Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. A permutation on the set \{1, \ldots, k\} is a one-to-one, onto function on this set. When \( p \) is a permutation, \( p^t \) means the composition of \( p \) with itself \( t \) times. Let

\[
\text{PERM-POWER} = \{ \langle p, q, t \rangle : p = q^t \text{ where } p \text{ and } q \text{ are permutations on } \{1, \ldots, k\} \text{ and } t \text{ is a binary integer} \}.
\]

Show that \( \text{PERM-POWER} \in P \). (Note that the most obvious algorithm doesn’t run within polynomial time. Hint: First try it where \( t \) is a power of 2.)

2. Prove that the following language is undecidable:

\[
A = \{ \langle M \rangle : M \text{ is a TM that runs in time } 2^{O(n)} \}.
\]

3. Let \( \text{coNP} \) be the class of languages whose complement is in \( NP \). Show that \( P \subseteq NP \cap \text{coNP} \). Show that if \( P = NP \) then \( P = \text{coNP} \).

(Warning: \( \text{coNP} \) is not the complement of \( NP \).)

4. Let

\[
\text{DOUBLE} - SAT = \{ \langle \phi \rangle : \phi \text{ is a boolean formula that has at least two satisfying assignments} \}.
\]

Show that \( \text{DOUBLE} - SAT \) is \( NP \)-complete.