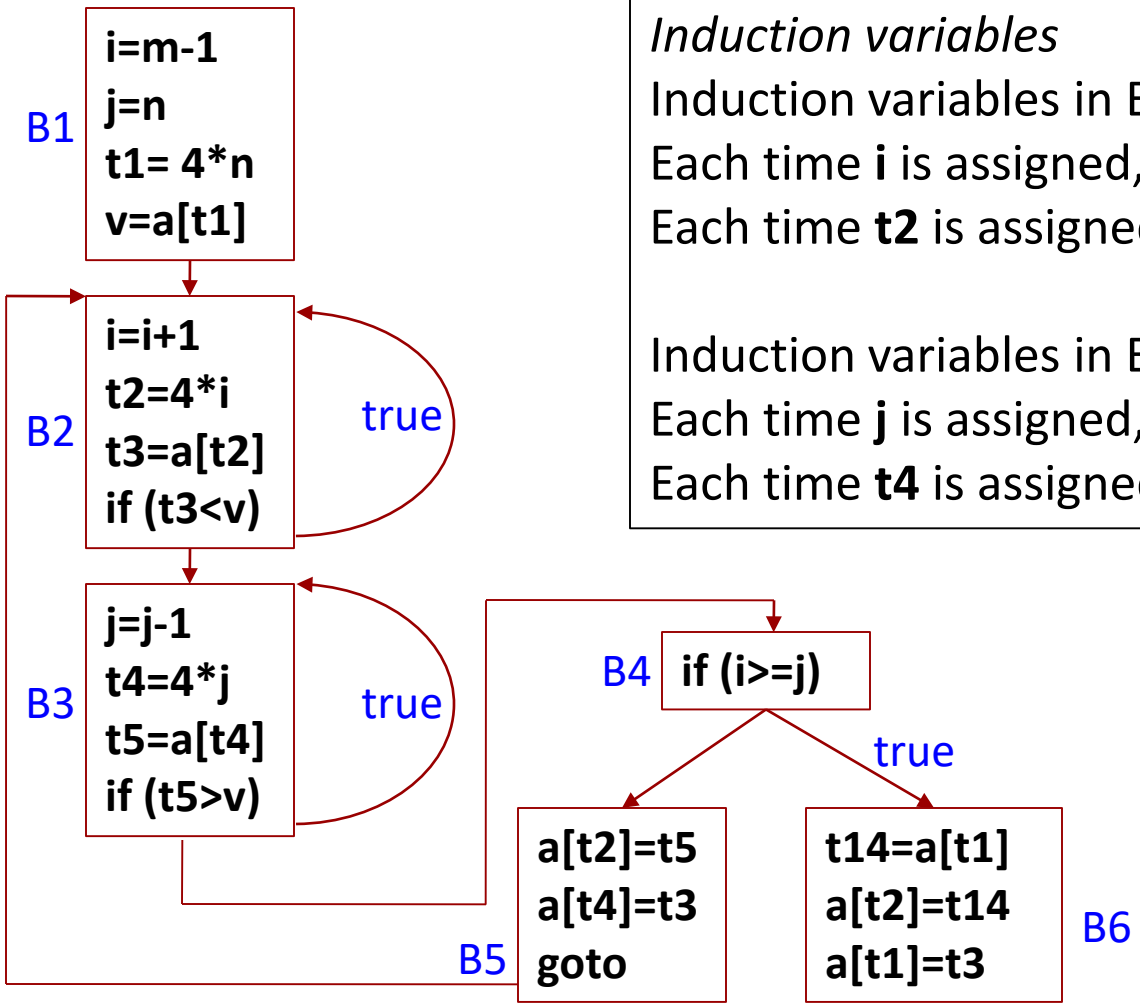
a[t2]=t9 becomes **a[t2]=t5**
t9 is eliminated


B1
i=m-1
j=n
t1= 4*n
v=a[t1]
B2
i=i+1
t2=4*i
t3=a[t2]
if (t3<v)
true
B3
j=j-1
t4=4*j
t5=a[t4]
if (t5>v)
true
B4
if (i>=j)
true
a[t2]=t5
a[t4]=t3
goto
B5
t14=a[t1]
a[t2]=t14
a[t1]=t3
B6



Additional common subexpressions?

t14=a[t1] : value already available in **v**?
Not really …



Induction variables

Induction variables in B2:

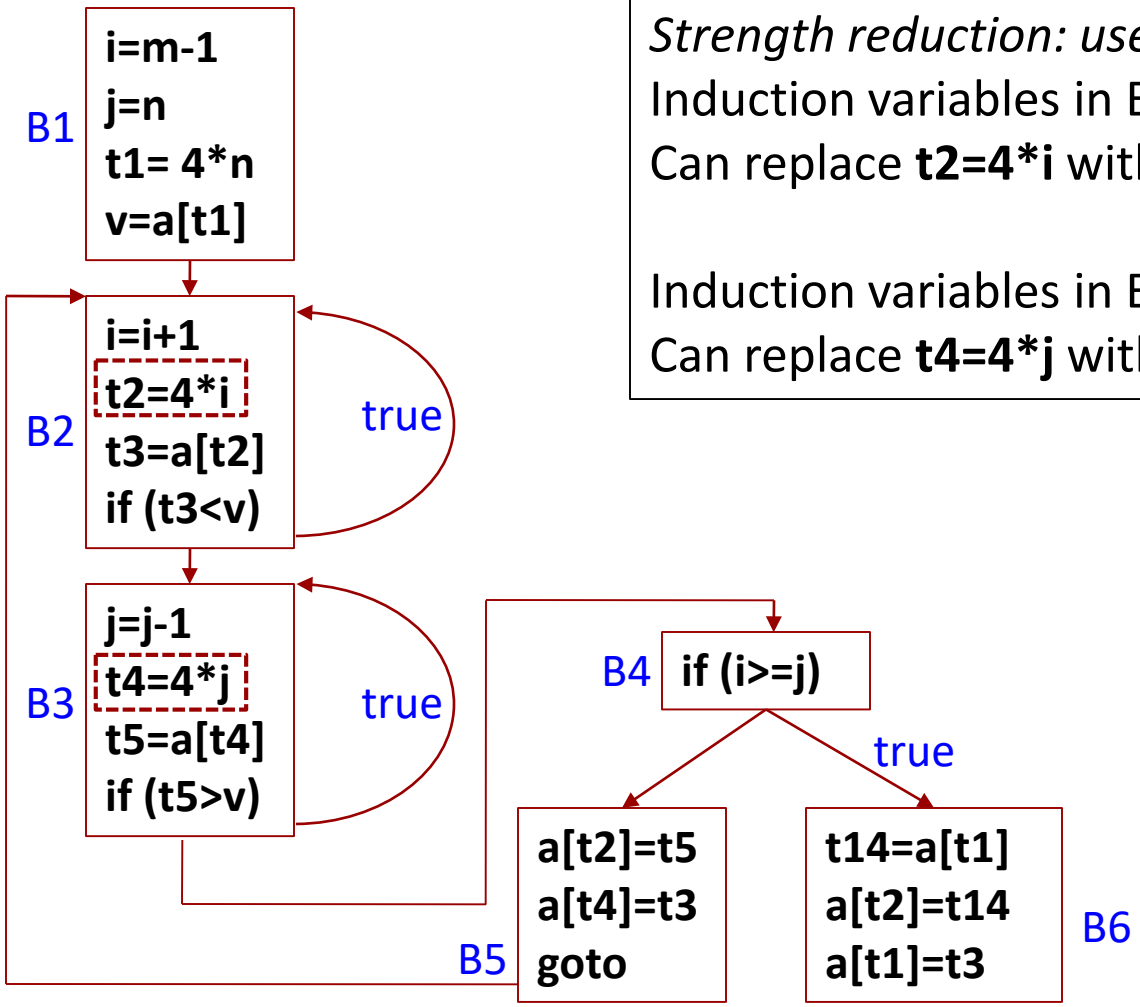
Each time **i** is assigned, its value increases by 1

Each time **t2** is assigned, its value increases by 4

Induction variables in B3:

Each time **j** is assigned, its value decreases by 1

Each time **t4** is assigned, its value decreases by 4



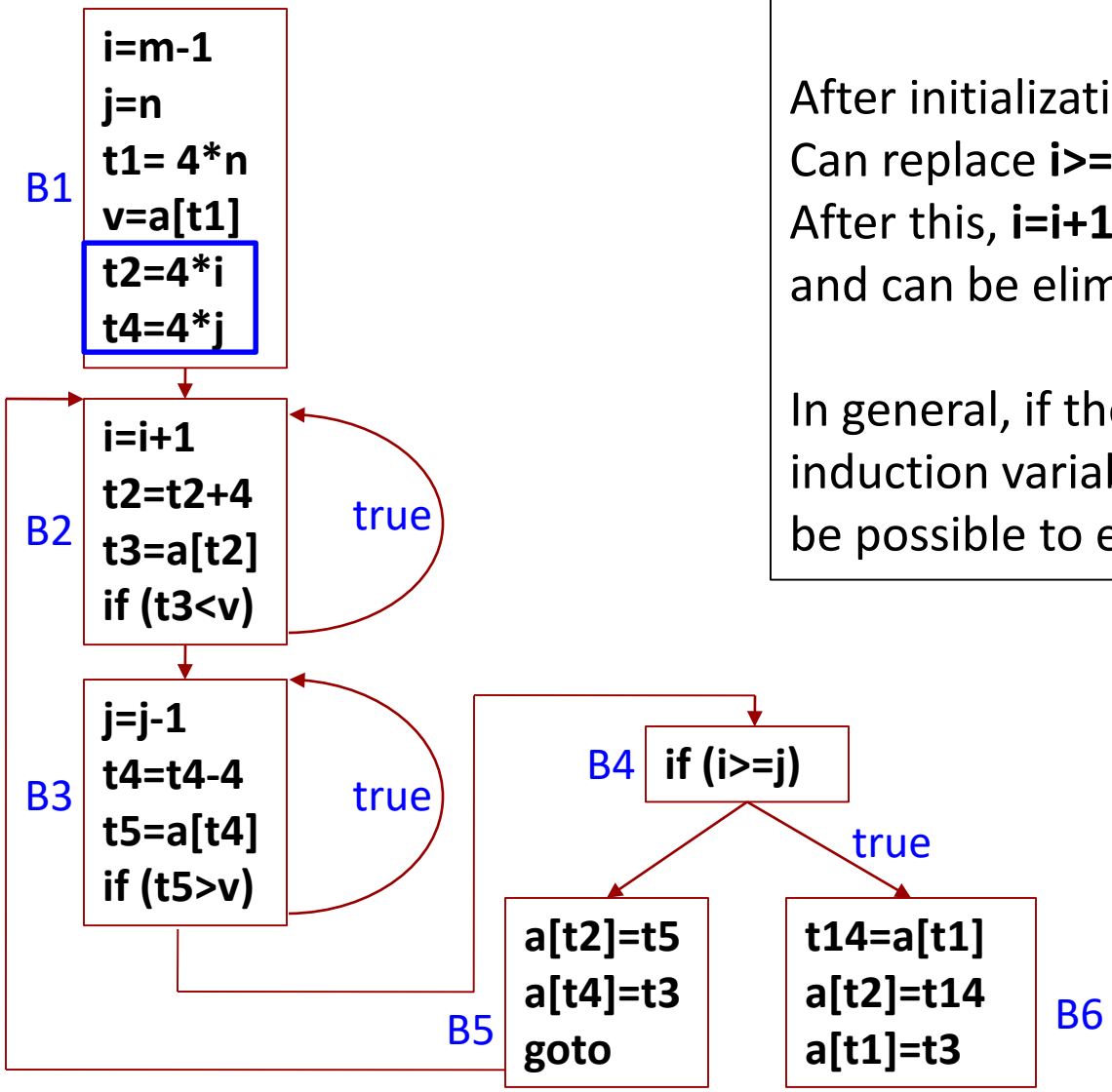
*Strength reduction: use +/- instead of **

Induction variables in B2:

Can replace **t2=4*i** with **t2=t2+4**

Induction variables in B3:

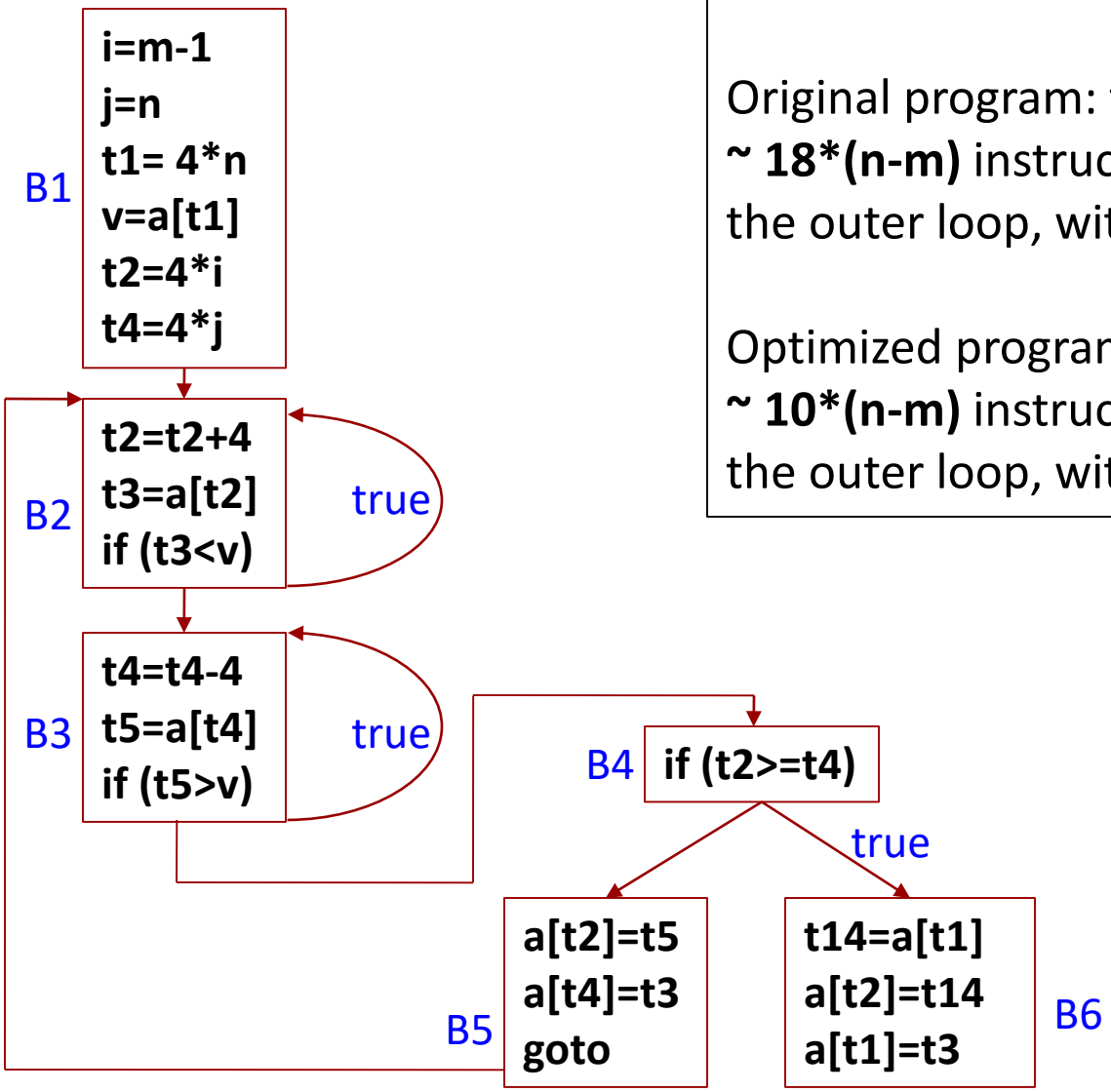
Can replace **t4=4*j** with **t4=t4-4**



Elimination of induction variables

After initialization, **i** and **j** are used only in B4
Can replace **i>=j** with **t2>=t4** in B4
After this, **i=i+1** and **j=j-1** become dead code and can be eliminated

In general, if there are two or more induction variables in the same loop, it may be possible to eliminate all but one of them



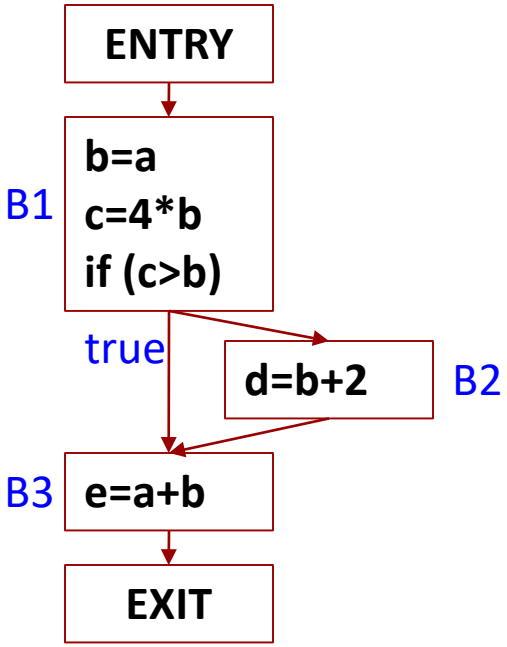
Final program after all optimizations

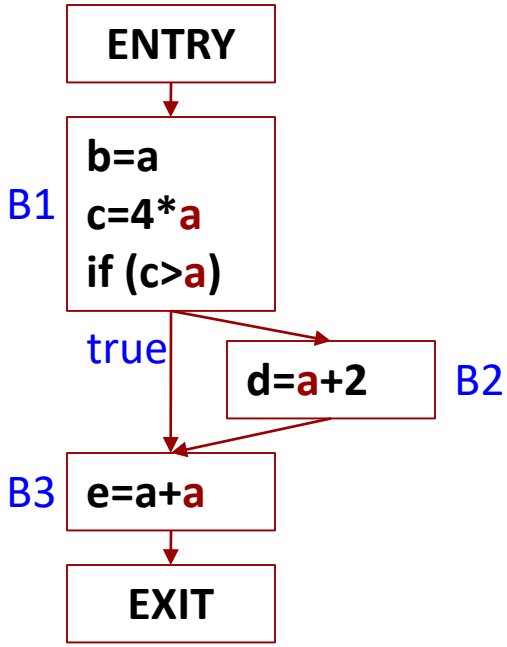
Original program: for the worst-case input, **~ 18*(n-m)** instructions would be executed in the outer loop, with **~ 6*(n-m)** multiplications

Optimized program: for the worst-case input, **~ 10*(n-m)** instructions would be executed in the outer loop, without any multiplications

(start detour) Another Dataflow Analysis

- Copy propagation: for **x = y**, replace subsequent uses of **x** with **y**, as long as x and y have not changed along the way
  - Creates opportunities for dead code elimination: e.g., after copy propagation we may find that **x** is not live





1) Dead code elimination: **b=a**
2) Strength reduction: **e=a+a** use left shift instead of addition

Formulation as a System of Equations

- For each CFG node *n* (assume nodes = instructions)

$$\mathrm{IN}[n] = \bigsqcap_{m \in \mathrm{Predecessors}(n)} \mathrm{OUT}[m] \qquad \mathrm{OUT}[\mathrm{ENTRY}] = \varnothing$$

$$\mathrm{OUT}[n] = (\mathrm{IN}[n] - \mathrm{KILL}[n]) \cup \mathrm{GEN}[n]$$

  - IN[*n*] is a set of copy instructions **x=y** such that nether **x** nor **y** is assigned along any path from x=y to *n*
  - GEN[*n*] is
    - A singleton set containing the copy instruction, when *n* is indeed a copy instruction
    - The empty set, otherwise
  - KILL[*n*]: if *n* assigns to **x**, kill every **y=x** and **x=y**
  - Note that we must use *intersection* of OUT[*m*]

Worklist Algorithm (end detour)

IN[*n*] = *the set of all copy instructions*, for all *n*

Put the successor of ENTRY on *worklist*

While (*worklist* is not empty)

1. Remove a CFG node *m* from the worklist
2. OUT[*m*] = (IN[*m*] – KILL[*m*]) ∪ GEN[*m*]
3. For each successor *n* of *m*

   old = IN[*n*]

   IN[*n*] = IN[*n*] ∩ OUT[*m*]

   If (*old* != IN[*n*]) add *n* to *worklist*

In Reaching Definitions, we initialized IN[*n*] to the empty set; here we cannot do this, because of IN[*n*] = IN[*n*] ∩ OUT[*m*]

- Here the "merge" operation is *set intersection*
- In Reaching Definitions, "merge" is *set union*

Part 4: Loop Optimizations

- Loops are important for performance
- Parallelization
  - May need scalar expansion
- Loop peeling
- Loop unrolling
- Loop fusion
- Many more [CSE 5441 - Introduction to Parallel Computing]
  - Loop permutation (interchange)
  - Loop distribution (fission)
  - Loop tiling
  - Loop skewing
  - Index set splitting (generalization of peeling)

Loop Parallelization

- When all iterations of a loop are independent of each other

```
for ( i = 0 ; i < 4096 ; i++ )
    c[i] = a[i] + b[i];
```

- Needs some form of loop dependence analysis, which often involves reasoning about arrays
- May require enabling pre-transformations to make it parallel (e.g., scalar expansion or privatization)

```
for ( i = 0 ; i < 4096 ; i++ ) {
    t = a[i] + b[i];
    c[i] = t*t; }
```

```
double tx[4096];
for ( i = 0 ; i < 4096 ; i++ ) {
    tx[i] = a[i] + b[i];
    c[i] = tx[i]*tx[i]; }
t = tx[4095];
```

Scalar expansion example

Loop Peeling

- Goal: extract the first (or last) iteration
  - E.g. wraparound variables for cylindrical coordinates

```
j = N;
for ( i = 0 ; i < N ; i++ ) {
   b[i] = (a[i] + a[j]) / 2;
   j = i; } // assume  j is not live here
```

```
b[0] = (a[0] + a[N]) / 2;
b[1] = (a[1] + a[0]) / 2;
b[2] = (a[2] + a[1]) / 2
...
```

- Peel-off the first iteration, then do induction variable analysis and copy propagation

```
j = N;
if (N>=1) {
  b[0] = (a[0] + a[j]) / 2;
  j = 0;
  for ( i = 1 ; i < N ; i++ ) {
     b[i] = (a[i] + a[j]) / 2;
     j = i; } }
```

```
if (N>=1) {
  b[0] = (a[0] + a[N]) / 2;
  for ( i = 1 ; i < N ; i++ ) {
     b[i] = (a[i] + a[i-1]) / 2; } }
// now we can do unrolling
```

Loop Unrolling

- Loop unrolling: extend the body

```
for ( i = 0 ; i < 4096 ; i++ )
    c[i] = a[i] + b[i];
```

```
for ( i = 0 ; i < 4095 ; i +=2 ) {
    c[i] = a[i] + b[i];
    c[i+1] = a[i+1] + b[i+1];
} // unroll factor of 2
```

  - Reduces the "control overhead" of the loop: makes the loop exit test (i < 4096) execute less frequently
  - Hardware advantages: instruction-level parallelism; fewer pipeline stalls
  - Issue: loop bound may not be a multiple of unroll factor
  - Problem: high unroll factors may degrade performance due to register pressure and spills (more later)

Loop Fusion

- Merge two loops with compatible bounds

```
for ( i = 0 ; i < N ; i++ )
   c[i] = a[i] + b[i];
for ( j = 0 ; j < N ; j++)
   d[j] = c[j] * 2;
```

```
for ( k = 0 ; k < N ; k++ ) {
   c[k] = a[k] + b[k];
   d[k] = c[k] * 2;
}
```

  - Reduces the loop control overhead (i.e., loop exit test)
  - May improve cache use (e.g. the reuse of **c[k]** above) – especially producer/consumer loops [↓capacity misses]
  - Fewer parallelizable loops and increased work per loop: reduces parallelization overhead (cost of spawn/join)
  - If the loop bounds are "±1 off" – use peeling
  - Not always legal – need loop dependence analysis