Computer Animation Algorithms and Techniques

Physically Based Animation

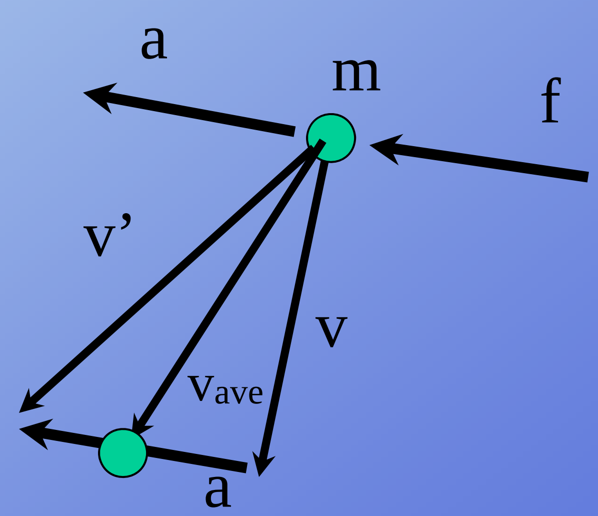
Physics Review: force, mass, acceleration velocity, position

$$f = ma$$

$$a = f / m$$

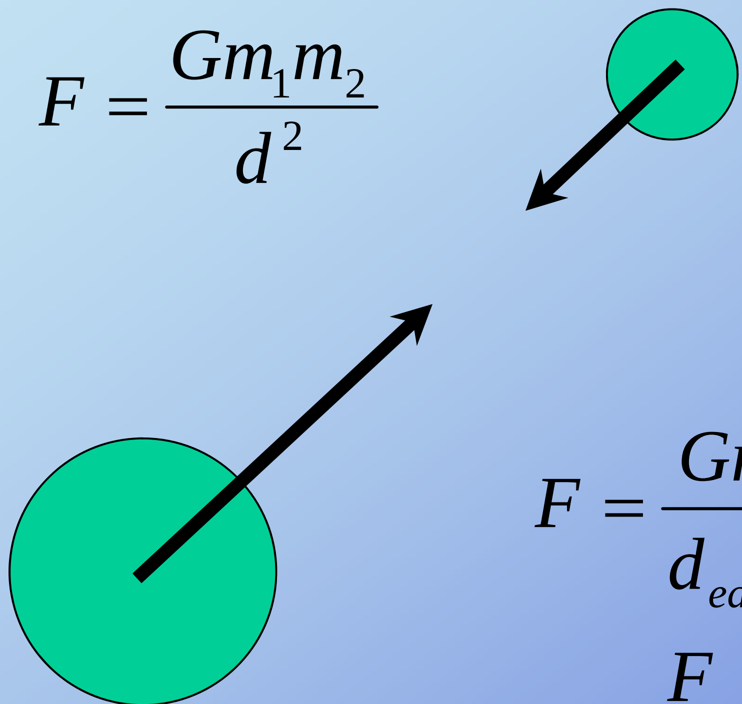
$$v' = v + a \cdot \Delta t$$

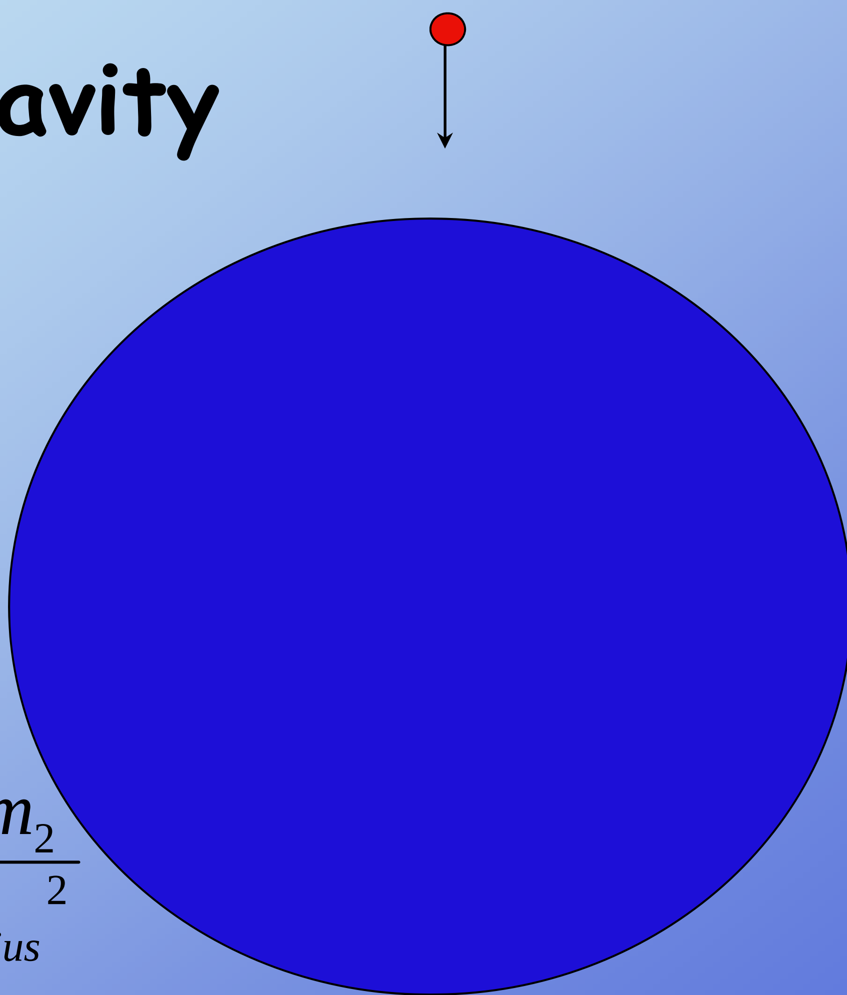
$$p' = p + \frac{(v + v')}{2}\Delta t = p + v\Delta t + \frac{1}{2}a\Delta t^2$$



Physics Review: Gravity

$$F = \frac{Gm_1 m_2}{d^2}$$





$$F = \frac{Gm_{earth} m_2}{d_{earth-radius}^{\;2}}$$

$$a = \frac{F}{m_2} = 9.8m/s^2$$

Physics Review: Other forces

$$f_{st} = k_{st} f_N$$

Static friction

$$f_k = k_k f_N$$

Kinetic friction

$$f_{vis} = -K_{vis} nv$$

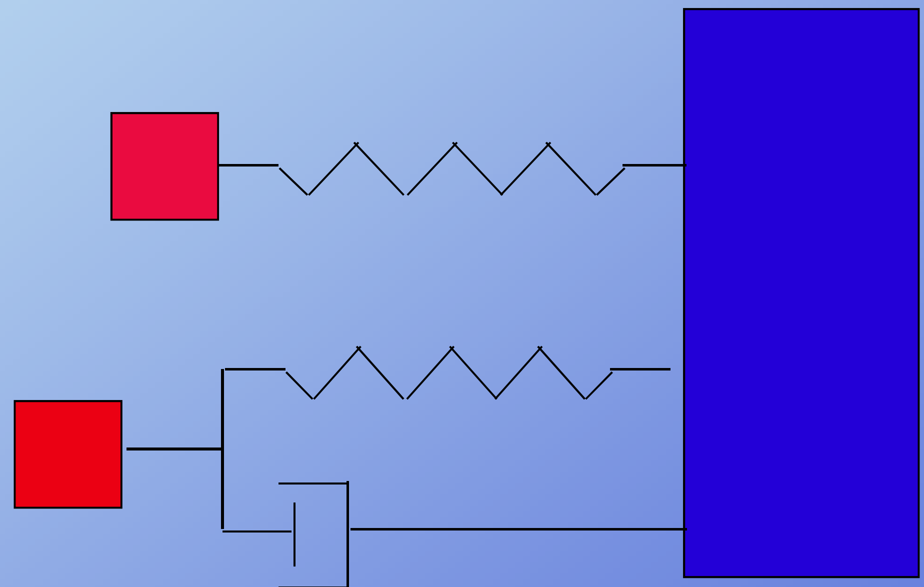
$$K_{vis} = 6\pi r$$

Viscosity
for small objects
No turbulence

For sphere

Physics Review: Spring-damper

$$F = k_s(L_{current} - L_{rest})$$



$$F = k_s(L_{current} - L_{rest}) - k_d V_{spring}$$

Hooke's Law

Physics Review: Momentum

$$P = mv$$

conservation of momentum (mv)

In a closed system, momentum is conserved

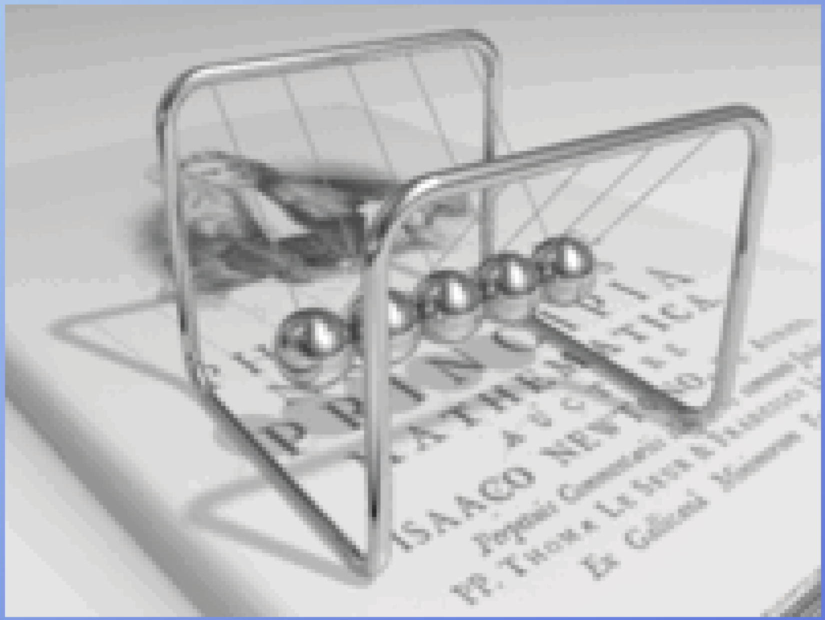
$$P = \Sigma mv = \Sigma m'v'$$

After collision has same momentum as before collision

Elastic Collisions

No energy lost (e.g. deformations, heat)

$$\Sigma mv = \Sigma m'v'$$



$$F = ma = \frac{d(mv)}{dt} = \frac{dP}{dt}$$

Elastic Collisions & Kinetic Energy

As in all collisions: momentum is conserved

$$P = \Sigma mv = \Sigma m'v'$$

In elastic collisions, <u>kinetic energy</u> is also conserved

$$P = \Sigma \frac{1}{2} mv^2 = \Sigma \frac{1}{2} m'v'^2$$

Solve for velocities

Inelastic Collisions

Kinetic energy is NOT conserved

Inelastic Collisions

Kinetic energy lost to deformation and heat

Momentum is conserved

<u>Coefficient of restitution</u>
ratio of velocities before and after collision

Center of Mass

$$C_{mass} = \frac{1}{M}\int (r)dm = \frac{1}{M}\int (\rho(r)r)dV = \frac{\int (\rho(r)r)d\rho V}{\int \rho(r)dV}$$

$$C_{mass} = \frac{\Sigma m_i r_i}{\Sigma m_i}$$

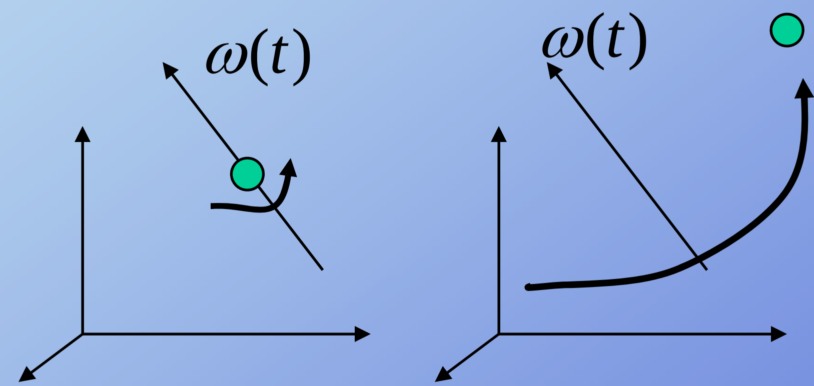
Physics Review
Linear v. angular terms

Linear	Angular
postion	Rotation, orientation
velocity	rotational (angular) velocity
acceleration	rotational (angular) acceleration
mass	Inertia tensor
Force	Torque
momentum	Angular momentum

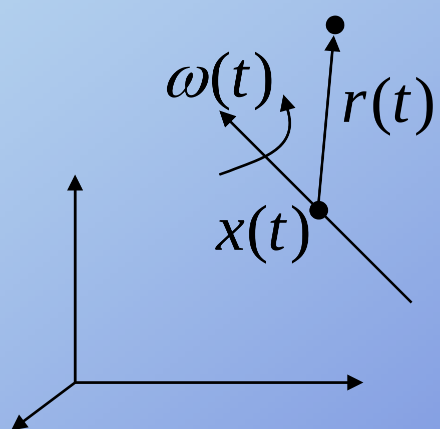
Change in point due to rotation

$\omega(t)$ - Angular velocity

On-axis or off axis rotation
Angular velocity is the same



$$p(t) = x(t) + r(t)$$



$$\dot{r}(t) = \omega(t) \times r(t)$$

$$|\dot{r}(t)| = |\omega(t)||r(t)|\sin\theta$$

Delta-orientation due to rotation

$$R(t) = \begin{bmatrix} R_1(t) & R_2(t) & R_3(t) \end{bmatrix}$$

$$\dot{R}(t) = \begin{bmatrix} \omega(t) \times R_1(t) & \omega(t) \times R_2(t) & \omega(t) \times R_3(t) \end{bmatrix}$$

$$A \times B = \begin{bmatrix} A_y B_z - A_z B_y \\ A_z B_x - A_x B_z \\ A_x B_y - A_y B_x \end{bmatrix} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = A^* B$$

$$\dot{R}(t) = \omega(t)^* R(t)$$

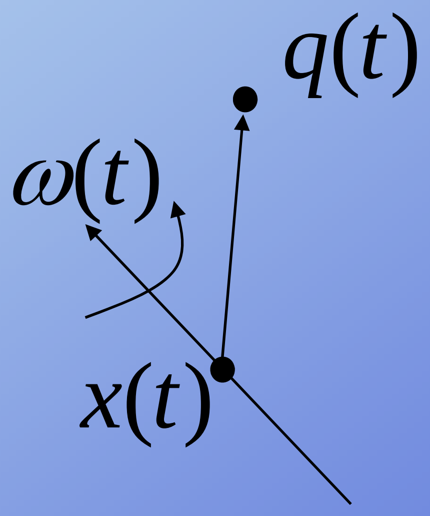
Change to a point on rotating object

$$q(t) = R(t)q + x(t)$$

$$R(t)q = q(t) - x(t)$$

$$\dot{q}(t) = \omega(t)^{*} R(t)q + v(t)$$

$$\dot{q}(t) = \omega(t) \times (q(t) - x(t)) + v(t)$$



Physics Review: Angular Stuff

$$f = ma$$

$$P = mv$$

$$f = \frac{dP}{dt}$$

$$\Sigma P_i = c$$

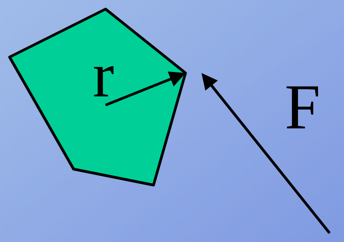
$$\tau = r \times F$$

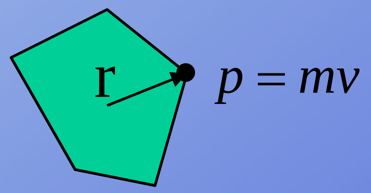
$$L = r \times p$$

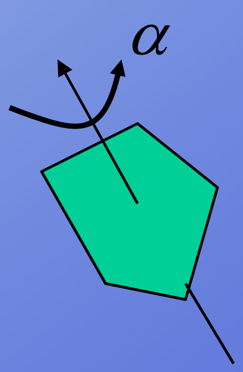
$$\tau = \frac{dL}{dt}$$

$$\Sigma L_i = c$$

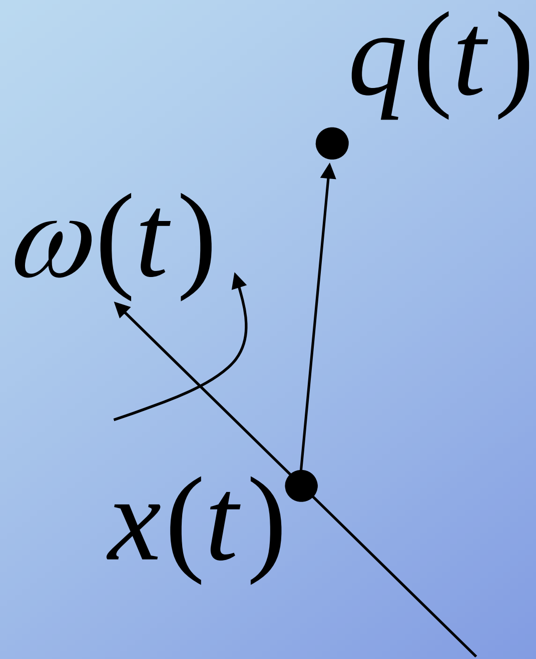
$$\tau = I\alpha$$







Angular Momentum



$$L(t) = \Sigma((q(t) - x(t)) \times m_i(\dot{q}(t) - v(t)))$$

$$= \Sigma(R(t)q \times m_i(\omega(t) \times (q(t) - x(t))))$$

$$= \Sigma(m_i(R(t)q \times (\omega(t) \times R(t)q)))$$

Inertia Tensor

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

Aka Angular mass

$$I_{xx} = \int (y^2 + z^2) dm \qquad I_{xy} = -\int xydm$$

$$I_{yy} = \int (x^2 + z^2) dm \qquad I_{xz} = -\int xzdm$$

$$I_{zz} = \int (x^2 + y^2) dm \qquad I_{yz} = -\int yzdm$$

Inertia Tensor of particles

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

Discrete version
For particle collection

$$I_{xx} = \Sigma m_i (y_i{}^2 + z_i{}^2) \qquad I_{xy} = -\Sigma m_i x_i y_i$$

$$I_{yy} = \Sigma m_i (x_i{}^2 + z_i{}^2) \qquad I_{xz} = -\Sigma m_i x_i z_i$$

$$I_{zz} = \Sigma m_i (x_i{}^2 + y_i{}^2) \qquad I_{yz} = -\Sigma m_i y_i z_i$$

Standard Inertia Tensors

Symmetric wrt axes

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Cuboid

$$I = \frac{1}{12}\begin{bmatrix} M(b^2+c^2) & 0 & 0 \\ 0 & M(a^2+c^2) & 0 \\ 0 & 0 & M(a^2+b^2) \end{bmatrix}$$

Sphere

$$I = \frac{2MR^2}{5}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

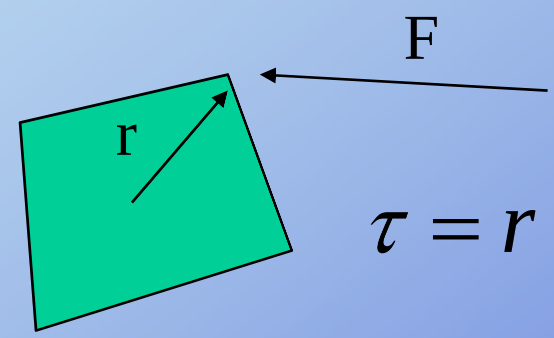
Inertia Tensor transformations

$$I_{translated} = \begin{bmatrix} I_{xx} + M(Y^2 + Z^2) & -I_{xy} - MXY & -I_{xz} - MXZ \\ -I_{xy} - MXY & I_{yy} + M(X^2 + Z^2) & -I_{yz} - MYZ \\ -I_{xz} - MXZ & -I_{yz} - MYZ & I_{zz} + M(X^2 + Y^2) \end{bmatrix}$$

$$I_{rotated} = R I_{object} R^{-1}$$

The Equations

$$S(t) = \begin{bmatrix} x(t) \\ R(t) \\ P(t) \\ L(t) \end{bmatrix}$$



$$\tau = r \times F$$

<u>Object attributes</u>

M, I_{object}^{-1}

$$I_{rotated}^{\ -1} = R I_{object}^{\ -1} R^{-1}$$

$$v(t) = \frac{P(t)}{M}$$

$$w(t) = I(t)^{-1} L(t)$$

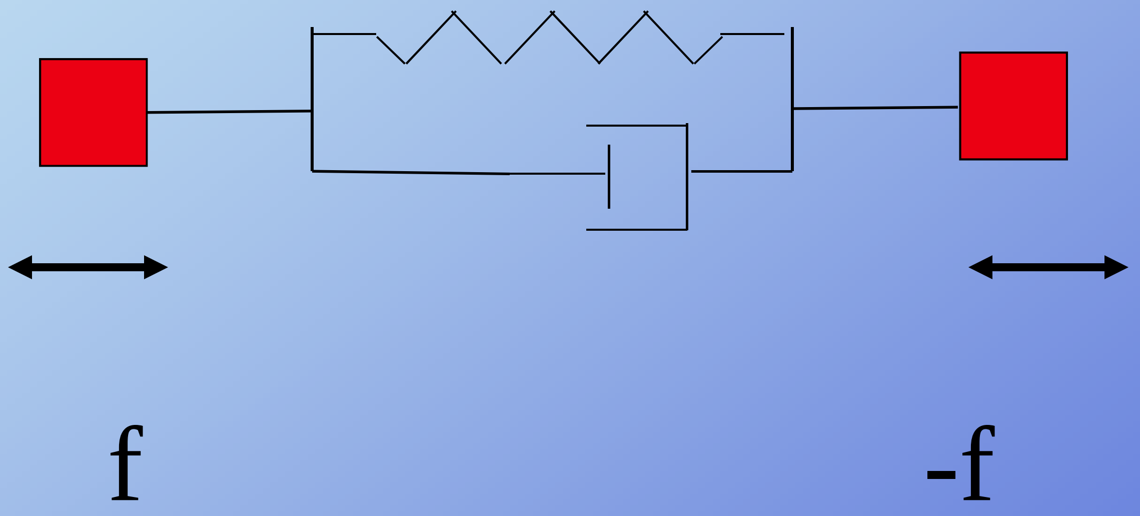
$$\frac{d}{dt} S(t) = \frac{d}{dt} \begin{bmatrix} x(t) \\ R(t) \\ P(t) \\ L(t) \end{bmatrix} = \begin{bmatrix} v(t) \\ w(t)^{*} R(t) \\ F(t) \\ \tau(t) \end{bmatrix}$$

If using rotation matrix, will need to orthonormalize updated rotation matrix

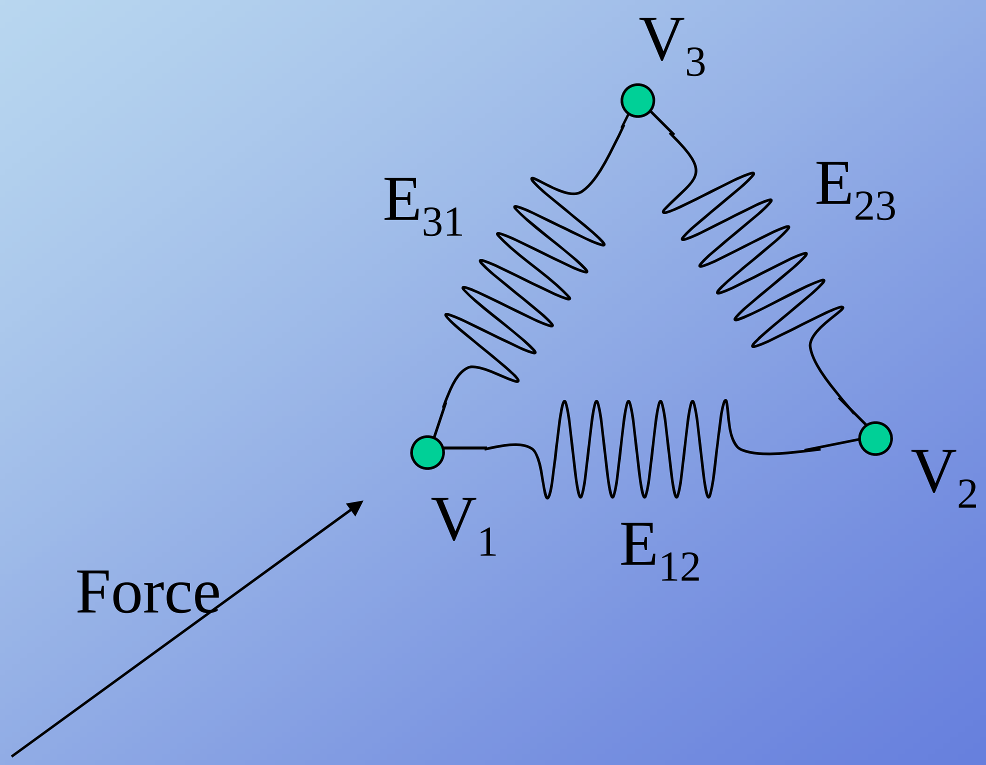
Springs

Flexible objects
Cloth
Virtual springs
Proportional derivative controllers (PDCs)

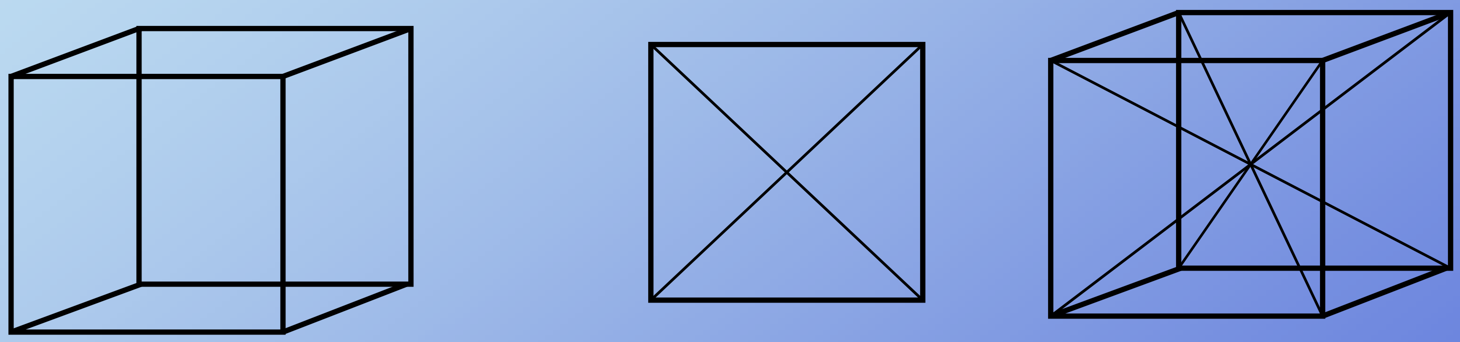
Spring-mass-damper system



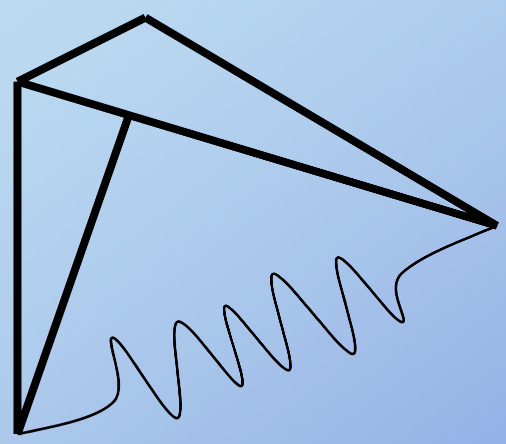
Spring-mass system



“Virtual” edge springs system



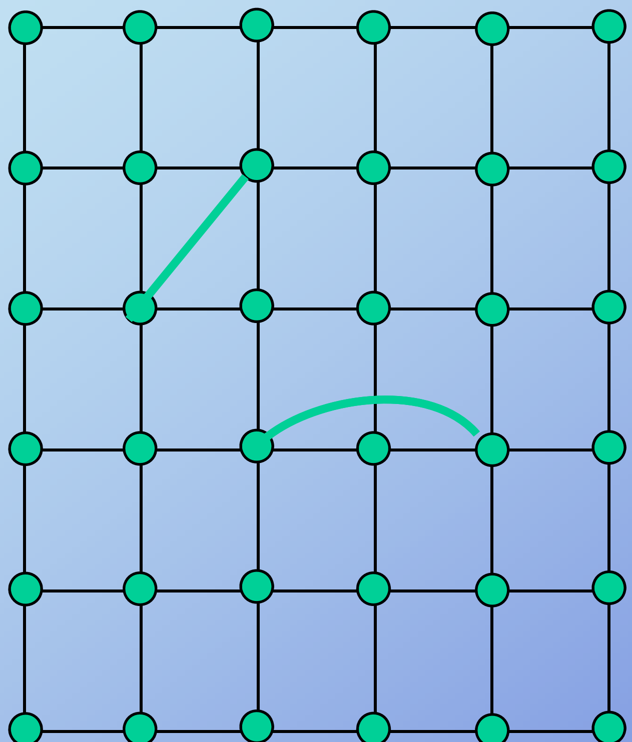
Angular springs



Linear spring between vertices

Dihedral angular spring

Spring mesh



Each vertex is a point mass

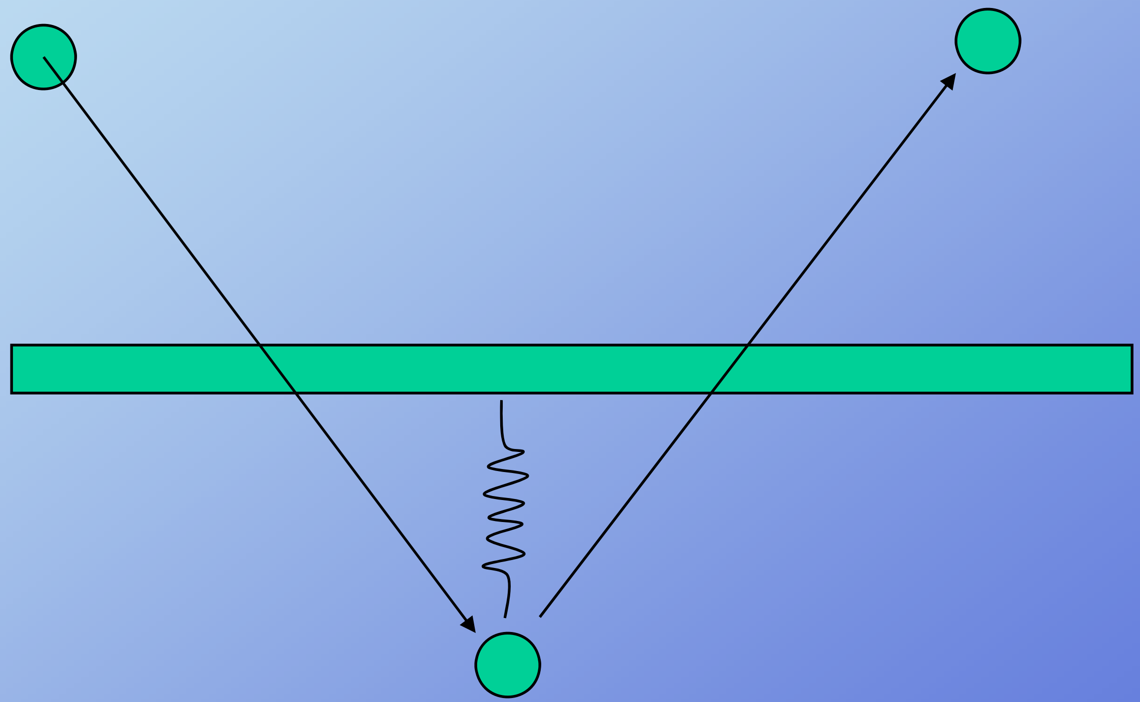
Each edge is a spring-damper

Diagonal springs for rigidity

Angular springs connect every other mass point

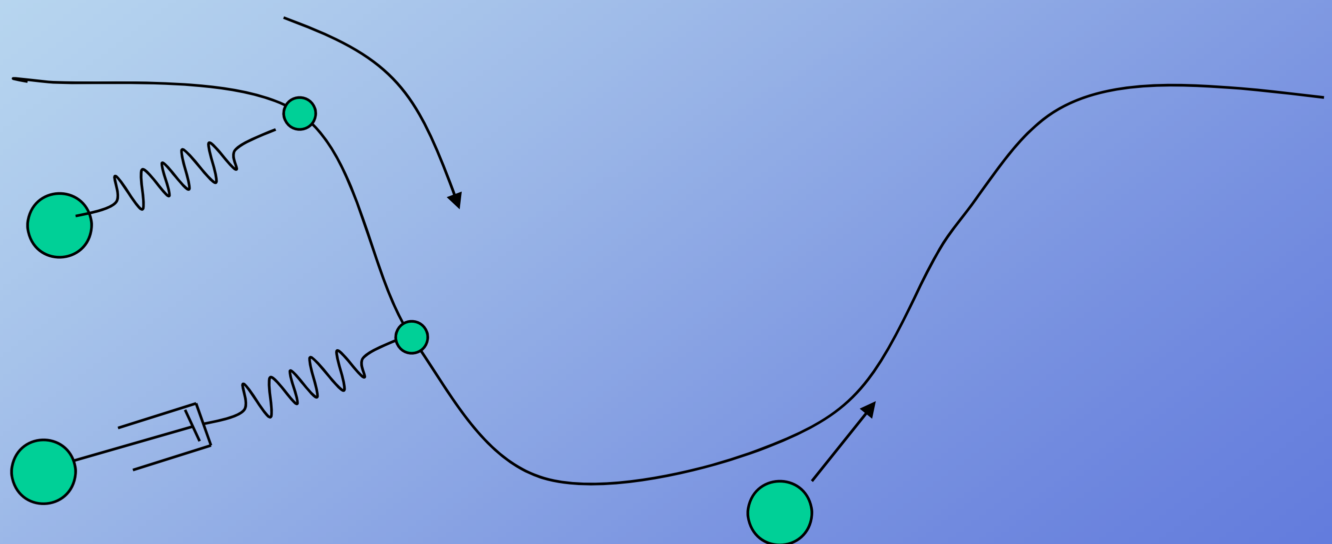
Global forces: gravity, wind

Virtual springs - soft constraints



Proportional (Derivative) Controllers

e.g., particle reacts to other forces while trying to maintain position on curve – virtual spring



$$d = x(t) - p(t)$$

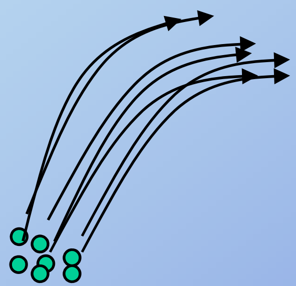
$$F = k_s d$$

$$F = k_s d - k_d v_{relative}$$

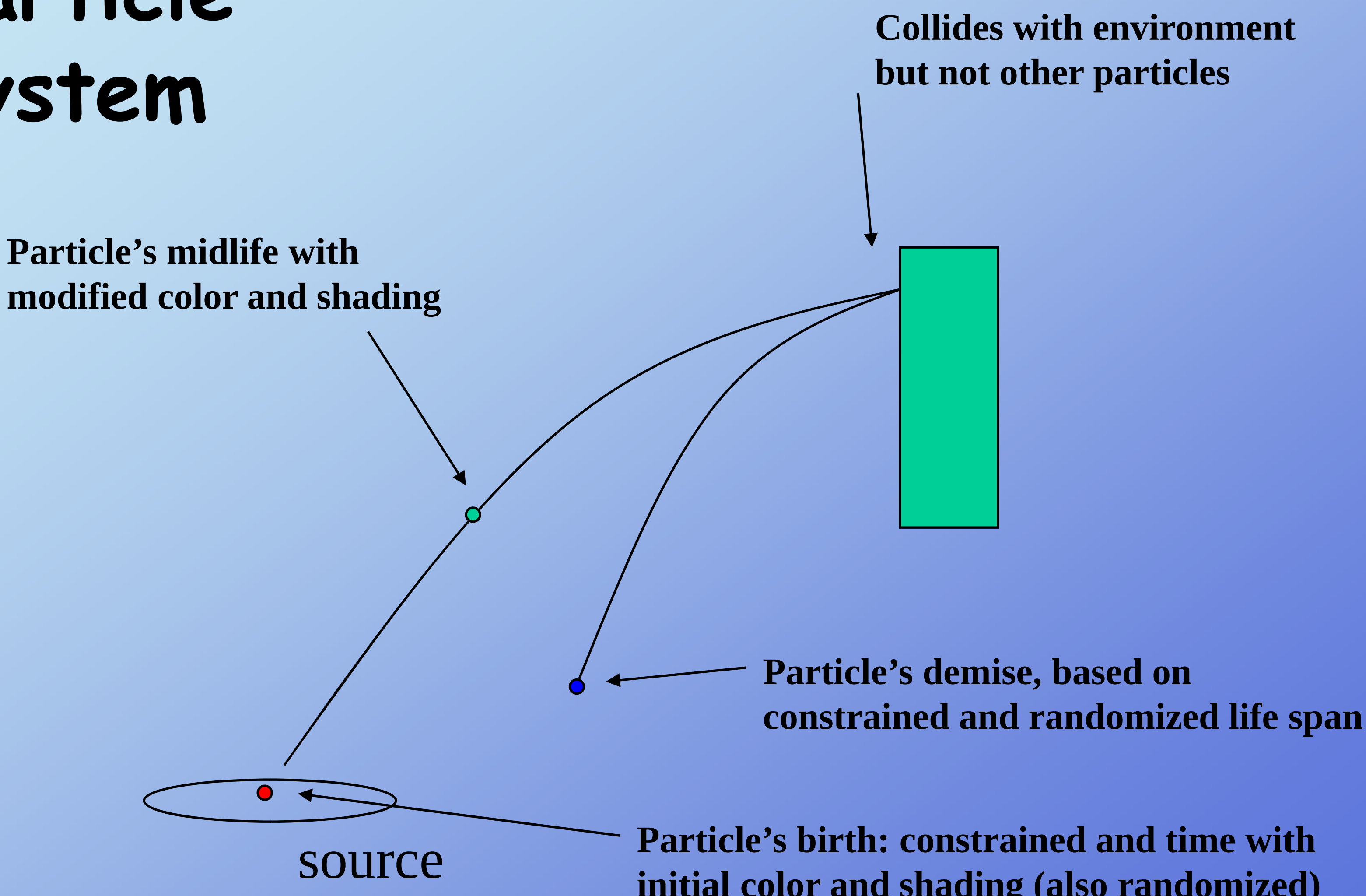
Particle systems

Lots of small particles - local rules of behavior
Create ‘emergent’ element
Particles:

- Do collide with the environment
- Do not collide with other partcles
- Do not cast shadows on other particles
- Might cast shadows on environment
- Do not reflect light - usually emit it



Particle system



Particle system implementation

<u>STEPS</u>

1. for each particle

    1. if dead, reallocate and assign new attributes

    2. animate particle, modify attributes

2. render particles

Use constrained randomization to keep control of the simulation while adding interest to the visuals