Exploring Flow Fields Using Fractal Analysis of Field Lines

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ABSTRACT

We present a novel technique for analyzing the geometry of streamlines representing large scale flow fields produced in scientific simulations. We introduce the box counting ratio, a metric related to the Kolmogorov capacity or box counting dimension, for quantifying geometric complexity of streamlines (or streamline segments). We utilize this metric to drive a visual analytic framework for extracting, organizing and representing features of varying sizes from large number of streamlines. This framework allows the user to easily visualize and interact with the features otherwise hidden in large data. We present case studies using combustion and climate simulation datasets.

1 INTRODUCTION

Modern scientific simulations often produce enormous vector fields. Analysis and effective visualization of these fields play a key role in scientific knowledge discovery. Direct exploration of the field is often not feasible due to sheer data size. However, large number of streamlines can be computed very fast from the field. The task of analyzing the streamlines to identify and select the important ones, which actually represent interesting features of the field, remains for the user.

We introduce a metric called *box counting ratio*, related to Kolmogorov capacity or box counting dimension, for effective geometric analysis of streamlines. In fluid dynamics area, box counting dimension is applied as a standard technique to measure and analyze turbulent flow. Box counting dimension, which is one way of defining "fractal dimension", is applicable to streamlines due to their self-similar behavior near vortex-like points. [3].

Box counting ratio quantifies the geometric complexity of a streamline with a value between 0 and 3. We primarily exploit its ability to detect features of varying sizes. We also present a visual analytic framework which extracts complex segments from streamlines based on the box counting ratio, transforms them into high dimensional features and presents them on an interactive 2D space. Without having to navigate through the cluttered and less navigation-friendly 3D spatial domain of streamlines, the user can brush regions of potential interest on this 2D space to see the corresponding streamlines in a linked display.

2 BOX COUNTING RATIO

Fractal dimension measures the extent of space-filling by a fractal object. Since a fractal replicates itself at different scales, its conventional measurements such as length or area vary with scale. Fractal dimension captures the limiting value of the rate of growth of such measurements at an infinitesimally small scale.

Box counting dimension is one way of defining the fractal dimension of a set of points. If $N_\delta(F)$ is the minimum number of boxes of edge length δ which cover a set F, then the box counting dimension of the set is obtained by:

$$D = \lim_{\delta \to 0} \frac{log(N_\delta(F))}{-log\delta} \quad (1)$$

Most physical objects are not fractals at all, or exhibit fractal behavior only within a range of δ. Thus the above limit does not truly exist, or it is impossible to compute. Instead, it is practical to compute $\log(N_\delta(F))$ for a set of δ within a given range, fit a line to the points $(\log(1/\delta), \log(N_\delta(F))$, and then compute its slope. For a fractal, the slope converges to its box counting dimension as δ goes to zero. Interestingly, a related measure can be useful for real non-fractal data as well. Khoury and Wenger [2] show that complexity of isosurfaces can be quantified by computing $N_\delta(F)$ only for two values, $\delta_1 = \delta$ and $\delta_2 = 2\delta$, and then taking their logarithmic ratio. This related metric is defined as below:

$$B = log_2 \frac{N_{\delta_1}(F)}{N_{\delta_2}(F)} \quad (2)$$

In this work, we show that this quantity, which we call *box counting ratio*, can be computed for streamlines as well and is useful for their effective geometric analysis. Details about the work with a complete reference can be found here [1].

We first compute how many grid cubes of a fixed length δ intersect with the streamline. This value is then computed in a grid of cubes of length 2δ. The logarithmic ratio of these two counts gives the *box counting ratio* of the streamline. Since a streamline can visit a grid cell multiple times, Amanatides-Woo algorithm [1] is employed for accurately counting boxes avoiding duplicates. Since this value has a geometric interpretation, streamlines can be ordered and filtered based on their box counting ratios (Figure 1).

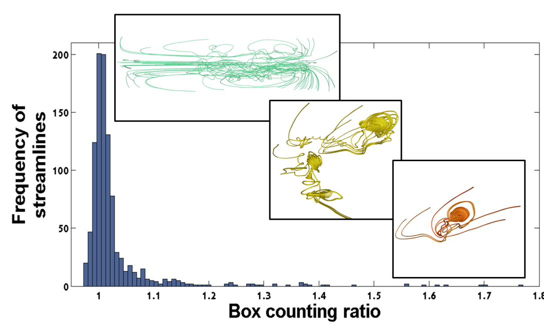

Figure 1: Distribution of box counting ratio B for Solar Plume. **Right inset.** Streamlines satisfying $B > 1.6$; all of them contain complex feature(s). **Middle inset.** Streamlines satisfying $1.35 < B < 1.6$ also have complex segments. **Left inset.** Few of the mostly linear streamlines having $B \sim 1$.

3 MULTI-SCALE FEATURE DETECTION

The uniqueness of box counting ratio lies in its ability to detect features at different scales. Unlike box counting dimension of true fractals, box counting ratio depends on the scale of measurement (δ). For linear segments, the scale has no significant effect (ratio

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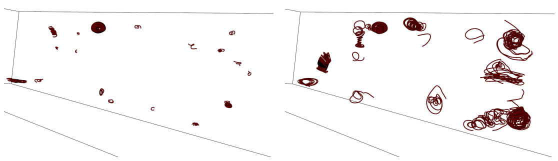
always close to 1). But for winding segments, the space-filling behavior is not captured unless δ is large enough to contain multiple parts of the geometry in one grid cell. Hence, as different δ-pairs are used on a set of streamlines, grids of larger cells (higher δ) detect sparser and bigger features.



(a) (b)

Figure 2: Features extracted at different scales of measurement (δ-pairs (1,2) and (8,16)) clearly have different size and sparseness.

For each resolution pair, we also compute various estimates of the average size and sparseness of the extracted top-scoring (we use top 50) features. As long as varying the resolution pair extracts a considerable number of new features, positive correlation of both size and sparseness with the scale of measurement is observed.

4 Feature Exploration Framework

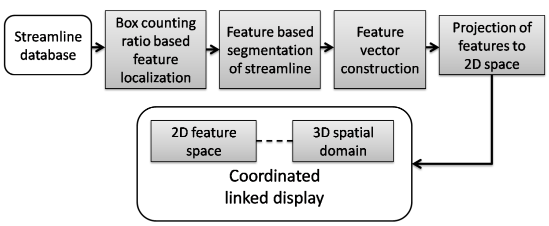


Figure 3: Proposed feature exploration framework.

We propose a novel visual analytic framework (Figure 3) for exploring features from large streamline data. It first identifies the complex regions using box counting ratio (*localization*) and extracts segments preserving the features (*segmentation*). Then, feature vectors are constructed comprising feature location (center of bounding box), size (diagonal length of bounding box) and box counting ratio. Finally, the features are projected on an interactive 2D space using principal component analysis. Size-based color coding enables the user to select features based on size.

5 Results

Case studies from real datasets corroborate the utility of the proposed framework. For large fields, a static visualization with all streamlines is seldom useful due to clutter (Figure 4). Clutter may not reduce by merely changing viewpoint. Placing seeds sparsely may reduce it at the cost of missing some features. Our proposed linked display approach is particularly useful for presenting features without clutter from dense streamlines.

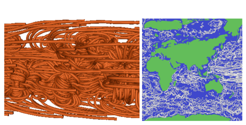


Figure 4: Solar Plume ($126 \times 126 \times 512$) and Ocean ($3600 \times 2400 \times 40$) datasets.

Figure 5 displays a triage of the extracted feature segments from **Solar Plume** dataset (courtesy: NCAR) in the spatial domain (top row). Points are color coded by feature size on the feature space (bottom row) so that the user can make size-based selection.

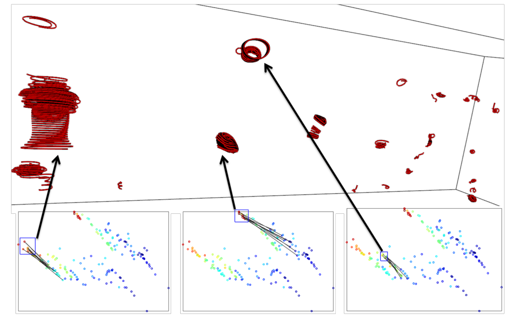


Figure 5: **Top.** A large number of complex features extracted from Solar Plume data set. **Bottom.** Three selected regions on 2D space and the corresponding features.

Exploring **Ocean** (courtesy: Mathew Maltrud, LANL) is challenging due to its large spatial extent. For such cases, we propose *feature links* which connect segments from the same streamline. Feature links help reveal connection between spatially remote features. By tracking a long feature link (as in Figure 6 left) which stands out in the 2D space, the user finds two different sized vortices (Figure 6 right) connected through a long streamline. It would be cumbersome to identify such connections without such cues.

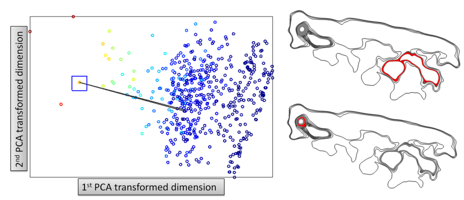


Figure 6: Exploring Ocean dataset. **Left.** A large feature corresponding to the enclosed points. **Right.** Smaller feature corresponding to the other end of the link.

6 Discussion and Future Work

This work presents a novel metric for identifying and organizing flow features from large number of streamlines. We achieve reasonably fast performance on an Intel(R) Core(TM) i7-2600 CPU @ 3.40GHz, 16GHz machine, where box counting ratio computation of 1000 streamlines (each having 3419 steps on an average) from Ocean takes only 14 seconds. As future work, we intend to study the fractal behavior of path lines and stream surfaces.

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