Control-Flow Analysis

“Dragon book” [Ch. 8, Section 8.4; Ch. 9, Section 9.6]
Compilers: Principles, Techniques, and Tools, 2nd ed. by
Alfred V. Aho, Monica S. Lam, Ravi Sethi, and Jerey D. Ullman
on reserve in 18th Ave Library (ask for CSE 5343 textbook)

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Control-Flow Graphs

• Control-flow graph (CFG) for a procedure/method
  – A node is a **basic block**: a single-entry-single-exit sequence of three-address instructions
  – An edge represents the potential flow of control from one basic block to another

• Uses of a control-flow graph
  – Inside a basic block: **local code optimizations**; done as part of the code generation phase
  – Across basic blocks: **global code optimizations**; done as part of the code optimization phase
  – Aspects of code generation: e.g., **global register allocation**
Control-Flow Analysis

• Part 1: Constructing a CFG
• Part 2: Finding dominators and post-dominators
• Part 3: Finding loops in a CFG
  – What exactly is a loop? We cannot simply say “whatever CFG subgraph is generated by while, do-while, and for statements” – need a general graph-theoretic definition
• Part 4: Static single assignment form (SSA)
• Part 5: Finding control dependences
  – Necessary as part of constructing the program dependence graph (PDG), a popular IR for software tools for slicing, refactoring, testing, and debugging
Part 1: Constructing a CFG

• Basic block: maximal sequence of consecutive three-address instructions such that
  – The flow of control can enter only through the first instruction (i.e., no jumps into the middle of the block)
  – The flow of control can exit only at the last instruction

• Given: the entire sequence of instructions

• First, find the leaders (starting instructions of all basic blocks)
  – The first instruction
  – The target of any conditional/unconditional jump
  – Any instruction that immediately follows a conditional or unconditional jump
Constructing a CFG

• Next, find the basic blocks: for each leader, its basic block contains itself and all instructions up to (but not including) the next leader.

1. \( i = 1 \)
2. \( j = 1 \)
3. \( t1 = 10 \times i \)
4. \( t2 = t1 + j \)
5. \( t3 = 8 \times t2 \)
6. \( t4 = t3 - 88 \)
7. \( a[t4] = 0.0 \)
8. \( j = j + 1 \)
9. if \( j \leq 10 \) goto (3)
10. \( i = i + 1 \)
11. if \( i \leq 10 \) goto (2)
12. \( i = 1 \)
13. \( t5 = i - 1 \)
14. \( t6 = 88 \times t5 \)
15. \( a[t6] = 1.0 \)
16. \( i = i + 1 \)
17. if \( i \leq 10 \) goto (13)

First instruction
Target of 11
Target of 9
Follows 9
Follows 11
Target of 17

Note: this example sets array elements \( a[i][j] \) to 0.0, for \( 1 \leq i,j \leq 10 \) (instructions 1-11). It then sets \( a[i][i] \) to 1.0, for \( 1 \leq i \leq 10 \) (instructions 12-17). The array accesses in instructions 7 and 15 are done with offsets computed as described in Section 6.4.3, assuming row-major order, 8-byte array elements, and array indexing that starts from 1, not from 0.
Artificial ENTRY and EXIT nodes are often added for convenience.

There is an edge from $B_p$ to $B_q$ if it is possible for the first instruction of $B_q$ to be executed immediately after the last instruction of $B_p$. This is conservative: e.g., if $(3.14 > 2.78)$ still generates two edges.
Practical Considerations

• The usual data structures for graphs can be used
  – The graphs are sparse (i.e., have relatively few edges), so an adjacency list representation is the usual choice
  • Number of edges is at most $2 \times$ number of nodes

• Nodes are basic blocks; edges are between basic blocks, not between instructions
  – Inside each node, some additional data structures for the sequence of instructions in the block (e.g., a linked list of instructions)
  – Often convenient to maintain both a list of successors (i.e., outgoing edges) and a list of predecessors (i.e., incoming edges) for each basic block
Part 2: Dominance

• A CFG node $d$ dominates another node $n$ if every path from ENTRY to $n$ goes through $d$
  – Implicit assumption: every node is reachable from ENTRY (i.e., there is no dead code)
  – A dominance relation $dom \subseteq \text{Nodes} \times \text{Nodes}$: $d \ dom \ n$
  – The relation is trivially reflexive: $d \ dom \ d$

• Node $m$ is the immediate dominator of $n$ if
  – $m \neq n$
  – $m \ dom \ n$
  – For any $d \neq n$ such $d \ dom \ n$, we have $d \ dom \ m$

• Every node has a unique immediate dominator
  – Except ENTRY, which is dominated only by itself
ENTRY dom n for any n
1 dom n for any n except ENTRY
2 does not dominate any other node
3 dom 3, 4, 5, 6, 7, 8, 9, 10, EXIT
4 dom 4, 5, 6, 7, 8, 9, 10, EXIT
5 does not dominate any other node
6 does not dominate any other node
7 dom 7, 8, 9, 10, EXIT
8 dom 8, 9, 10, EXIT
9 does not dominate any other node
10 dom 10, EXIT

Immediate dominators:
1 → ENTRY 2 → 1
3 → 1 4 → 3
5 → 4 6 → 4
7 → 4 8 → 7
9 → 8 10 → 8
EXIT → 10
A Few Observations

• Dominance is a **transitive** relation: \( a \ dom \ b \) and \( b \ dom \ c \) means \( a \ dom \ c \)

• Dominance is an **anti-symmetric** relation: \( a \ dom \ b \) and \( b \ dom \ a \) means that \( a \) and \( b \) must be the same
  – Reflexive, anti-symmetric, transitive: **partial order**

• If \( a \) and \( b \) are two dominators of some \( n \), either \( a \ dom \ b \) or \( b \ dom \ a \)
  – Therefore, \( dom \) is a **total order** for \( n \)’s dominator set
  – Corollary: for any acyclic path from ENTRY to \( n \), all dominators of \( n \) appear along the path, always in the same order; the last one is the immediate dominator
Dominator Tree

• The parent of $n$ is its immediate dominator

The path from $n$ to the root contains all and only dominators of $n$


Post-Dominance

• A CFG node \( d \) post-dominates another node \( n \) if every path from \( n \) to EXIT goes through \( d \)
  – Implicit assumption: EXIT is reachable from every node
  – A relation \( pdom \subseteq \text{Nodes} \times \text{Nodes} : d \ pdom \ n \)
  – The relation is trivially reflexive: \( d \ pdom \ d \)

• Node \( m \) is the immediate post-dominator of \( n \) if
  – \( m \neq n; m \ pdom \ n; \forall d \neq n. \ d \ pdom \ n \Rightarrow d \ pdom \ m \)
  – Every \( n \) has a unique immediate post-dominator

• Post-dominance on a CFG is equivalent to dominance on the reverse CFG (all edges reversed)

• Post-dominator tree: the parent of \( n \) is its immediate post-dominator; root is EXIT
ENTRY does not post-dominate any other \( n \)
1 \( pdom \) ENTRY, 1, 9
2 does not post-dominate any other \( n \)
3 \( pdom \) ENTRY, 1, 2, 3, 9
4 \( pdom \) ENTRY, 1, 2, 3, 4, 9
5 does not post-dominate any other \( n \)
6 does not post-dominate any other \( n \)
7 \( pdom \) ENTRY, 1, 2, 3, 4, 5, 6, 7, 9
8 \( pdom \) ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9
9 does not post-dominate any other \( n \)
10 \( pdom \) ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
EXIT \( pdom \) \( n \) for any \( n \)

Immediate post-dominators:
ENTRY \( \rightarrow \) 1 1 \( \rightarrow \) 3
2 \( \rightarrow \) 3
4 \( \rightarrow \) 7
6 \( \rightarrow \) 7
8 \( \rightarrow \) 10
8 \( \rightarrow \) 10
9 \( \rightarrow \) 1
10 \( \rightarrow \) EXIT
The path from $n$ to the root contains all and only post-dominators of $n$

Constructing the post-dominator tree: use any algorithm for constructing the dominator tree; just “pretend” that the edges are reversed
Computing the Dominator Tree

• Theoretically superior algorithms are not necessarily the most desirable in practice
• Our choice: Cooper et al., 2001
• Formulation and algorithm based on insights from dataflow analysis
  – Essentially, solving a system of mutually-recursive equations – more later ...
• You should read the paper carefully and implement the algorithm for computing the dominator tree
Details on the Algorithm

• Given: CFG \(G=(N,E,n_0)\), compute \(\text{DOM}(n)\) for each \(n\)
  – All nodes dominating \(n\), including \(n\) itself

• Assumption: all nodes are reachable from \(n_0\)
  – Issue: unreachable code in \(\text{catch}(\text{Exception } e) \ldots\)

• Visit the nodes in reverse postorder
  – Recall depth-first search: it grows a DFS spanning tree
    • Postorder in this DSF spanning tree
    • During DSF, whenever a node becomes “black” (p. 604 of CLRS-3), it is postorder-visited
  – Do DFS from ENTRY, put the nodes on a list (e.g., ArrayList in Java) in the reverse of this order
Details on the Algorithm

\[
\text{DOM}(n_0) = \{n_0\}
\]

\[
\text{DOM}(n) = \left( \bigcap_{p \in \text{preds}(n)} \text{DOM}(p) \right) \cup \{n\}
\]
Details on the Algorithm

for all nodes, n
    $\text{DOM}[n] \leftarrow \{1 \ldots N\}$
    Changed $\leftarrow$ true
while (Changed)
    Changed $\leftarrow$ false
    for all nodes, n, in reverse postorder
        new_set $\leftarrow \left( \bigcap_{p \in \text{preds}(n)} \text{DOM}[p] \right) \cup \{n\}$
        if (new_set $\neq \text{DOM}[n]$)
            $\text{DOM}[n] \leftarrow$ new_set
            Changed $\leftarrow$ true

Note: $\text{DOM(ENTRY)} = \{ \text{ENTRY} \}$ and this node is never processed by the algorithm (so, it should be “for all nodes other than ENTRY ....”)
Details on the Algorithm

• (Re-)compute DOM(n) as the intersection of DOM(m) for all predecessor nodes m, union \{ n \}
  – If any DOM set changes, recompute everything

• Reverse postorder guarantees efficient algorithm
  – d(G)+2 iterations of the while(Changed) loop; d(G) = max number “retreating” edges on any acyclic path

• **Problem**: representation and intersection of sets are expensive, in terms of time and memory
  – Also, we do not find the immediate dominators

• **Solution**: careful algorithm design (Fig 3 in the paper)
  – You will implement this algorithm in a project
Part 3: Loops in CFGs

- **Cycle**: sequence of edges that starts and ends at the same node
  - Example:
    
- **Strongly-connected component (SCC)**: a maximal set of nodes such as each node in the set is reachable from every other node in the set
  - Example:
    
- **Loop**: informally, a strongly-connected component with a single entry point
  - An SCC that is not a loop:
Back Edges and Natural Loops

• Back edge: a CFG edge \((n,h)\) where \(h\) dominates \(n\)
  – Easy to see that \(n\) and \(h\) belong to the same SCC

• Natural loop for a back edge \((n,h)\)
  – The set of all nodes \(m\) that can reach node \(n\) without going through node \(h\) (trivially, this set includes \(h\))
  – Easy to see that \(h\) dominates all such nodes \(m\)
  – Node \(h\) is the header of the natural loop

• Trivial algorithm to find the natural loop of \((n,h)\)
  – Mark \(h\) as visited
  – Perform depth-first search (or breadth-first) starting from \(n\), but follow the CFG edges in reverse direction
  – All and only visited nodes are in the natural loop
Immediate dominators:
1 → ENTRY  2 → 1  3 → 1
4 → 3  5 → 4  6 → 4
7 → 4  8 → 7  9 → 8
10 → 8  EXIT → 10

Back edges: 4 → 3, 7 → 4, 8 → 3, 9 → 1, 10 → 7

Loop(10 → 7) = { 7, 8, 10 }

Loop(7 → 4) = { 4, 5, 6, 7, 8, 10 }
Note: Loop(10 → 7) ⊆ Loop(7 → 4)

Loop(4 → 3) = { 3, 4, 5, 6, 7, 8, 10 }
Note: Loop(7 → 4) ⊆ Loop(4 → 3)

Loop(8 → 3) = { 3, 4, 5, 6, 7, 8, 10 }
Note: Loop(8 → 3) = Loop(4 → 3)

Loop(9 → 1) = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
Note: Loop(4 → 3) ⊆ Loop(9 → 1)
Loops in the CFG

• Find all back edges; each target $h$ of at least one back edge defines a loop $L$ with $\text{header}(L) = h$

• $\text{body}(L)$ is the union of the natural loops of all back edges whose target is $\text{header}(L)$
  – Note that $\text{header}(L) \in \text{body}(L)$

• Example: this is a single loop with header node 1

• For two CFG loops $L_1$ and $L_2$
  – $\text{header}(L_1)$ is different from $\text{header}(L_2)$
  – $\text{body}(L_1)$ and $\text{body}(L_2)$ are either disjoint, or one is a proper subset of the other (nesting – inner/outer)
Graph Algorithms

• DFS again (p. 604 of CLRS-3)
  – Set each node’s color as white
  – Call DFS(ENTRY)
  – DFS(n)
    • Set the color of n to grey
    • For each successor m: if color is white, call DFS(m)
    • Set the color of n to black

• Inside DFS(n), seeing a grey successor m means that (n,m) is a retreating edge
  – Note: m could be n itself, if there is an edge (n,n)

• The order in which we consider the successors matters: the set of retreating edges depends on it
Reducible Control-Flow Graphs

• For reducible CFGs, the retreating edges discovered during DFS are all and only back edges
  – The order during DFS traversal is irrelevant: all DFS traversals produce the same set of retreating edges

• For irreducible CFGs: a DFS traversal may produce retreating edges that are not back edges
  – Each traversal may produce different retreating edges
  – Example:

    • No back edges
    • One traversal produces the retreating edge 3 → 2
    • The other one produces the retreating edge 2 → 3
Reducibility

• A number of equivalent definitions
  – One of them we already saw

• The graph can be reduced to a single node with the application of the following two rules
  – Given a node $n$ with a single predecessor $m$, merge $n$ into $m$; all successors of $n$ become successors of $m$
  – Remove an edge $n \rightarrow n$

• Try this on the graphs from the previous slides
Reducibility

• The essence of irreducibility: a SCC with multiple possible entry points
  – If the original program was written using if-then, if-then-else, while-do, do-while, break, and continue, the resulting CFG is always reducible
  – If goto was used by the programmer, the CFG could be irreducible (but, in practice, it typically is reducible)

• Optimizations of the intermediate code, done by the compiler, could introduce irreducibility

• Code obfuscation: e.g., Java bytecode can be transformed to be irreducible, making it impossible to reverse-engineer a valid Java source program
Part 4: Static Single Assignment (SSA) Form

- Source: Cytron et al., ACM TOPLAS, Oct. 1991
  - Section 1 (ignore Section 1.1)
  - Section 2
  - Section 3 (ignore Section 3.1)
  - Section 4 (ignore the detailed proofs in Section 4.3)

- Key ideas
  - Insert \( \varphi \)-functions at join points (Sections 3 and 4)
    - Based on dominance frontiers
  - Rename the variables so that each use (read) of a variable is reached by exactly one definition (write) of that variable – i.e., by a single assignment
    - Section 5.2 discusses this issue, but we will not
Examples

\[
\begin{align*}
V & \leftarrow 4 \\
& \leftarrow V + 5 \\
V & \leftarrow 6 \\
& \leftarrow V + 7
\end{align*}
\]

\[
\begin{align*}
V_1 & \leftarrow 4 \\
& \leftarrow V_1 + 5 \\
V_2 & \leftarrow 6 \\
& \leftarrow V_2 + 7
\end{align*}
\]

Fig. 2. Straight-line code and its single assignment version.

\[
\begin{align*}
\text{if } P \\
\quad \text{then } V & \leftarrow 4 \\
\quad \text{else } V & \leftarrow 6 \\
\end{align*}
\]

/* Use V several times. */

\[
\begin{align*}
\quad \text{if } P \\
\quad \text{then } V_1 & \leftarrow 4 \\
\quad \text{else } V_2 & \leftarrow 6 \\
\quad V_3 & \leftarrow \phi(V_1, V_2) \\
\end{align*}
\]

/* Use V_3 several times. */

Fig. 3. if-then-else and its single assignment version.
I ← 1
J ← 1
K ← 1
L ← 1
repeat

if (P)
    then do
        J ← I
        if (Q)
            then L ← 2
            else L ← 3
        K ← K + 1
    end
else K ← K + 2

print(I, J, K, L)
repeat

if (R)
    then L ← L + 4
until (S)
I ← I + 6
until (T)
Placement of $\phi$ Functions

- $\phi$ functions are used in “fake” assignments of the form $V_k \leftarrow \phi(V_i, V_j)$
  - Along one edge, variable $V$ has the value of $V_i$; along the other, the value of $V_j$
  - If multiple incoming edges: $\phi(V_i, V_j, ..., V_m)$

- Naïve: for each $V$, check each pair of assignments to $V$; do they reach a common join point?

- Better: for each $V$, consider each assignment to $V$ and find its **dominance frontier**; place $\phi$ for $V$
  - And then find the dominance frontier of each $\phi$, and place $\phi$ there as well, and so on ...
Dominance Frontier (DF)

• Suppose node $x$ is an assignment to $V$
• $DF(x) = \{ y \mid \text{for some edge } z \rightarrow y, x \text{ dominates } z \text{ but } x \text{ does not strictly dominate } y \}$

• A few observations
  – $y$ must be a join point. Why?
  – If the flow of control reaches $y$ from $z$, the value of $V$ is either the one assigned at $x$, or at some node “between” $x$ and $z$
  – If the flow of control reaches $y$ from some other predecessor (not $z$), the value of $V$ may come from a different assignment to $V$

• DF algorithm: Sec 5; alternative: Cooper et al. 2001
Part 5: Control Dependence: Informally

- A node $n$ is control dependent on a node $c$ if
  - There exists an edge $e_1$ coming out of $c$ that definitely causes $n$ to execute
  - There exists some edge $e_2$ coming out of $c$ that is the start of some path that avoids the execution of $n$

- The decision made at $c$ affects whether $n$ gets executed: if $e_1$ is followed, $n$ definitely is executed; if $e_2$ is followed, there is the possibility that $n$ is not executed at all
  - Thus, $n$ is control dependent on $c$ – the control-flow leading to $n$ depends on what $c$ does
Control Dependence: Formally

• (part 1) $n$ is control dependent on $c$ if
  – $n \neq c$
  – $n$ does not post-dominate $c$
  – there exists a path from $c$ to $n$ such that $n$ post-dominates every node on the path except $c$

• (part 2) $n$ is control dependent on $n$ if
  – there exists a path from $n$ to $n$ (with at least one edge)
    such that $n$ post-dominates every node on the path
    • this implies that $n$ has two outgoing edges
    • this case applies to the header of a loop

• See Cytron et al., 1991, Section 6 for more details
  – $c$ belongs to DF($n$) but computed on the reverse CFG
Consider all branch nodes $c$: 1, 4, 7, 8, 10

ENTRY does not post-dominate any other $n$
1 $pdom$ ENTRY, 1, 9
2 does not post-dominate any other $n$
3 $pdom$ ENTRY, 1, 2, 3, 9
4 $pdom$ ENTRY, 1, 2, 3, 4, 9
5 does not post-dominate any other $n$
6 does not post-dominate any other $n$
7 $pdom$ ENTRY, 1, 2, 3, 4, 5, 6, 7, 9
8 $pdom$ ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9
9 does not post-dominate any other $n$
10 $pdom$ ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
EXIT $pdom$ $n$ for any $n$

2 is control dependent on 1
3, 4, 5, 6 are control dependent on 4
4, 7 are control dependent on 7
9, 1, 3, 4, 7, 8 are control dependent on 8
7, 8, 10 are control dependent on 10
Finding All Control Dependences

• Consider all CFG edges \((c,x)\) such that \(x\) does not post-dominate \(c\) (therefore, \(c\) is a branch node)

• Traverse the post-dominator tree bottom-up
  – \(n = x\)
  – while \((n \neq \text{parent of } c\) in the post-dominator tree\)
    • report that \(n\) is control dependent on \(c\)
    • \(n = \text{parent of } n\) in the post-dominator tree
  – Example: for CFG edge \((8,9)\) from the previous slide, traverse and report \(9, 1, 3, 4, 7, 8\) (stop before 10)

• Other algorithms exist, but this one is simple and works quite well