Control-Flow Analysis

"Dragon book" [Ch. 8, Section 8.4; Ch. 9, Section 9.6]

Compilers: Principles, Techniques, and Tools, 2nd ed. by

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Control-Flow Graphs

- Control-flow graph (CFG) for a procedure/method
  - A node is a basic block: a single-entry-single-exit sequence of three-address instructions
  - An edge represents the potential flow of control from one basic block to another
- Uses of a control-flow graph
  - Inside a basic block: local code optimizations; done as part of the code generation phase
  - Across basic blocks: global code optimizations; done as part of the code optimization phase
  - Aspects of code generation: e.g., global register allocation

Control-Flow Analysis

- Part 1: Constructing a CFG
- Part 2: Finding dominators and post-dominators
- Part 3: Finding loops in a CFG
  - What exactly is a loop?We cannot simply say “whatever CFG subgraph is generated by *while*, *do-while*, and *for* statements” – need a general graph-theoretic definition
- Part 4: Static single assignment form (SSA)
- Part 5: Finding control dependences
  - Necessary as part of constructing the program dependence graph (PDG), a popular IR for software tools for slicing, refactoring, testing, and debugging

Part 1: Constructing a CFG

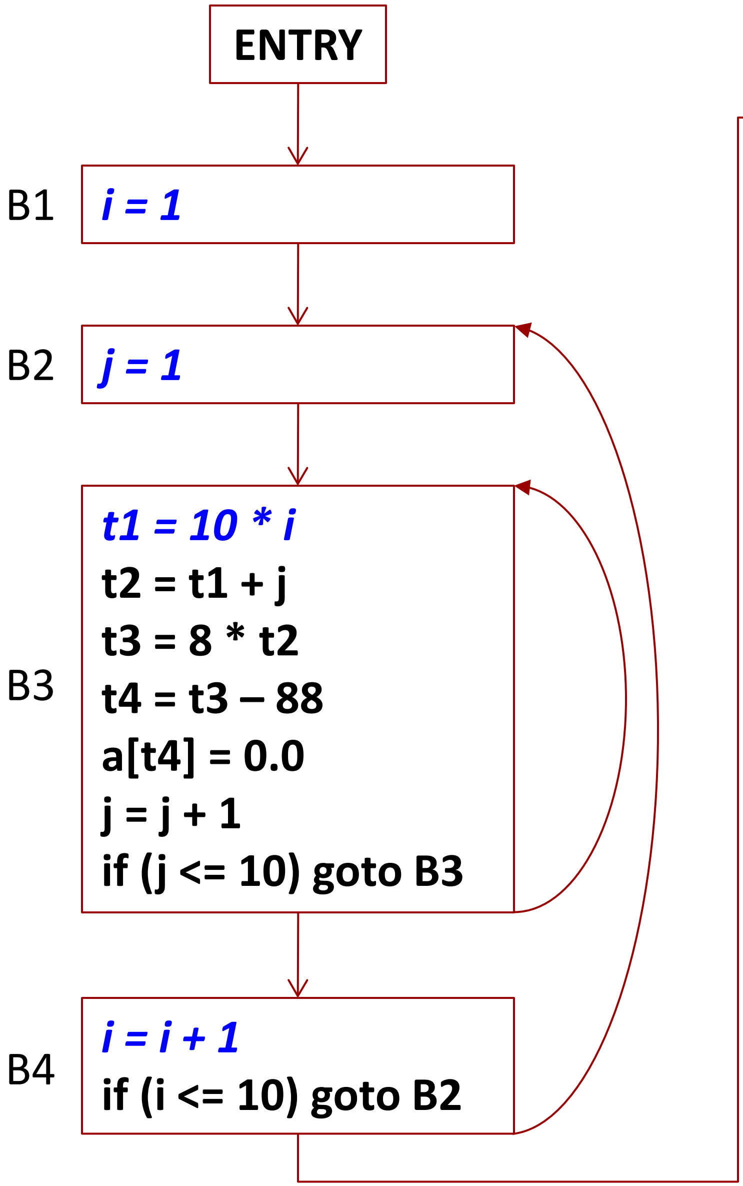
- Basic block: maximal sequence of consecutive three-address instructions such that
  - The flow of control can enter only through the first instruction (i.e., no jumps into the middle of the block)
  - The flow of control can exit only at the last instruction
- Given: the entire sequence of instructions
- First, find the leaders (starting instructions of all basic blocks)
  - The first instruction
  - The target of any conditional/unconditional jump
  - Any instruction that immediately follows a conditional or unconditional jump

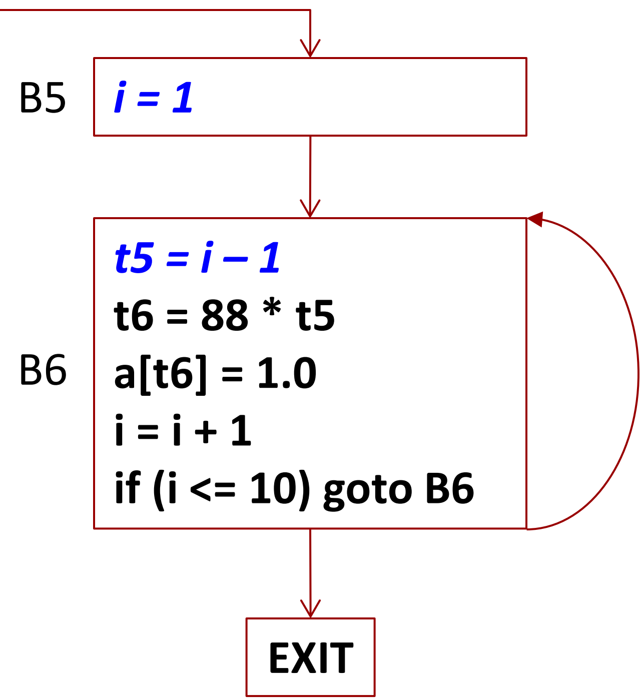
Constructing a CFG

- Next, find the basic blocks: for each leader, its basic block contains itself and all instructions up to (but not including) the next leader

Instruction	Leader
1. i = 1	First instruction
2. j = 1	Target of 11
*3. t1 = 10 * i*	Target of 9
4. t2 = t1 + j	
5. t3 = 8 * t2	
6. t4 = t3 – 88	
7. a[t4] = 0.0	
8. j = j + 1	
9. if (j <= 10) goto (3)	
10. i = i + 1	Follows 9
11. if (i <= 10) goto (2)	
12. i = 1	Follows 11
13. t5 = i – 1	Target of 17
14. t6 = 88 * t5	
15. a[t6] = 1.0	
16. i = i + 1	
17. if (i <= 10) goto (13)	

Note: this example sets array elements a[i][j] to 0.0, for 1 <= i,j <= 10 (instructions 1-11). It then sets a[i][i] to 1.0, for 1 <= i <= 10 (instructions 12-17). The array accesses in instructions 7 and 15 are done with offsets computed as described in Section 6.4.3, assuming row-major order, 8-byte array elements, and array indexing that starts from 1, not from 0.





Artificial ENTRY and EXIT nodes are often added for convenience.

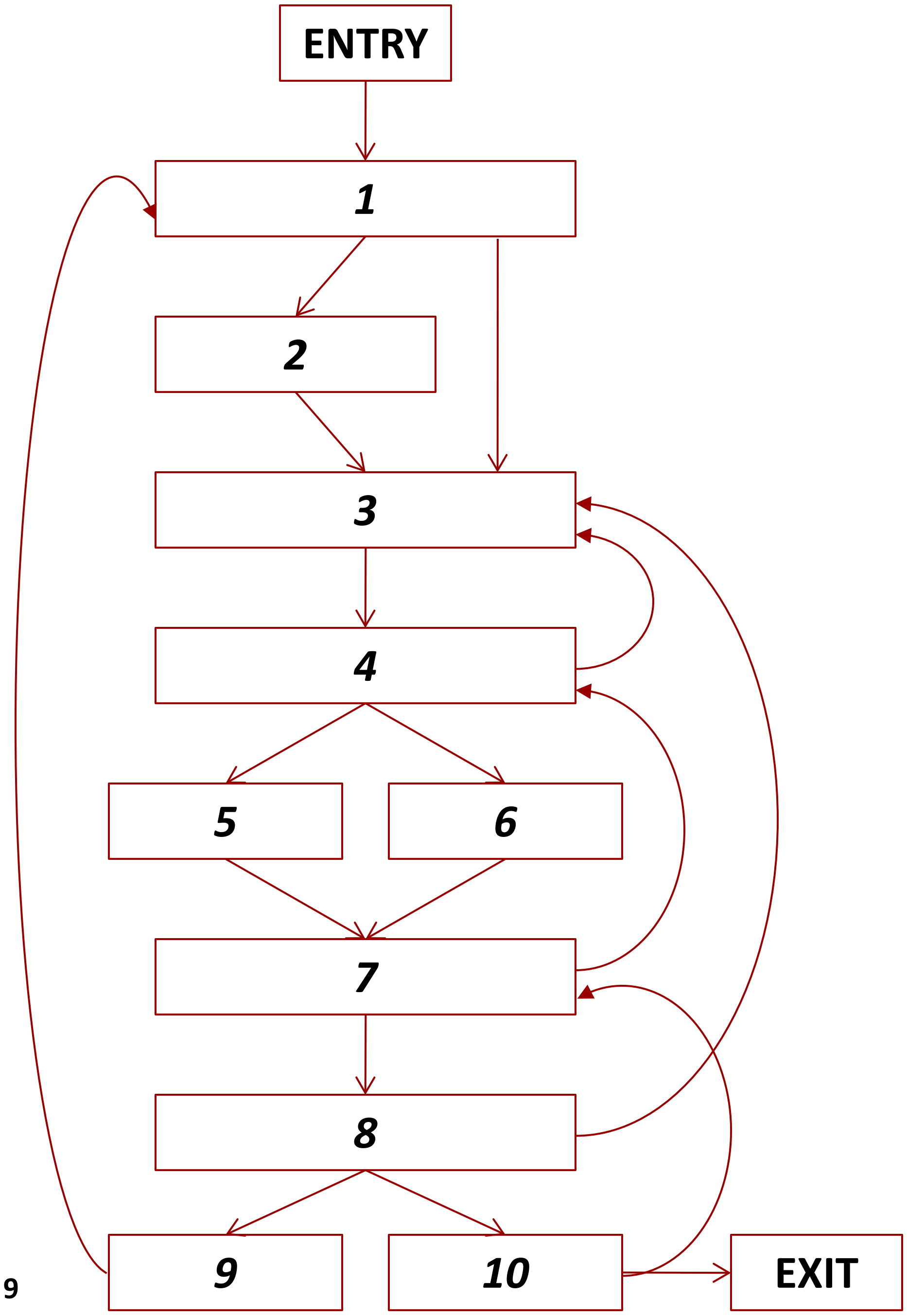
There is an edge from B_p to B_q if it is possible for the first instruction of B_q to be executed immediately after the last instruction of B_p. This is conservative: e.g., **if (3.14 > 2.78)** still generates two edges.

Practical Considerations

- The usual data structures for graphs can be used
  - The graphs are sparse (i.e., have relatively few edges), so an adjacency list representation is the usual choice
    - Number of edges is at most 2 * number of nodes
- Nodes are basic blocks; edges are between basic blocks, not between instructions
  - Inside each node, some additional data structures for the sequence of instructions in the block (e.g., a linked list of instructions)
  - Often convenient to maintain both a list of successors (i.e., outgoing edges) and a list of predecessors (i.e., incoming edges) for each basic block

Part 2: Dominance

- A CFG node d dominates another node n if every path from ENTRY to n goes through d
  - Implicit assumption: every node is reachable from ENTRY (i.e., there is no dead code)
  - A dominance relation $dom \subseteq$ Nodes × Nodes: d dom n
  - The relation is trivially reflexive: d dom d
- Node m is the immediate dominator of n if
  - $m \neq n$
  - m dom n
  - For any $d \neq n$ such d dom n, we have d dom m
- Every node has a unique immediate dominator
  - Except ENTRY, which is dominated only by itself



ENTRY *dom n* for any *n*

1 dom n for any *n* except ENTRY

2 does not dominate any other node

3 *dom* 3, 4, 5, 6, 7, 8, 9, 10, EXIT

4 *dom* 4, 5, 6, 7, 8, 9, 10, EXIT

5 does not dominate any other node

6 does not dominate any other node

7 *dom* 7, 8, 9, 10, EXIT

8 *dom* 8, 9, 10, EXIT

9 does not dominate any other node

10 *dom* 10, EXIT

Immediate dominators:

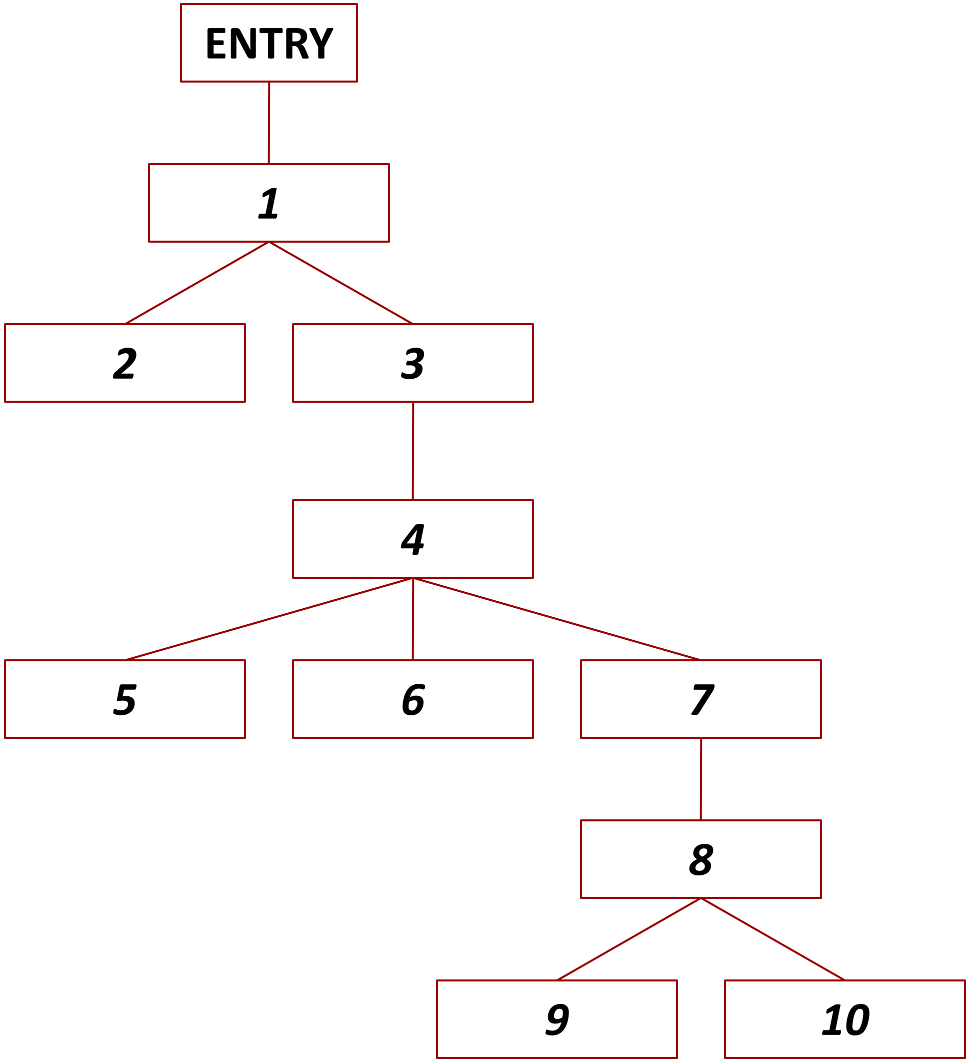
1 → ENTRY	2 → 1
3 → 1	4 → 3
5 → 4	6 → 4
7 → 4	8 → 7
9 → 8	10 → 8
	EXIT → 10

A Few Observations

- Dominance is a transitive relation: *a dom b* and *b dom c* means *a dom c*
- Dominance is an anti-symmetric relation: *a dom b* and *b dom a* means that *a* and *b* must be the same
  - Reflexive, anti-symmetric, transitive: **partial order**
- If *a* and *b* are two dominators of some *n*, either *a dom b* or *b dom a*
  - Therefore, *dom* is a **total order** for *n*'s dominator set
  - Corollary: for any acyclic path from ENTRY to *n*, all dominators of *n* appear along the path, always in the same order; the last one is the immediate dominator

Dominator Tree

- The parent of *n* is its immediate dominator



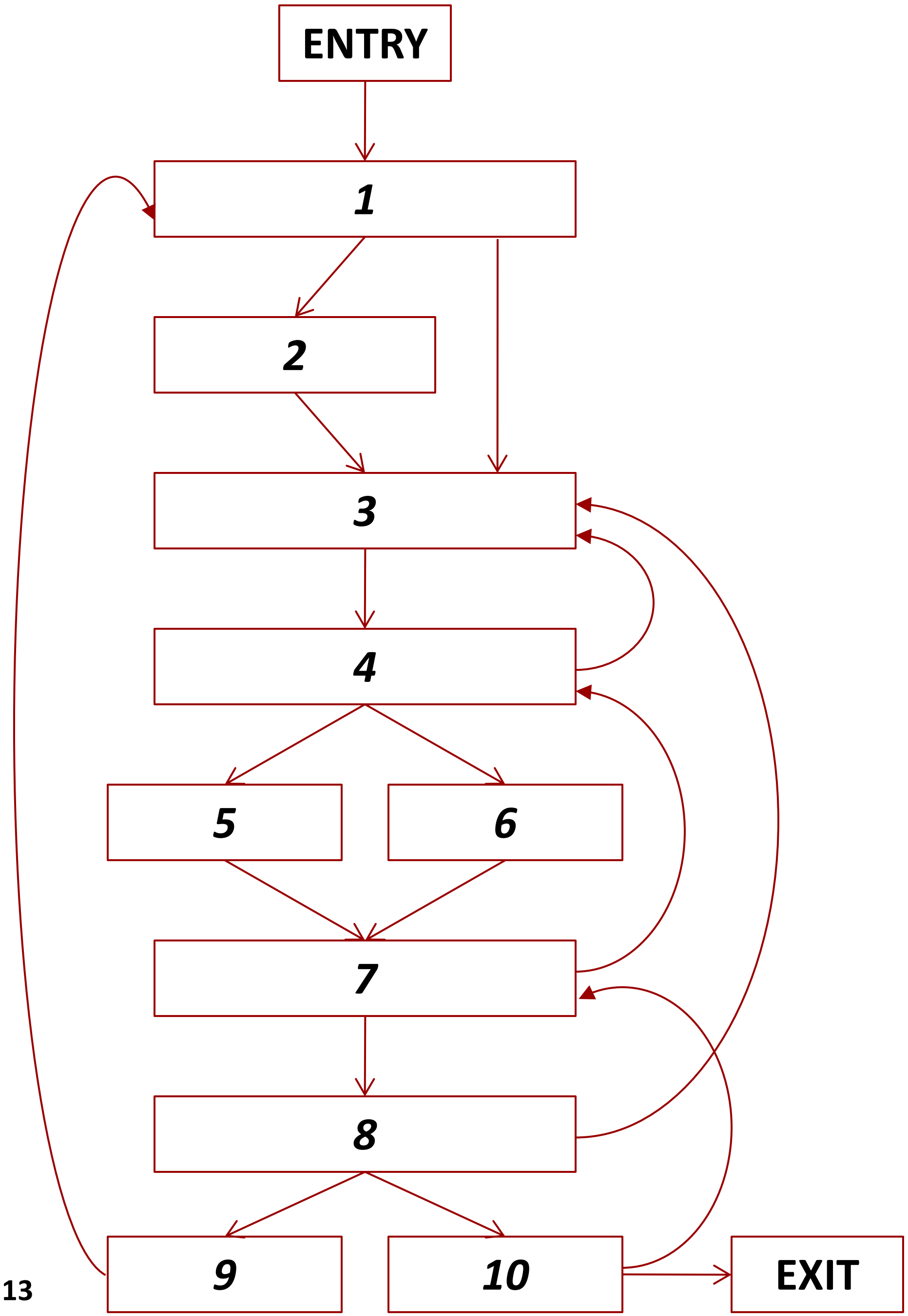
The path from *n* to the root contains all and only dominators of *n*

Constructing the dominator tree: the classic O(Nα(N)) approach is from *T. Lengauer and R. E. Tarjan. A fast algorithm for finding dominators in a flowgraph. ACM Transactions on Programming Languages and Systems, 1(1): 121–141, July 1979.*

Many other algorithms: e.g., see *K. D. Cooper, T. J. Harvey and K. Kennedy. A simple, fast dominance algorithm. Software – Practice and Experience, 4:1–10, 2001.*

Post-Dominance

- A CFG node *d* post-dominates another node *n* if every path from *n* to EXIT goes through *d*
  - Implicit assumption: EXIT is reachable from every node
  - A relation *pdom* $\subseteq$ Nodes × Nodes: *d pdom n*
  - The relation is trivially reflexive: *d pdom d*
- Node *m* is the immediate post-dominator of *n* if
  - $m \neq n$; m *pdom* n; $\forall d \neq n.\ d$ *pdom* $n \Rightarrow d$ *pdom* m
  - Every *n* has a unique immediate post-dominator
- Post-dominance on a CFG is equivalent to dominance on the reverse CFG (all edges reversed)
- Post-dominator tree: the parent of *n* is its immediate post-dominator; root is EXIT



ENTRY does not post-dominate any other *n*

1 *pdom* ENTRY, 1, 9

2 does not post-dominate any other *n*

3 *pdom* ENTRY, 1, 2, 3, 9

4 *pdom* ENTRY, 1, 2, 3, 4, 9

5 does not post-dominate any other *n*

6 does not post-dominate any other *n*

7 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 9

8 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9

9 does not post-dominate any other *n*

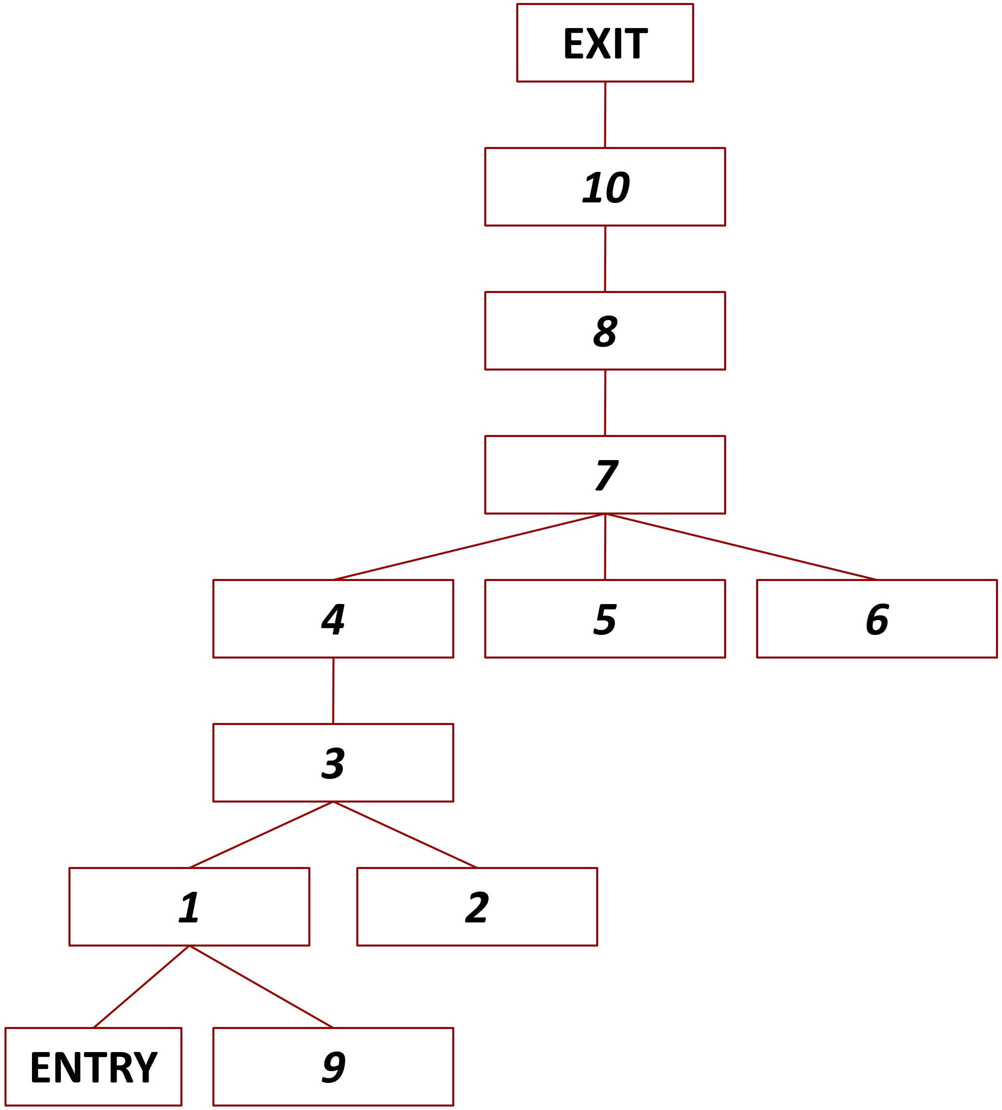
10 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

EXIT *pdom* *n* for any *n*

Immediate post-dominators:

ENTRY → 1	1 → 3
2 → 3	3 → 4
4 → 7	5 → 7
6 → 7	7 → 8
8 → 10	9 → 1
10 → EXIT	

Post-Dominator Tree



The path from *n* to the root contains all and only post-dominators of *n*

Constructing the post-dominator tree: use any algorithm for constructing the dominator tree; just “pretend” that the edges are reversed

Computing the Dominator Tree

- Theoretically superior algorithms are not necessarily the most desirable in practice
- Our choice: Cooper et al., 2001
- Formulation and algorithm based on insights from dataflow analysis
  - Essentially, solving a system of mutually-recursive equations – more later …
- You should read the paper carefully and implement the algorithm for computing the dominator tree

Details on the Algorithm

- Given: CFG $G=(N,E,n_0)$, compute DOM(n) for each n
  - All nodes dominating n, including n itself
- Assumption: all nodes are reachable from n_0
  - Issue: unreachable code in *catch(Exception e) …*
- Visit the nodes in reverse postorder
  - Recall depth-first search: it grows a DFS spanning tree
    - Postorder in this DSF spanning tree
    - During DSF, whenever a node becomes “black” (p. 604 of CLRS-3), it is postorder-visited
  - Do DFS from ENTRY, put the nodes on a list (e.g., ArrayList in Java) in the reverse of this order

Details on the Algorithm

$$\textsc{Dom}(n_0) = \{n_0\}$$

$$\textsc{Dom}(n) = \left(\bigcap_{p \in preds(n)} \textsc{Dom}(p) \right) \cup \{n\}$$

Details on the Algorithm

$$
\begin{array}{l}
\textit{for all nodes, } n \\
\quad \textsc{Dom}[n] \leftarrow \{1 \ldots N\} \\
\textit{Changed} \leftarrow \textit{true} \\
\textit{while (Changed)} \\
\quad \textit{Changed} \leftarrow \textit{false} \\
\quad \textit{for all nodes, } n, \textit{ in reverse postorder} \\
\quad\quad \textit{new\_set} \leftarrow \left(\bigcap_{p \in preds(n)} \textsc{Dom}[p] \right) \bigcup \; \{n\} \\
\quad\quad \textit{if } (\textit{new\_set} \neq \textsc{Dom}[n]) \\
\quad\quad\quad \textsc{Dom}[n] \leftarrow \textit{new\_set} \\
\quad\quad\quad \textit{Changed} \leftarrow \textit{true}
\end{array}
$$

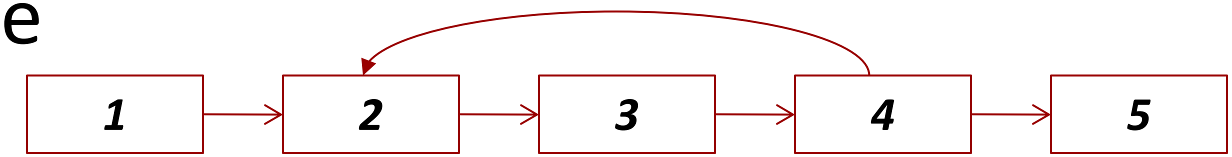
Note: DOM(ENTRY) = { ENTRY } and this node is never processed by the algorithm (so, it should be *"for all nodes other than ENTRY …."*)

Details on the Algorithm

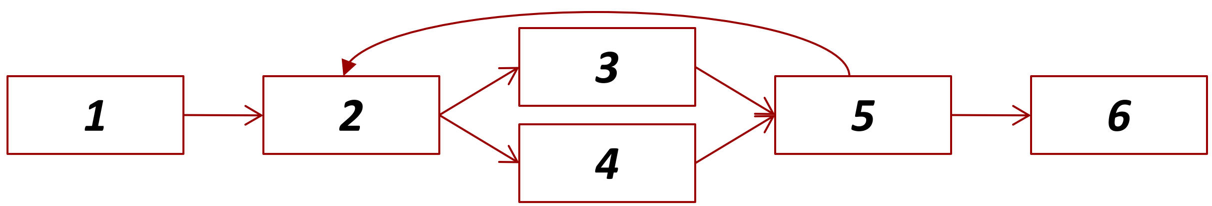
- (Re-)compute DOM(n) as the intersection of DOM(m) for all predecessor nodes m, union { n }
  - If any DOM set changes, recompute everything
- Reverse postorder guarantees efficient algorithm
  - d(G)+2 iterations of the *while(Changed)* loop; d(G) = max number “retreating” edges on any acyclic path
- **Problem**: representation and intersection of sets are expensive, in terms of time and memory
  - Also, we do not find the immediate dominators
- **Solution**: careful algorithm design (Fig 3 in the paper)
  - You will implement this algorithm in a project

Part 3: Loops in CFGs

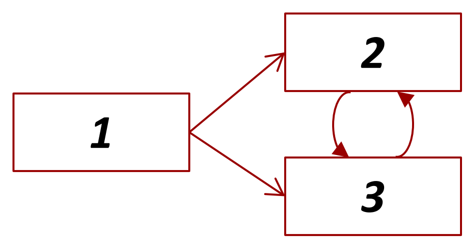
- Cycle: sequence of edges that starts and ends at the same node
  - Example:



- Strongly-connected component (SCC): a maximal set of nodes such as each node in the set is reachable from every other node in the set
  - Example:

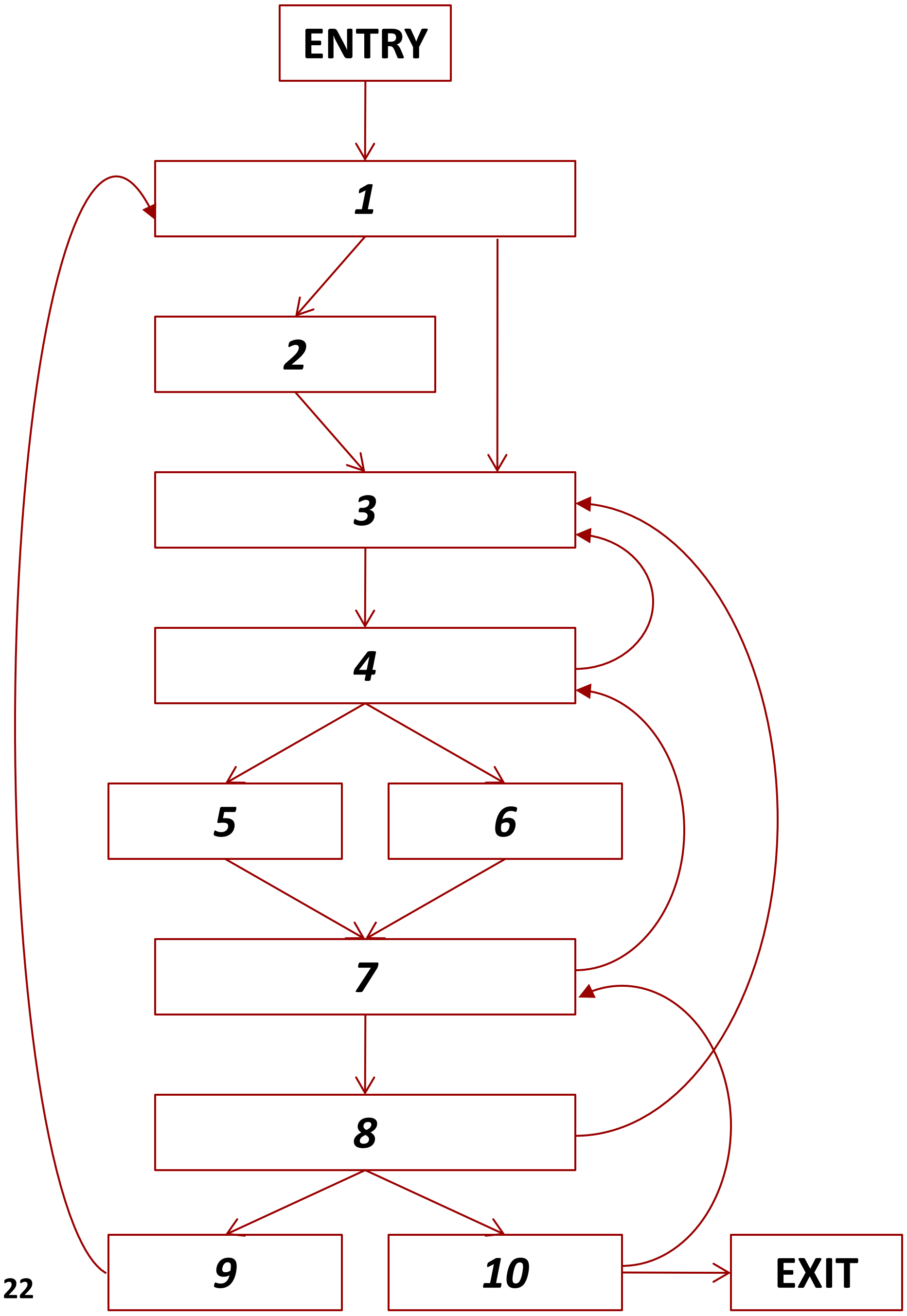


- Loop: informally, a strongly-connected component with a single entry point
  - An SCC that is not a loop:



Back Edges and Natural Loops

- Back edge: a CFG edge (*n*,*h*) where *h* dominates *n*
  - Easy to see that *n* and *h* belong to the same SCC
- Natural loop for a back edge (*n*,*h*)
  - The set of all nodes *m* that can reach node *n* without going through node *h* (trivially, this set includes *h*)
  - Easy to see that *h* dominates all such nodes *m*
  - Node *h* is the header of the natural loop
- Trivial algorithm to find the natural loop of (*n*,*h*)
  - Mark *h* as visited
  - Perform depth-first search (or breadth-first) starting from *n*, but follow the CFG edges in reverse direction
  - All and only visited nodes are in the natural loop



Immediate dominators:

$1 \rightarrow$ ENTRY	$2 \rightarrow 1$	$3 \rightarrow 1$
$4 \rightarrow 3$	$5 \rightarrow 4$	$6 \rightarrow 4$
$7 \rightarrow 4$	$8 \rightarrow 7$	$9 \rightarrow 8$
$10 \rightarrow 8$	EXIT $\rightarrow 10$	

Back edges: **$4 \rightarrow 3$**, **$7 \rightarrow 4$**, **$8 \rightarrow 3$**, **$9 \rightarrow 1$**, **$10 \rightarrow 7$**

Loop(**$10 \rightarrow 7$**) = { 7, 8, 10 }

Loop(**$7 \rightarrow 4$**) = { 4, 5, 6, 7, 8, 10 }

Note: Loop(**$10 \rightarrow 7$**) $\subseteq$ Loop(**$7 \rightarrow 4$**)

Loop(**$4 \rightarrow 3$**) = { 3, 4, 5, 6, 7, 8, 10 }

Note: Loop(**$7 \rightarrow 4$**) $\subseteq$ Loop(**$4 \rightarrow 3$**)

Loop(**$8 \rightarrow 3$**) = { 3, 4, 5, 6, 7, 8, 10 }

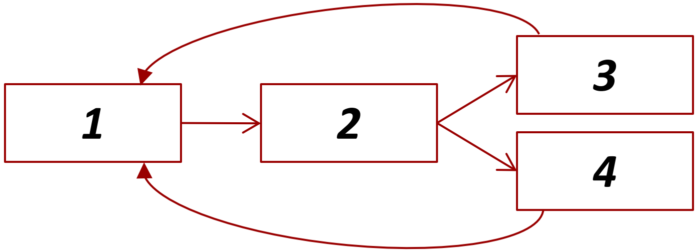
Note: Loop(**$8 \rightarrow 3$**) = Loop(**$4 \rightarrow 3$**)

Loop(**$9 \rightarrow 1$**) = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

Note: Loop(**$4 \rightarrow 3$**) $\subseteq$ Loop(**$9 \rightarrow 1$**)

Loops in the CFG

- Find all back edges; each target *h* of at least one back edge defines a loop *L* with *header(L) = h*
- *body(L)* is the union of the natural loops of all back edges whose target is *header(L)*
  - Note that *header(L)* ∈ *body(L)*
- Example: this is a single loop with header node 1



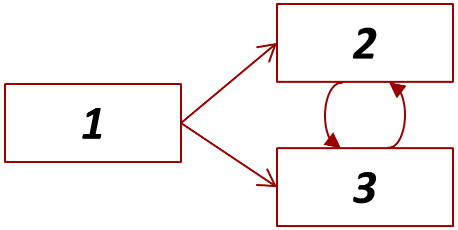
- For two CFG loops L_1 and L_2
  - *header*(L_1) is different from *header*(L_2)
  - *body*(L_1) and *body*(L_2) are either disjoint, or one is a proper subset of the other (nesting – inner/outer)

Graph Algorithms

- DFS again (p. 604 of CLRS-3)
  - Set each node's color as *white*
  - Call DFS(ENTRY)
  - DFS(*n*)
    - Set the color of *n* to *grey*
    - For each successor *m*: if color is *white*, call DFS(*m*)
    - Set the color of *n* to *black*
- Inside DFS(*n*), seeing a grey successor *m* means that (*n*,*m*) is a retreating edge
  - Note: *m* could be *n* itself, if there is an edge (*n*,*n*)
- The order in which we consider the successors matters: the set of retreating edges depends on it

Reducible Control-Flow Graphs

- For reducible CFGs, the retreating edges discovered during DFS are all and only back edges
  - The order during DFS traversal is irrelevant: all DFS traversals produce the same set of retreating edges
- For irreducible CFGs: a DFS traversal may produce retreating edges that are not back edges
  - Each traversal may produce different retreating edges
  - Example:



    - No back edges
    - One traversal produces the retreating edge 3 $\rightarrow$ 2
    - The other one produces the retreating edge 2 $\rightarrow$ 3

Reducibility

- A number of equivalent definitions
  - One of them we already saw
- The graph can be reduced to a single node with the application of the following two rules
  - Given a node *n* with a single predecessor *m*, merge *n* into *m*; all successors of *n* become successors of *m*
  - Remove an edge n → n
- Try this on the graphs from the previous slides

Reducibility

- The essence of irreducibility: a SCC with multiple possible entry points
  - If the original program was written using **if-then**, **if-then-else**, **while-do**, **do-while**, **break**, and **continue**, the resulting CFG is always reducible
  - If **goto** was used by the programmer, the CFG could be irreducible (but, in practice, it typically is reducible)
- Optimizations of the intermediate code, done by the compiler, could introduce irreducibility
- Code obfuscation: e.g., Java bytecode can be transformed to be irreducible, making it impossible to reverse-engineer a valid Java source program

Part 4: Static Single Assignment (SSA) Form

- Source: Cytron et al., ACM TOPLAS, Oct. 1991
  - Section 1 (ignore Section 1.1)
  - Section 2
  - Section 3 (ignore Section 3.1)
  - Section 4 (ignore the detailed proofs in Section 4.3)
- Key ideas
  - Insert ϕ-functions at join points (Sections 3 and 4)
    - Based on **dominance frontiers**
  - Rename the variables so that **each use (read)** of a variable is reached by **exactly one definition (write)** of that variable – i.e., by a **single assignment**
    - Section 5.2 discusses this issue, but we will not

Examples

```
V ← 4
  ← V + 5
V ← 6
  ← V + 7
```

$$\begin{aligned} V_1 &\leftarrow 4 \\ &\leftarrow V_1 + 5 \\ V_2 &\leftarrow 6 \\ &\leftarrow V_2 + 7 \end{aligned}$$

Fig. 2. Straight-line code and its single assignment version.

```
if P
   then V ← 4
   else V ← 6

/* Use V several times. */
```

```
if P
   then V1 ← 4
   else V2 ← 6
V3 ← φ(V1, V2)
/* Use V3 several times. */
```

Fig. 3. **if-then-else** and its single assignment version.

```
I ← 1
J ← 1
K ← 1
L ← 1
repeat




  if (P)
    then do
      J ← I
      if (Q)
        then L ← 2
        else L ← 3

      K ← K + 1
    end
    else K ← K + 2




  print(I,J,K,L)
  repeat

    if (R)
      then L ← L + 4

  until (S)
  I ← I + 6
until (T)
```

```
I1 ← 1
J1 ← 1
K1 ← 1
L1 ← 1
repeat
  I2 ← φ(I3,I1)
  J2 ← φ(J4,J1)
  K2 ← φ(K5,K1)
  L2 ← φ(L9,L1)
  if (P)
    then do
      J3 ← I2
      if (Q)
        then L3 ← 2
        else L4 ← 3
      L5 ← φ(L3,L4)
      K3 ← K2 + 1
    end
    else K4 ← K2 + 2
  J4 ← φ(J3,J2)
  K5 ← φ(K3,K4)
  L6 ← φ(L2,L5)
  print(I2,J4,K5,L6)
  repeat
    L7 ← φ(L9,L6)
    if (R)
      then L8 ← L7 + 4
    L9 ← φ(L8,L7)
  until (S)
  I3 ← I2 + 6
until (T)
```

Placement of ϕ Functions

- ϕ functions are used in “fake” assignments of the form $V_k \leftarrow \phi(V_i,V_j)$
  - Along one edge, variable V has the value of V_i; along the other, the value of V_j
  - If multiple incoming edges: $\phi(V_i,V_j,\ldots, V_m)$
- Naïve: for each V, check each pair of assignments to V; do they reach a common join point?
- Better: for each V, consider each assignment to V and find its **dominance frontier**; place ϕ for V
  - And then find the dominance frontier of each ϕ, and place ϕ there as well, and so on …

Dominance Frontier (DF)

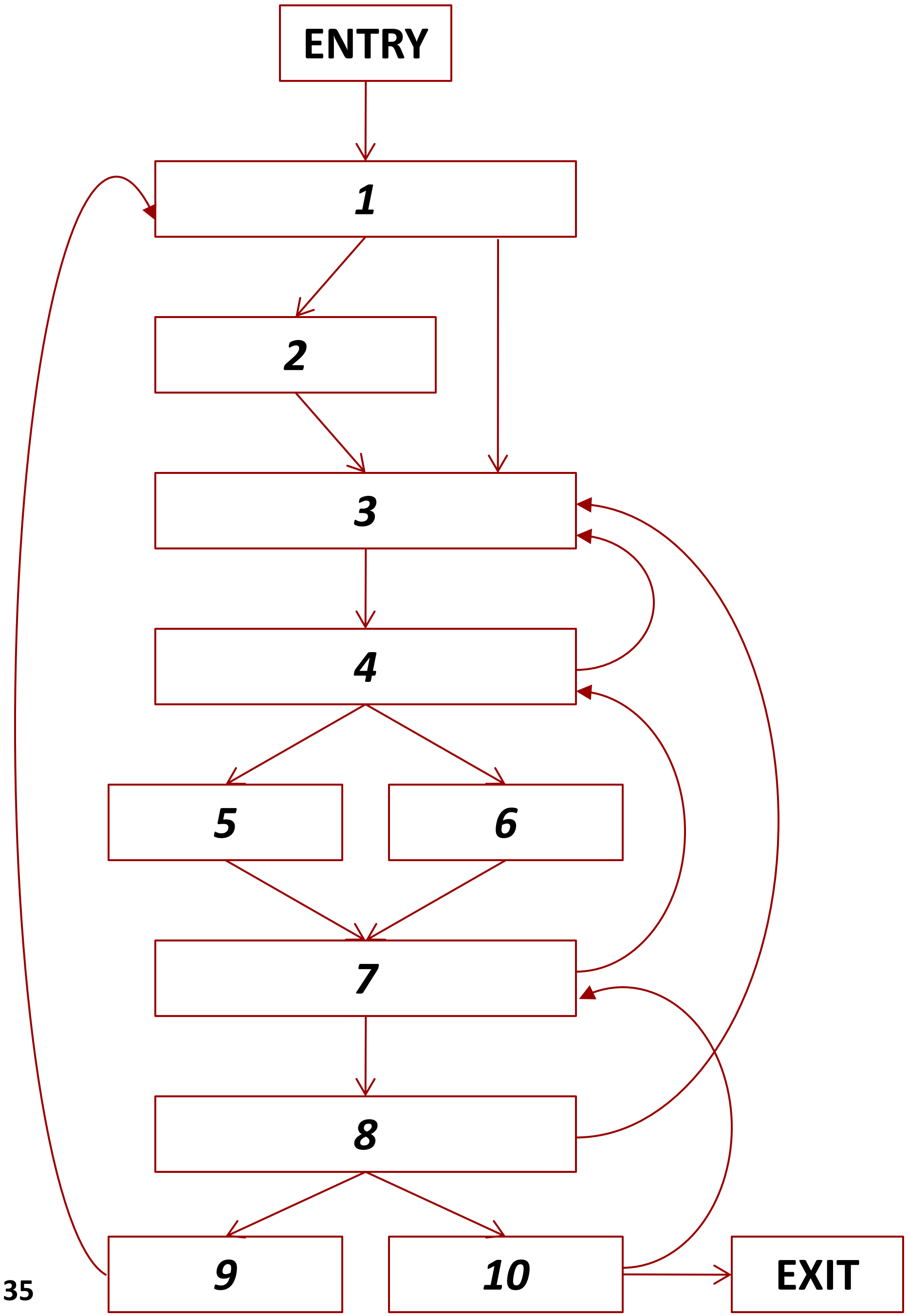
- Suppose node x is an assignment to V
- DF(**x**) = { **y** | for some edge **z** → **y**, **x** dominates **z** but **x** does not strictly dominate **y** }
- A few observations
  – **y** must be a join point. Why?
  – If the flow of control reaches **y** from **z**, the value of V is either the one assigned at **x**, or at some node "between" **x** and **z**
  – If the flow of control reaches **y** from some other predecessor (not **z**), the value of V may come from a different assignment to V
- DF algorithm: Sec 5; alternative: Cooper et al. 2001

Part 5: Control Dependence: Informally

- A node *n* is control dependent on a node *c* if
  - There exists an edge e_1 coming out of *c* that definitely causes *n* to execute
  - There exists some edge e_2 coming out of *c* that is the start of some path that avoids the execution of *n*
- The decision made at *c* affects whether *n* gets executed: if e_1 is followed, n definitely is executed; if e_2 is followed, there is the possibility that n is not executed at all
  - Thus, *n* is control dependent on *c* – the control-flow leading to *n* depends on what *c* does

Control Dependence: Formally

- (part 1) *n* is control dependent on *c* if
  - $n \neq c$
  - *n* does not post-dominate *c*
  - there exists a path from *c* to *n* such that *n* post-dominates every node on the path except *c*
- (part 2) *n* is control dependent on *n* if
  - there exists a path from *n* to *n* (with at least one edge) such that *n* post-dominates every node on the path
    - this implies that *n* has two outgoing edges
    - this case applies to the header of a loop
- See Cytron et al., 1991, Section 6 for more details
  - *c* belongs to DF(*n*) but computed on the *reverse* CFG



Consider all branch nodes *c*: 1, 4, 7, 8, 10

ENTRY does not post-dominate any other *n*
1 *pdom* ENTRY, 1, 9
2 does not post-dominate any other *n*
3 *pdom* ENTRY, 1, 2, 3, 9
4 *pdom* ENTRY, 1, 2, 3, 4, 9
5 does not post-dominate any other *n*
6 does not post-dominate any other *n*
7 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 9
8 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9
9 does not post-dominate any other *n*
10 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
EXIT *pdom n* for any *n*

2 is control dependent on 1
3, 4, 5, 6 are control dependent on 4
4, 7 are control dependent on 7
9, 1, 3, 4, 7, 8 are control dependent on 8
7, 8, 10 are control dependent on 10

Finding All Control Dependences

- Consider all CFG edges (*c*,*x*) such that *x* does not post-dominate *c* (therefore, *c* is a branch node)
- Traverse the post-dominator tree bottom-up
  - *n* = *x*
  - while (*n* != parent of *c* in the post-dominator tree)
    - report that *n* is control dependent on *c*
    - *n* = parent of *n* in the post-dominator tree
  - Example: for CFG edge (8,9) from the previous slide, traverse and report 9, 1, 3, 4, 7, 8 (stop before 10)
- Other algorithms exist, but this one is simple and works quite well