1. Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$.

2. Let

$$M = \{ \langle a, b, c, d \rangle : a, b, c \text{ and } p \text{ are binary integers }$$
$$\text{such that } a^b \equiv c \pmod{p} \}.$$ 

Show that $M \in P$. (Note that the most obvious algorithm does not run in polynomial time. Hint: Try first where $b$ is a power of 2.)

3. Prove that the following language is undecidable:

$$A = \{ \langle M \rangle : M \text{ is a TM with running time } O(n) \}.$$ 

4. Prove that the following language is undecidable:

$$A = \{ \langle M \rangle : L(M) \in \text{TIME}(n) \}.$$