Assignment 2

CSE 5239 (Rountev)

Due: Dec 3 (Wednesday) by 2:20 pm

1. (2 pts) Consider a partially ordered set S with partial order $\leq$. Let $S^n = S \times S \times \ldots \times S$ for some given value of n — that is, S^n is the set of all n-tuples of elements of S. Consider a relation $\leq^n$ over S^n defined as

   $$(a_1, \ldots, a_n) \leq^n (b_1, \ldots, b_n) \Leftrightarrow \forall i \in \{1, \ldots, n\} : a_i \leq b_i$$

   Prove that $\leq^n$ is a partial order over S^n.

2. (2 pts) Consider a meet semilattice with a meet operator $\wedge$. Prove that $\wedge$ is associative: $(a \wedge b) \wedge c = a \wedge (b \wedge c)$

3. (2 pts) Prove that in a meet semilattice, $a \leq b \Leftrightarrow a \wedge b = a$. Prove both directions.

4. (2 pts) Prove that for a meet semilattice L and any distributive function $f : L \to L$, f is also monotone.

5. (8 pts) Recall that $\mathcal{P}(\mathit{Defs})$ is the lattice for the Reaching Definitions problem; this is the powerset of the set *Defs* of definitions in the CFG. As discussed in class, the transfer functions for this problem are of the form $f(x) = (x \cap P) \cup G$, where P ("preserved") and G ("generated") are known lattice elements. This formulation generalizes to a number of other dataflow analysis problems. This question is about proving some useful properties of such problems.

   Consider an arbitrary lattice $(L, \leq, \wedge, \vee, \top, \bot)$ where $L = \mathcal{P}(Z)$ for some set Z. Each lattice element $x \in L$ is $x \subseteq Z$. Recall that for such a lattice $\leq$ is $\supseteq$, $\wedge$ is $\cup$, $\vee$ is $\cap$, $\top$ is $\emptyset$, and $\bot$ is Z.

   Let F be the set of all functions $f : L \to L$ of the form $f(x) = (x \vee a) \wedge b$ where $a, b \in L$ are known lattice elements. Prove that

   - Each $f \in F$ is distributive (and thus monotone)
   - Each $f \in F$ is rapid (i.e., $f(x) \geq x \wedge f(\top)$)
   - F is closed under composition: for any $f, g \in F$, the function h defined by $h(x) = f(g(x))$ is also in F
   - F is closed under functional meets: for any $f, g \in F$, the function h defined by $h(x) = f(x) \wedge g(x)$ is also in F
   - F contains the identity function $f(x) = x$

6. (4 pts) Consider the constant propagation problem, as applied to the following program:

```
if (some condition) { b=1; c=2; } else { b=2; c=1; }
a=b+c;
```

Show that for this example, the meet-over-all-paths (MOP) solution is strictly more precise than the maximum fixed point (MFP) solution. This implies that the transfer functions for constant propagation are not distributive.

7. (0 pts)

   Bonus question, worth zero points: do it if you have time and are interested in points-to analysis.

   Consider a set of variables V and a lattice $L = \mathcal{P}(V \times V)$. Each lattice element $G \in L$ is a points-to graph: a set of points-to edges $x \to y$ showing that variable x may contain the address of variable y. We consider four types of statements, shown below with their transfer functions.

   - `x = &y`: $f(G) = G \cup \{x \to y\}$
   - `x = y`: $f(G) = G \cup \{x \to z \mid y \to z \in G\}$
   - `x = *y`: $f(G) = G \cup \{x \to z \mid y \to w \in G \wedge w \to z \in G\}$
   - `*x = y`: $f(G) = G \cup \{w \to z \mid x \to w \in G \wedge y \to z \in G\}$

   These are the no-kill transfer functions commonly used for flow-insensitive points-to analysis.

   - Prove that these functions are monotone
   - Prove that these functions are not distributive