Problem numbers are from the third edition of “Introduction to algorithms”. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Given a graph $G = (V, E)$, show that if $v_1, \ldots, v_k \in V$ is the sequence of vertices of a shortest path between $v_1$ and $v_k$, then $v_1, \ldots, v_{k-1}$ is the sequence of vertices of a shortest path between $v_1$ and $v_{k-1}$.

2. Describe an $O(|V| + |E|)$ algorithm for the following problem: Given an undirected graph $G = (V, E)$ as adjacency lists, determine whether we can paint each vertex red or blue so that adjacent vertices get different colors. If such a coloring exists, the algorithm outputs one such coloring. (Hint: Breadth-first search).

3. 22.3-2, but do not show the classification of each edge.