A Block-Adaptive Subspace Method Using Oblique Projections for Blind Separation of Convolutive Mixtures*

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Abstract. This paper presents a block-adaptive subspace algorithm via oblique projection for blind source separation (BSS) problem of convolutive mixtures. In the proposed algorithm, the problem is reformulated into the one of instantaneous mixtures through oblique projections within one block, and then the separation matrix and other model parameters are updated by any static separation approach in a block-adaptive scheme. Compared with other work, the proposed algorithm can obtain lower computational complexity, faster convergence and higher robustness. Simulation results of modulation signals and real speech sources validate the proposed algorithm.

1 Introduction

The BSS problem is to recover or estimate the non-Gaussian independent components from their combined observations, and many efficient algorithms have been proposed to solve this problem of instantaneous mixtures [1][2]. Recently, more attentions have been focused on the case of convolutive mixtures [3]-[6].

The existing BSS approaches for convolutive mixtures can be divided into two groups in frequency domain and time domain. The key of the frequency domain approaches is to reformulate convolutive mixtures in time domain into instantaneous mixtures in frequency domain by Fourier transformation. However, such algorithms would encounter the permutation problem as well as its huge computation for FFT and BSS at each frequency point [3]. The time domain approaches are based on higher-order statistics, e.g., [4], or second-order statistics in subspace methods, e.g., [5], [6]. In those time domain approaches, the optimization of high-dimension cost functions would inevitably cause a high complexity.

This paper proposes a new block-adaptive subspace algorithm without the high-dimension optimization for the BSS of convolutive mixtures. The basic

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idea behind the new method is to reformulate the problem into the instantaneous separation model via oblique projections in one block, and then applies an instantaneous separation algorithm to update the separate matrix and other corresponding parameters for the next block. The new method requires only direct matrix computation, so it can achieve lower complexity and faster convergence.

2 Problem Formulation

The convolutive mixing model with M sources and N mixtures is given by

$$\mathbf{x}(t) = \sum_{\tau=0}^{L} \mathbf{A}(\tau)\mathbf{s}(t-\tau) \tag{1}$$

where $\mathbf{x}(t)$ is an $N \times 1$ observed signal vector, $\mathbf{s}(t)$ is an $M \times 1$ source vector, and $\mathbf{A}(\tau)$ represents the FIR channel matrix with the maximum length of $L+1$.

Let $\mathbf{X}_t = [\mathbf{x}(t), \cdots, \mathbf{x}(t+j-1)]$ and $\mathbf{X}_{t|t+i-1} = [\mathbf{X}_t^T, \cdots, \mathbf{X}_{t+i-1}^T]^T$ denote the observation matrix and block representation at time t, respectively, and then (1) can be rewritten as

$$\mathbf{X}_{t|t+i-1} = \Gamma_i(\mathbf{A})\mathbf{S}_{t-L|t+i-1} \tag{2}$$

where $\Gamma_i(\mathbf{A})$ is a Toeplitz matrix

$$\Gamma_i(\mathbf{A}) = \begin{bmatrix} \mathbf{A}(L) & \cdots & \mathbf{A}(0) & & \\ & \ddots & \cdots & \ddots & \\ & & \mathbf{A}(L) & \cdots & \mathbf{A}(0) \end{bmatrix}_{Ni \times M(i+L)} .$$

For (2) to be solvable, we assume that

A1. The sources $s_m(t), m = 1, \cdots, M$ are mutually independent.

A2. The source sequence $s_m(t)$ has the linear complexity greater than $2i+2L$.

A3. $\Gamma_i(\mathbf{A})$ has full column rank.

Assumption A1 is fundamental in all BSS problems [2], and *Assumptions A2* and *A3* indicate an isomorphic relation between the output and input subspaces [8], namely, if some $\Gamma_{i_0}(\mathbf{A})$ has full column rank, then we have

$$\mathrm{R}\{\mathbf{X}_{t|t+i-1}\} \simeq \mathrm{R}\{\mathbf{S}_{t-L|t+i-1}\}, \quad \forall i > i_0 \tag{3}$$

where $\mathrm{R}\{\mathbf{X}_{t|t+i-1}\}$ and $\mathrm{R}\{\mathbf{S}_{t-L|t+i-1}\}$ denote the row subspaces spanned by the data block $\mathbf{X}_{t|t+i-1}$ and $\mathbf{S}_{t-L|t+i-1}$, respectively. *Assumption A3* also implies that $N > M$ (more sensors than sources).

3 Block-Adaptive Algorithm Using Oblique Projections

Consider two row subspaces $P = \mathrm{R}\{\mathbf{P}\}$ and $Q = \mathrm{R}\{\mathbf{Q}\}$, then the oblique projector along Q onto P is given by [7]

$$\mathbf{E}_{P|Q} = \begin{bmatrix} \mathbf{P}^H & \mathbf{Q}^H \end{bmatrix} \begin{bmatrix} \mathbf{P}\mathbf{P}^H & \mathbf{P}\mathbf{Q}^H \\ \mathbf{Q}\mathbf{P}^H & \mathbf{Q}\mathbf{Q}^H \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{P} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

where $\mathbf{B}^{\dagger}$ is the pseudo-inverse of $\mathbf{B}$. Obviously, the projector $\mathbf{E}_{P|Q}$ satisfies

$$\mathbf{P}\mathbf{E}_{P|Q} = \mathbf{P}, \mathbf{Q}\mathbf{E}_{P|Q} = \mathbf{0}, \tag{5}$$

namely, P and Q are the range and null spaces of the projector $\mathbf{E}_{P|Q}$.

We notice that oblique projectors should keep invariant in two isomorphic subspaces, as the following proposition:

Proposition 1. *If the matrices* $\mathbf{T}_1$ *and* $\mathbf{T}_2$ *have full column rank, then*

$$\mathbf{E}_{P|Q} = \mathbf{E}_{\mathrm{R}\{\mathbf{T}_1\mathbf{P}\}|\mathrm{R}\{\mathbf{T}_2\mathbf{Q}\}} \tag{6}$$

The proof can be easily derived from the oblique projector definition and found in [9]. By *Proposition 1* and (3), oblique projectors in source subspaces can be constructed from the those in observation subspaces.

Consider the BSS problem of convolutive mixtures (2). The algorithm aims to find some operator $\mathbf{E}$ to obtain instantaneous mixtures as follows

$$\mathbf{X}_{t|t+L}\mathbf{E} = \mathbf{F}(\mathbf{A})\mathbf{s}_t \tag{7}$$

where $\mathbf{F}(\mathbf{A})$ are composed of channel response parameters. In our multiple input multiple output (MIMO) system, construct the *present*, *past* and *future* observation subspaces and their corresponding isomorphic source subspaces as

$$\begin{aligned} X_{\mathrm{pr}} &= \mathrm{R}\{\mathbf{X}_{t|t+L}\} \simeq \mathrm{R}\{\mathbf{S}_{t-L|t+L}\} = S_{\mathrm{pr}}, \\ X_{\mathrm{pa1}} &= \mathrm{R}\{\mathbf{X}_{t-i|t-1}\} \simeq \mathrm{R}\{\mathbf{S}_{t-i-L|t-1}\} = S_{\mathrm{pa1}}, \\ X_{\mathrm{pa2}} &= \mathrm{R}\{\mathbf{X}_{t-i|t}\} \simeq \mathrm{R}\{\mathbf{S}_{t-i-L|t}\} = S_{\mathrm{pa2}}, \\ X_{\mathrm{fu1}} &= \mathrm{R}\{\mathbf{X}_{t+L+1|t+L+i}\} \simeq \mathrm{R}\{\mathbf{S}_{t+1|t+L+i}\} = S_{\mathrm{fu1}}, \\ X_{\mathrm{fu2}} &= \mathrm{R}\{\mathbf{X}_{t+L|t+L+i}\} \simeq \mathrm{R}\{\mathbf{S}_{t|t+L+i}\} = S_{\mathrm{fu2}}. \end{aligned}$$

Substituting $\mathbf{E}_{X_{\mathrm{fu2}}|X_{\mathrm{pa1}}} = \mathbf{E}_{S_{\mathrm{fu2}}|S_{\mathrm{pa1}}}$ and using (5), the projection of $\mathbf{S}_{t-L|t+L}$ by $\mathbf{E}_{X_{\mathrm{fu2}}|X_{\mathrm{pa1}}}$ can be simplified as

$$\mathbf{S}_{t-L|t+L}\mathbf{E}_{X_{\mathrm{fu2}}|X_{\mathrm{pa1}}} = \mathbf{S}_{t-L|t+L}\mathbf{E}_{\mathrm{R}\{\mathbf{S}_{t|t+L+i}\}|\mathrm{R}\{\mathbf{S}_{t-i-L|t-1}\}} = \left[\mathbf{0}^T, \mathbf{S}_{t|t+L}^T\right]^T, \tag{8}$$

Similarly,

$$\mathbf{S}_{t|t+L}\mathbf{E}_{X_{\mathrm{pa2}}|X_{\mathrm{fu1}}} = \left[\mathbf{s}_t^T, \mathbf{0}^T\right]^T, \tag{9}$$

then the joint projection of $\mathbf{X}_{t|t+L}$ by $\mathbf{E}_1 = \mathbf{E}_{X_{\mathrm{fu2}}|X_{\mathrm{pa1}}}\mathbf{E}_{X_{\mathrm{pa2}}|X_{\mathrm{fu1}}}$ is reduced to

$$\begin{aligned} \mathbf{X}_{t|t+L}\mathbf{E}_1 &= \mathit{\Gamma}_{L+1}(\mathbf{A})\mathbf{S}_{t-L|t+L}\mathbf{E}_{X_{\mathrm{fu2}}|X_{\mathrm{pa1}}}\mathbf{E}_{X_{\mathrm{pa2}}|X_{\mathrm{fu1}}} \\ &= \mathit{\Gamma}_{L+1}(\mathbf{A})\left[\mathbf{0}^T, \mathbf{s}_t^T, \mathbf{0}^T\right]^T \\ &= \mathit{\Gamma}_{L+1}(\mathbf{A})\mathbf{H}\mathbf{s}_t \end{aligned} \tag{10}$$

where $\mathbf{H} = [\mathbf{0}_{LM\times(LM+M)}^T, \mathbf{I}_{M\times M}^T, \mathbf{0}_{LM\times(LM+M)}^T]^T$ is used to select the middle M columns of $\mathit{\Gamma}_{L+1}(\mathbf{A})$.

Though the instantaneous mixture model (10) is obtained in a similar way to Yu's work on channel estimation in a single input-multiple output (SIMO) system [8], their differences lie in: $\mathbf{s}_t$ is a row vector and the rank of $\mathbf{X}_{t|t+L}\mathbf{E}_1$ equals one in Yu's work; while $\mathbf{s}_t$ is a $M \times j$ matrix and the rank of $\mathbf{X}_{t|t+L}\mathbf{E}_1$ equals M in this model, so the SVD of (10) yields instantaneous mixtures

$$\mathbf{X}_{t|t+L}\mathbf{E}_1 = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H, \tag{11}$$

$$\mathbf{A}_{ins}\mathbf{s}_t = \mathbf{\Sigma}(1:M,1:M)\mathbf{V}(:,1:M)^H; \tag{12}$$

furthermore, $\mathbf{A}_{ins}$ inevitably varies with different blocks for the uncertainty in a MIMO model. For convenience, the symbols with the upper sign $^{(k)}$, e.g. $\mathbf{U}^{(k)}, \mathbf{\Sigma}^{(k)}, \mathbf{V}^{(k)}$ and $\mathbf{A}_{ins}^{(k)}$, represent the corresponding components for the kth block.

Using (7), (11) and (12), we have

$$\mathbf{U}^{(k)}(:,1:M)\mathbf{A}_{ins}^{(k)} = \mathbf{F}(\mathbf{A}) = \mathbf{const}. \tag{13}$$

Based on (13), a block-adaptive scheme is developed to keep the same instantaneous mixtures model for each block in the below. Define the transform matrix $\mathbf{T}^{(k)}$ and instantaneous mixtures $\mathbf{Z}^{(k)}$ for the kth block as

$$\mathbf{T}^{(k)} = \mathbf{U}^{(1)}(:,1:M)^{\dagger}\mathbf{U}^{(k)}(:,1:M), \tag{14}$$

$$\mathbf{Z}^{(k)} = \mathbf{T}^{(k)}\mathbf{\Sigma}^{(k)}(1:M,1:M)\mathbf{V}^{(k)}(:,1:M)^H, \tag{15}$$

and simplify $\mathbf{Z}^{(k)}$ with (12) - (15)

$$\mathbf{Z}^{(k)} = \mathbf{U}^{(1)}(:,1:M)^{\dagger}\mathbf{U}^{(k)}(:,1:M)\mathbf{A}_{ins}^{(k)}\mathbf{s}_k = \mathbf{A}_{ins}^{(1)}\mathbf{s}_k \tag{16}$$

So the initial value of separation matrix $\mathbf{W}_{1,ini}^{(k)}$ for the kth block is given by the final value of separation matrix $\mathbf{W}_{1,fin}^{(k-1)}$ for the last block, i.e.,

$$\mathbf{W}_{1,ini}^{(k)} = \mathbf{W}_{fin}^{(k-1)} \tag{17}$$

Finally, the separated matrix $\mathbf{W}^{(k)}$ can be computed by any static BSS algorithm in each block.

The proposed algorithm can be seen as a sliding window method with the window size j. To make full use of each block, the well-known JADE in [1] is modified with the following update of higher-order cumulant $\mathbf{c}^{(k)}$ for the kth block

$$\mathbf{c}^{(k)} = \alpha\mathbf{c}^{(k-1)} + (1-\alpha)\mathbf{c}_k \tag{18}$$

where $\mathbf{c}_k$ denotes the higher-order cumulant calculated only from the kth block, and α acts as a forgetting factor. This modified JADE algorithm, called Block-JADE, can deal with a small number of block data.

Compared with other subspace algorithms, e.g. in [5][6], our algorithm has the following advantages: firstly, it reduces a high computational burden by

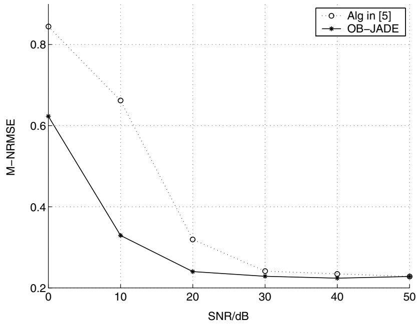


Fig. 1. M-NRMSE comparison versus different SNRs over 500 runs, where two modulation sources are given by $s_1(k) = sin(6\pi k/1000)sin(4\pi k/25)$ and $s_2(k) = sin(\pi k/50 + sin(\pi k/500))$

direct oblique projections avoiding high-dimension optimizations in their work; secondly, the proposed approach is block-adaptive and only requires a smaller number of data each time, while others are all batch algorithms; finally, the SVD in the proposed algorithm benefits the noise elimination and good performance at low SNR.

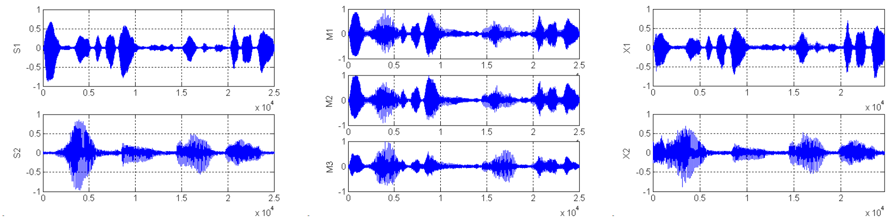
4 Experimental Results

To illustrate the performance of the proposed oblique projection algorithm, some simulations are presented in this section. The channel matrix is randomly generated, with the dimension $N \times M = 3 \times 2$ and the degree is $L = 4$. The data block parameters are chosen to be $i = 4, j = 200$, Block-JADE with $\alpha = (k-1)/k$ is taken as an instantaneous separation approach in our algorithm, called OB-JADE for short. For comparison, the mutually referenced algorithm in [5] is applied at the same time. To access the blind separation capability, multi-source normalized root mean square error (M-NRMSE) is given by:

$$\text{M-NRMSE} = \frac{1}{M}\sum_{i=1}^{M}\sqrt{\frac{1}{N_k\left\|s_i\right\|^2}\sum_{k=1}^{N_k}\left\|\hat{s}_{i(k)} - s_i\right\|^2} \tag{19}$$

where N_k is the number of independent runs, and $\hat{s}_{i(k)}$ is the ith separated signal sequence in the kth run.

Many simulations of different sources show that the proposed algorithm can efficiently reconstruct sources. Though both can achieve the separation of stationary signals, our algorithm can obtain a lower computation and better performance at low SNR, as shown in Fig. 1. Moreover, the proposed algorithm can obtain satisfactory results for non-stationary signals such as speech signals, while the algorithm in [5] can not. Fig. 2 shows the experiment results of two real speech signals by OB-JADE. It is observed that the speech signals are successfully reconstructed in about 10 blocks (2000 symbols).



(a) (b) (c)

Fig. 2. Blind separation of convolutive speech signals. (a) Speech sources. (b) Convolutive mixtures. (c) Separated signals

5 Conclusion

In this paper, we present a new subspace adaptive algorithm using oblique projections for blind separation of convolutive mixtures. The main idea behind this algorithm is the deconvolution via oblique projections and block-adaptive scheme on the relation between the instantaneous model and real channel parameters. Compared with other existing subspace methods, the proposed approach is a block-adaptive algorithm, with a lower computational cost, faster convergence and noise robustness.

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