Object Intersection

Object Representation

Implicit forms

$F(x,y,z) = 0$

testing

Explicit forms

Analytic form $x = F(y,z)$

generating

Parametric form $(x,y,z) = P(t)$

Ray-Object Intersection

Implicit forms

$F(x,y,z) = 0$

Ray: $P(t) = (x,y,z) =$ source + t*direction = $s + t*v$

Solve for t: $F(P(t)) = 0$

Ray-Sphere Intersection

Implicit form for sphere at origin of radius 1

$$F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$$

Ray: $P(t) = (x, y, z) = s + tv = (s_x + tv_x, s_y + tv_y, s_z + tv_z)$

Solve: …

$$\begin{aligned} F(P(t)) &= (s_x + tv_x)^2 + (s_y + tv_y)^2 + (s_z + tv_z)^2 - 1 = 0 \\ &= s_x{}^2 + s_y{}^2 + s_z{}^2 + 2t(s_x v_x + s_y v_y + s_z v_z) + t^2(v_x{}^2 + v_y{}^2 + v_z{}^2) - 1 = 0 \end{aligned}$$

Use quadratic equation…

Ray-Sphere Intersection

$At^2 + Bt + C = 0$

$A = |v|^2$

$B = 2s \cdot v$

$C = |s|^2 - 1$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$B^2 - 4AC < 0$ => no intersection

$= 0$ => just grazes

> 0 => two hits

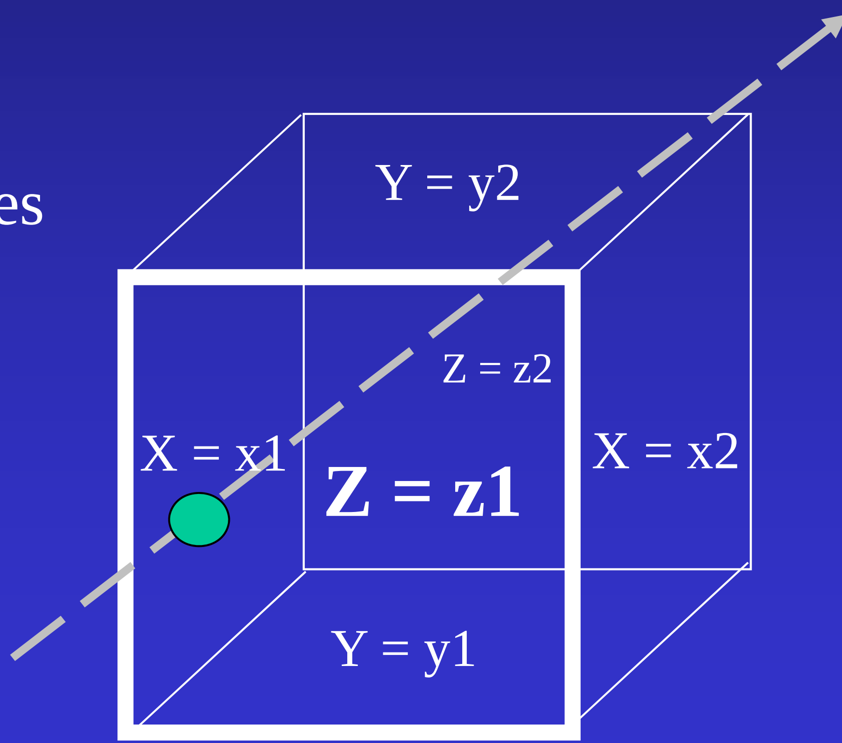
Axis-Aligned Cuboid

(rectangular solid, rectangular parallelpiped)

Ray equation

$P(t) = s + tv$

Planar equations



Solve for intersections with planes

$t_{x1} = (x1 - s_x)/v_x$

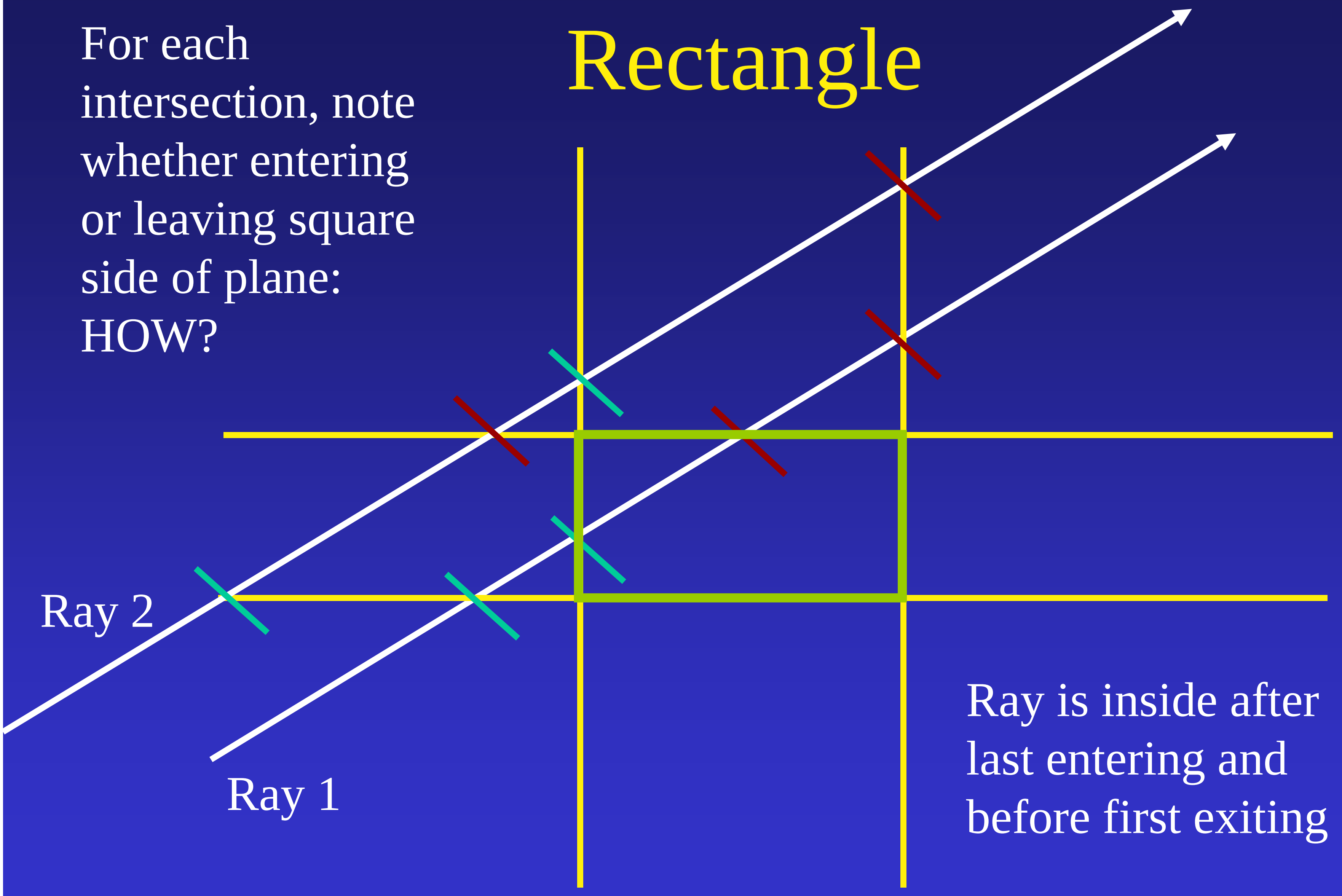
$t_{x2} = (x2 - s_x)/v_x$

$t_{y1} = (y1 - s_x)/v_x$

…

Rectangle

For each intersection, note whether entering or leaving square side of plane: HOW?



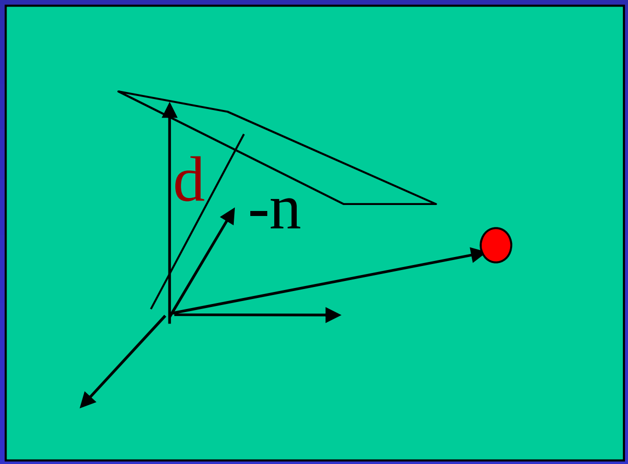
Ray is inside after last entering and before first exiting

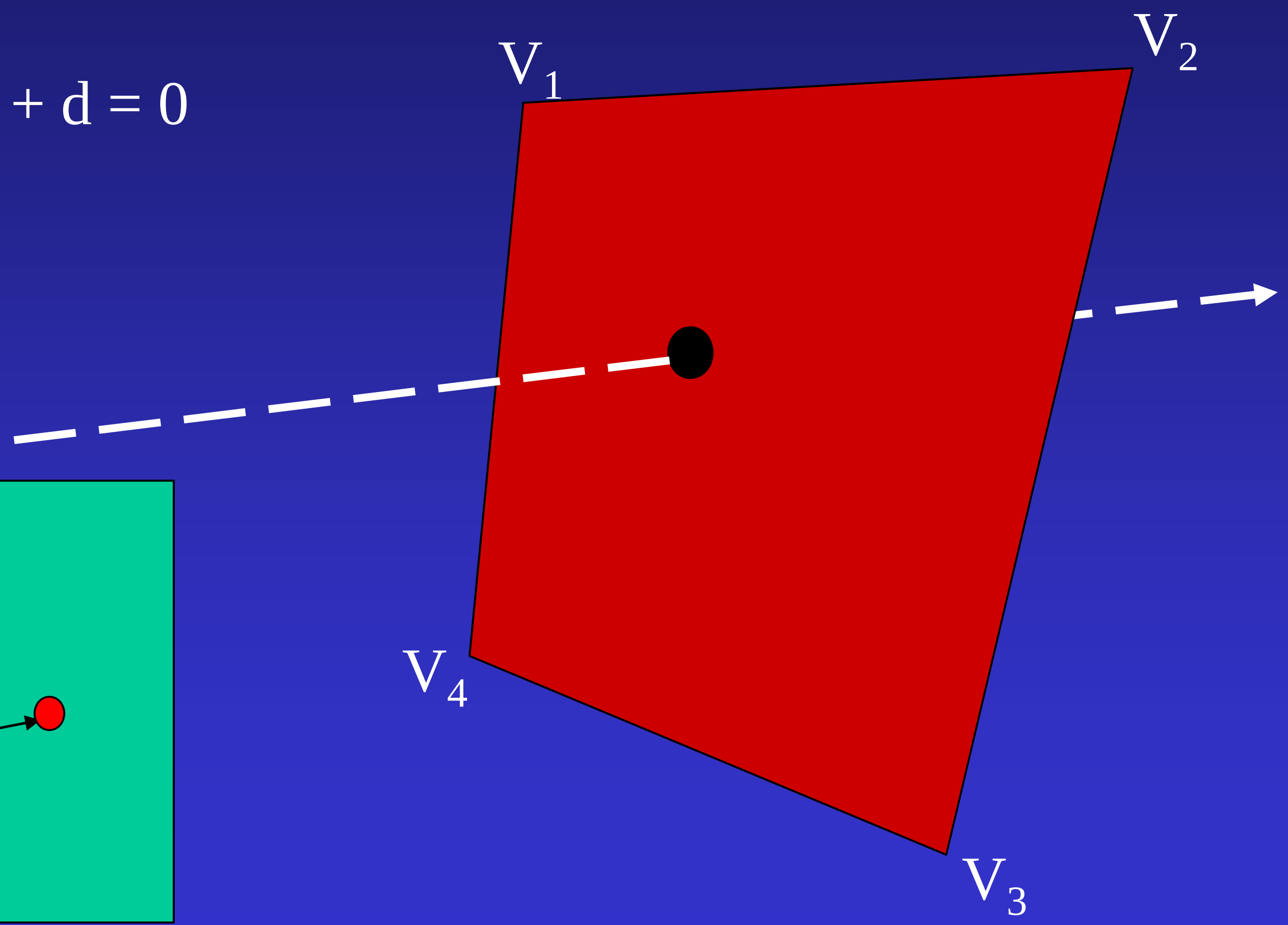
Ray-Plane for arbitrary plane

Generalize from axis-aligned planes to any plane:

$$ax + by + cz + d = 0$$

$$n \cdot P = -d$$





Normal Vector

Given ordered sequence of points defining a polygon how do you find a normal vector for the plane?

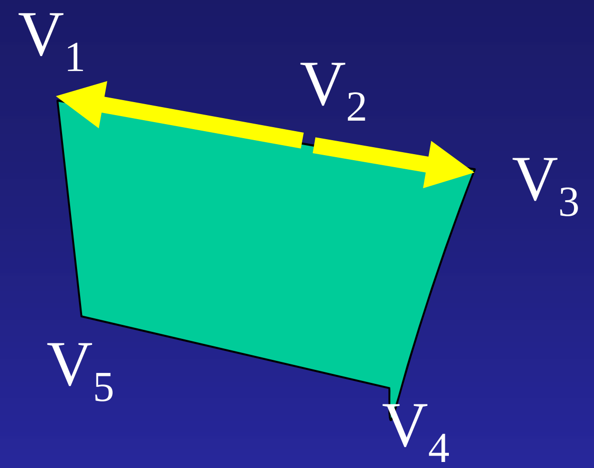
Note: 2 normal vectors to a plane, colinear and one is the negation of the other

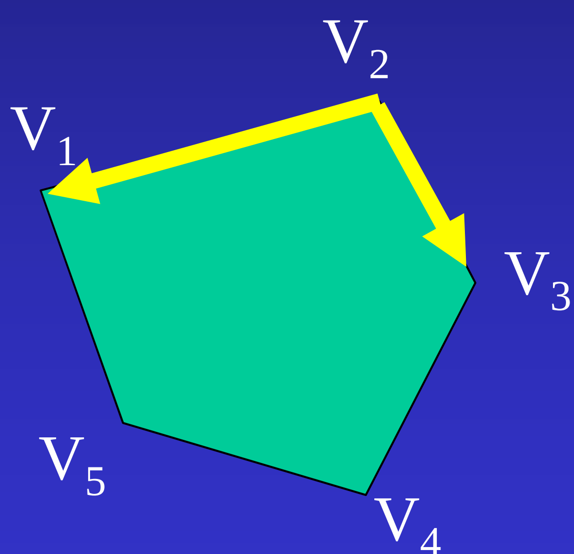
Ordered: e.g., clockwise when viewed from the front of the face

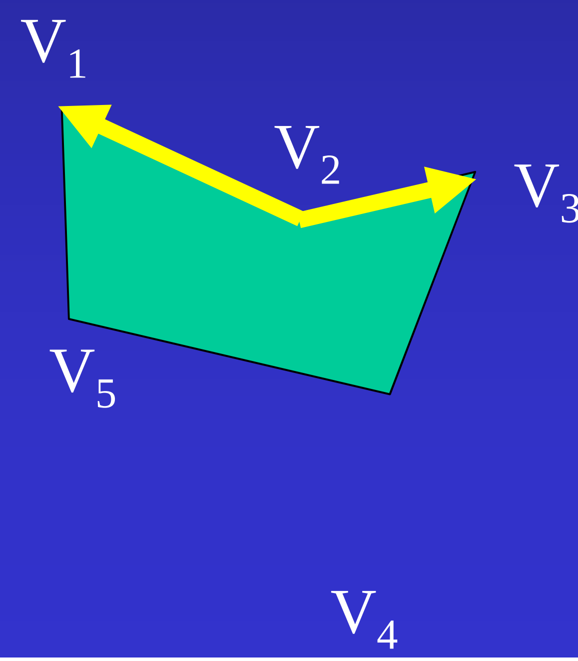
Right hand v. left hand space

Normal Vector

$$n = (V_1 - V_2) \times (V_3 - V_2)$$





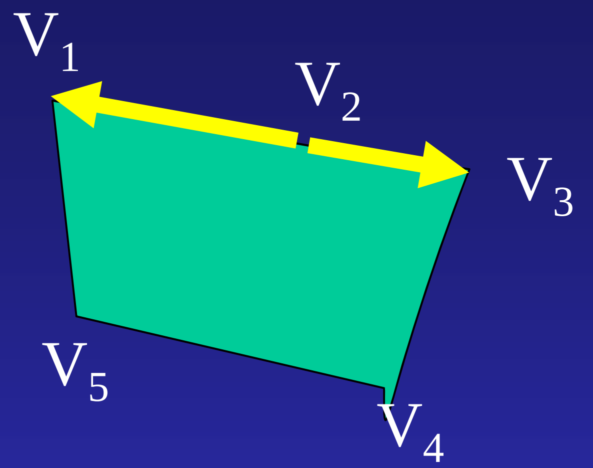


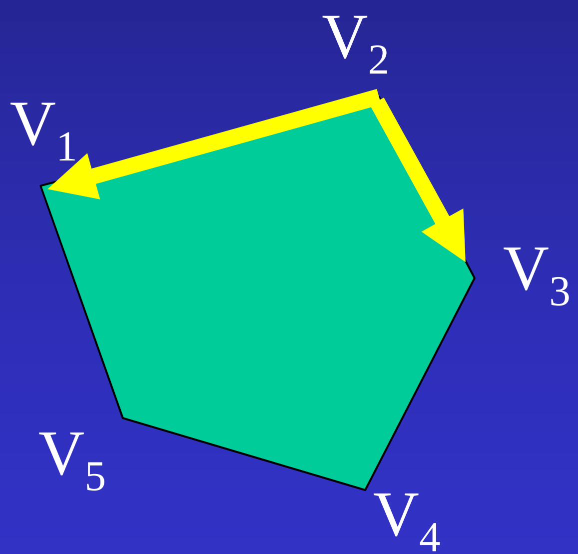
Normal Vector

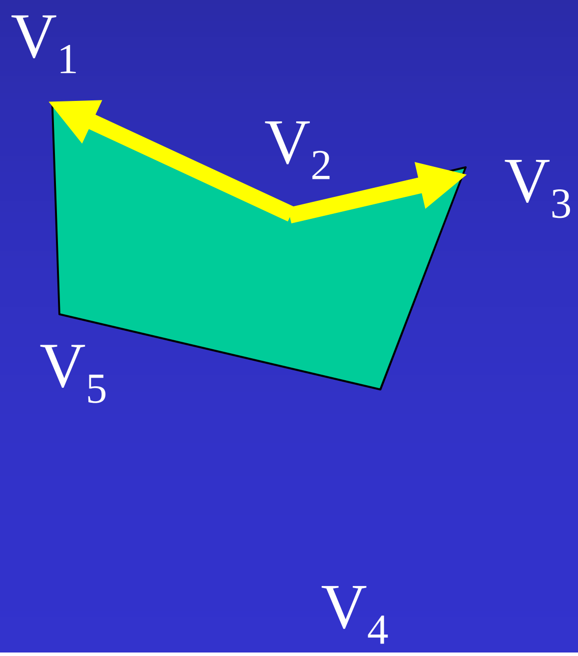
$$m_x = \Sigma(y_i - y_{next(i)})\,(z_i - z_{next(i)})$$

$$m_y = \Sigma(z_i - z_{next(i)})\,(x_i - x_{next(i)})$$

$$m_z = \Sigma(x_i - x_{next(i)})\,(y_i - y_{next(i)})$$



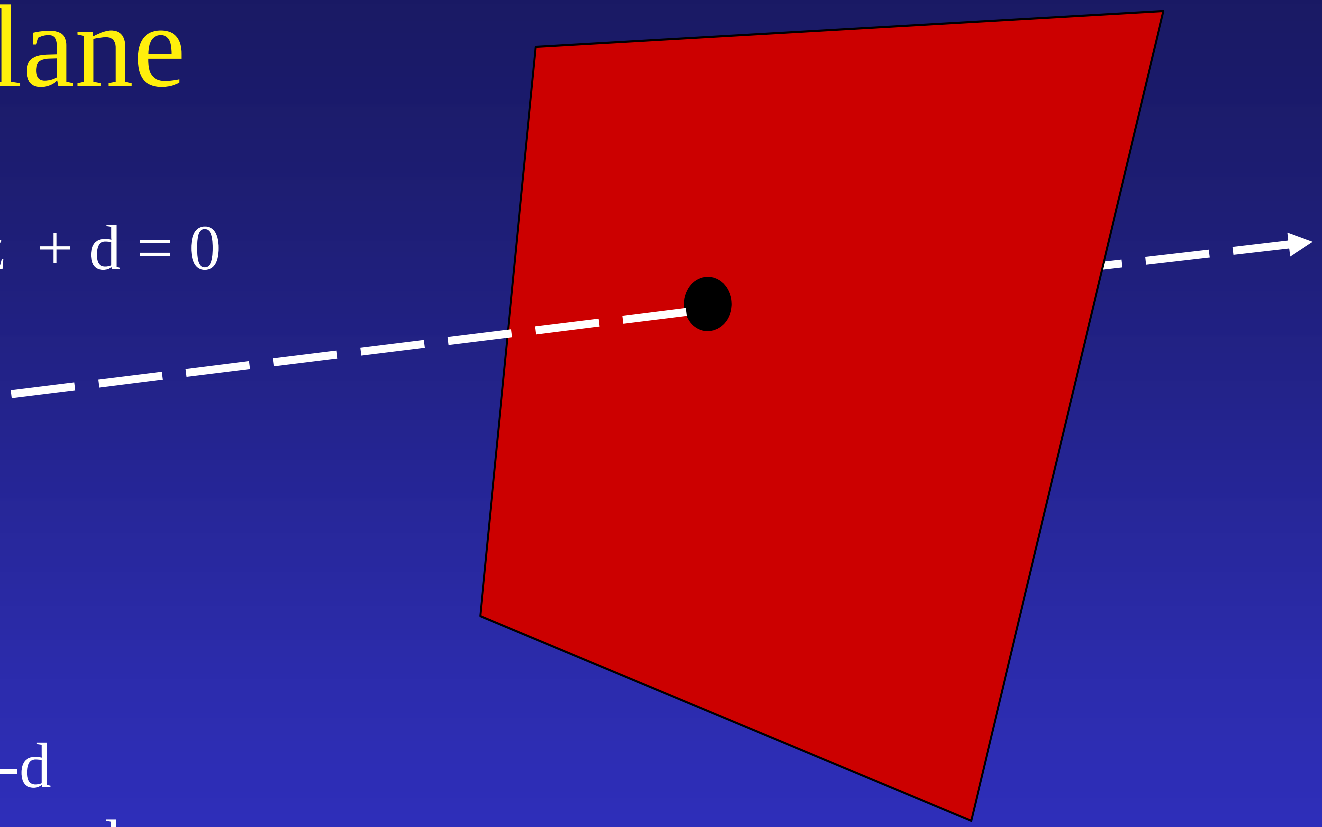




Ray-Plane

$Ax + by + cz + d = 0$

$n \cdot P = -d$



$P = s + t*v$

$n \cdot (s+t*v) = -d$

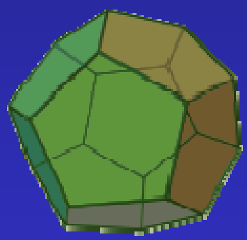
$n \cdot s + t*(n \cdot v) = -d$

$t = -(d + n \cdot s)/(n \cdot v)$

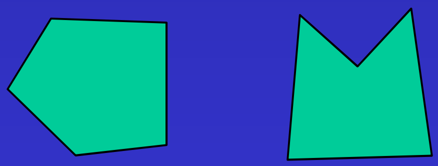
Ray-Polyhedron

Polyhedron - volume bounded by flat faces
Each face is defined by a ring of edges
Each edge is shared by 2 and only 2 faces

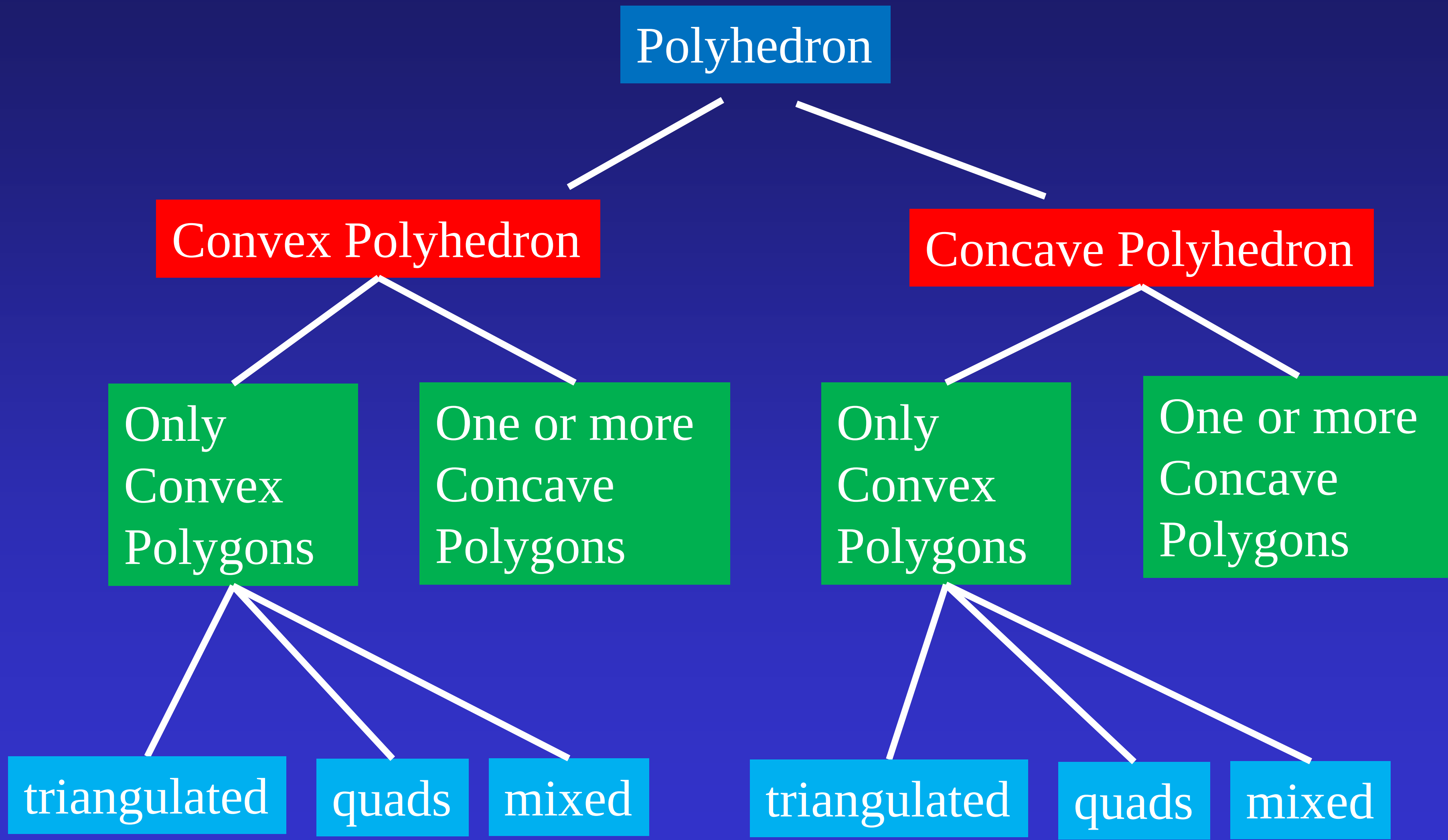
The polyhedron can be convex or concave



Faces can be convex or concave



Polyhedron Classification



Solid Modeling

Modeling of three-dimensional solids

Physically realizable objects

No infinitely thin sheets, no lines

Interior of object should 'hold water'
- Define a closed volume

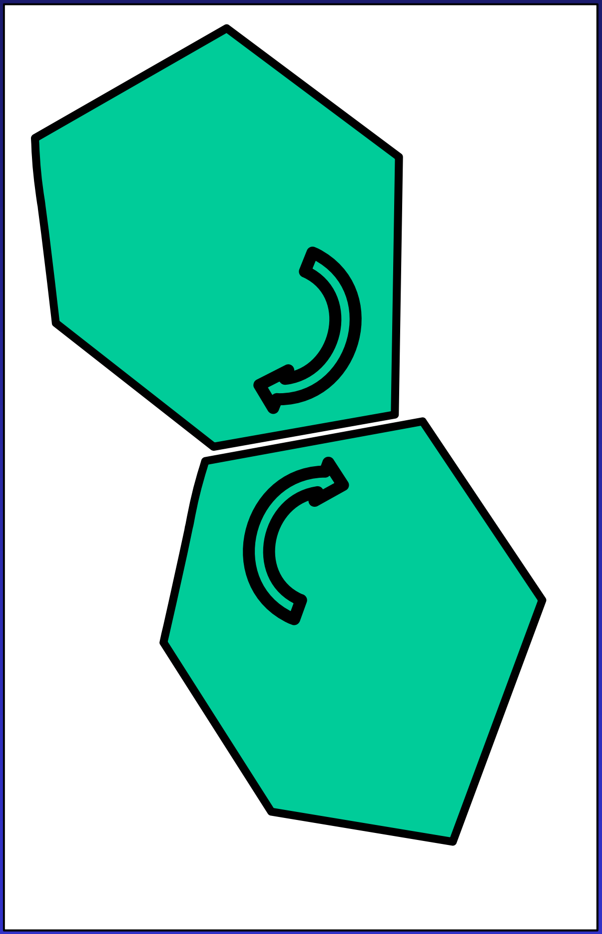
http://www.gvu.gatech.edu/~jarek/papers/SolidModelingWebster.pdf

Polygonal Solid Models

Vertices of a face have a consistent ordering (e.g. clockwise) when viewed from the outside side of the face

Each edge of a face is shared by one and only one other face

Each edge appears oriented one way in one face and the other way in the other face



EULER'S FORMULA

$$F - E + V = 2$$

$$F - E + V = 2 - 2P$$

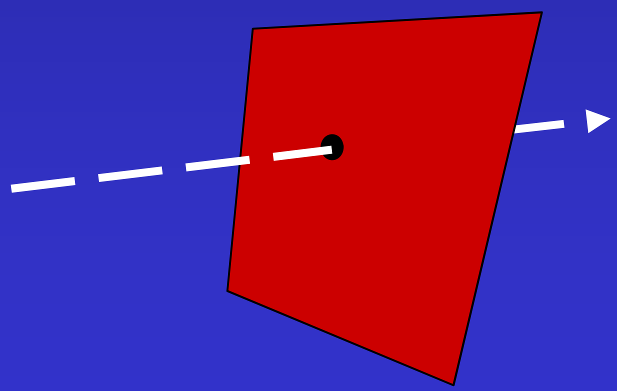
Convex Polyhedron

volume bounded by finite number of infinite planes

Computing intersections is similar to cube but using ray-plane intersection and arbitrary number of planes

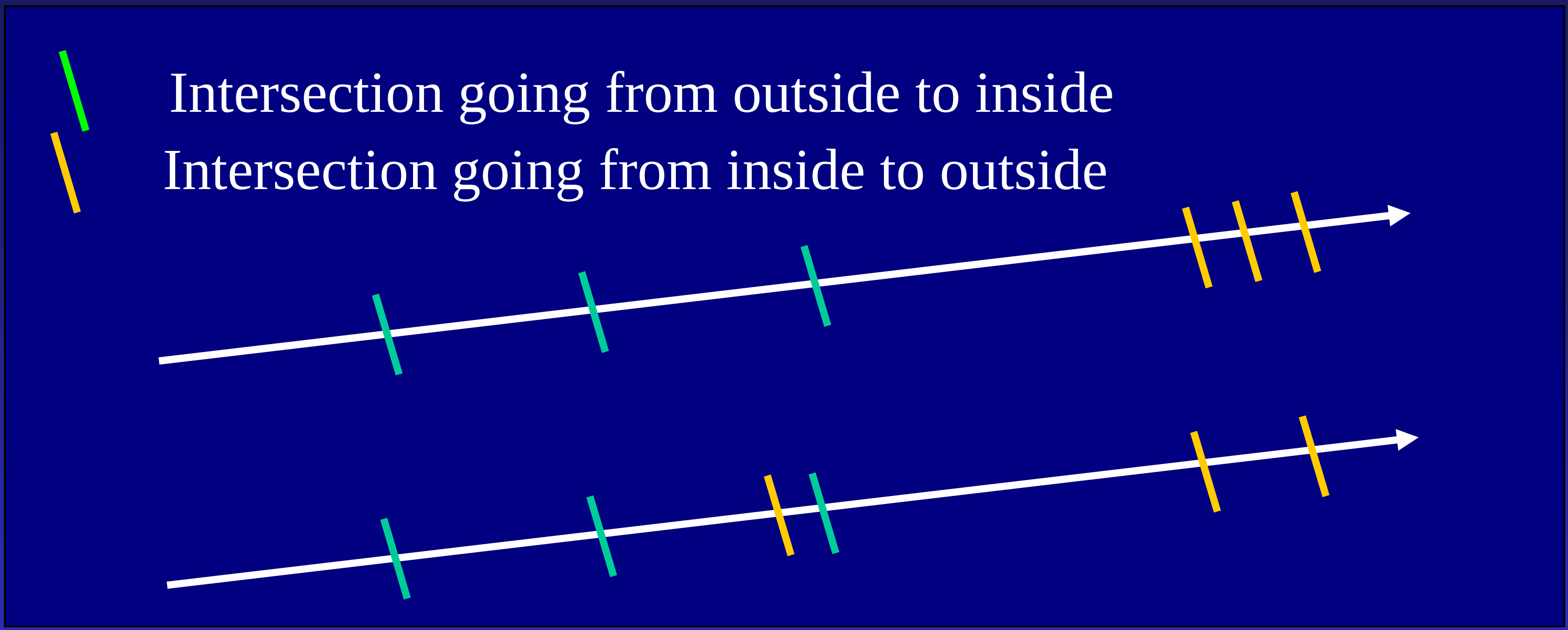
$n\cdot P = -d$
$P(t) = s + t*v$

Use $n\cdot v$ to determine Entering/exiting status



$n\cdot (s+t*v) = -d$
$n\cdot s + t*(n\cdot v) = -d$
$t = -(d + n\cdot s)/(n\cdot v)$

Convex Polyhedron

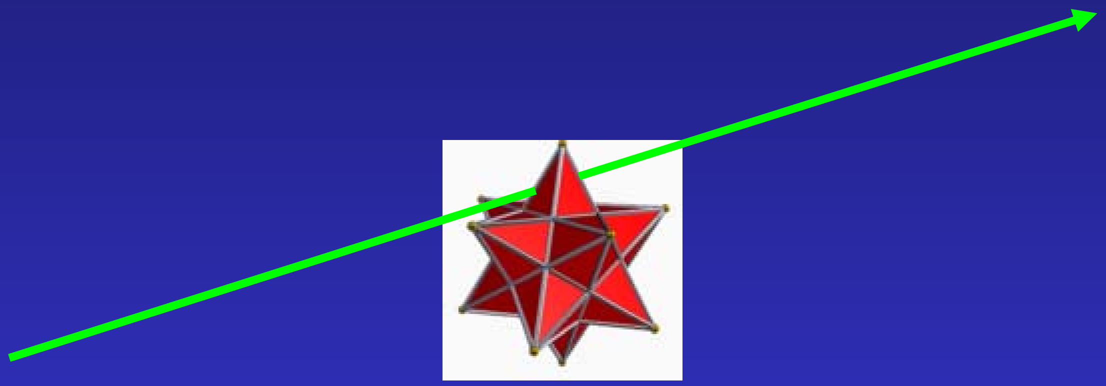


Record maximum entering intersection - enterMax
Record minimum exiting intersection - exitMin

If (enterMax < exitMin) polyhedron is intersected

Concave Polyhedron

Find closest face (if any) intersected by ray

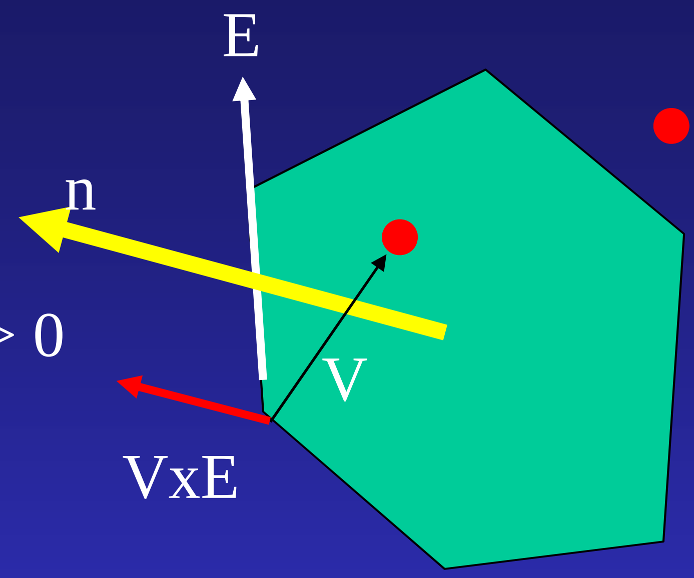


Need ray-face (ray-polygon) intersection test

Ray-Convex Polygon

Test to see if point is on
'inside' side of each edge

$n \cdot (V x E) > 0$



Dot product of
- normal
- Cross product of
  - ordered edge
  - vector from edge source to point of intersection

Ray-Concave Polyhedron

1. Intersect ray with plane

2. Determine if intersection point is inside of 2D polygon

   A) Convex polygon

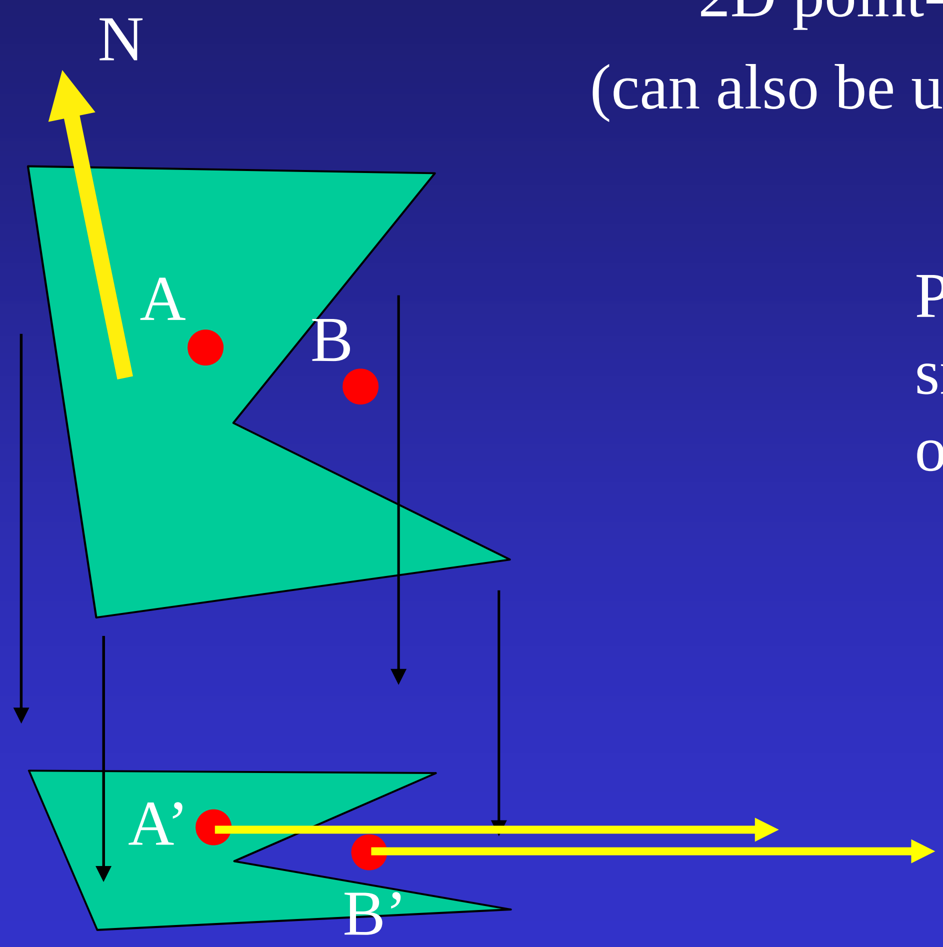
   B) Concave polygon

Ray-concave polygon

Project plane and point of intersection to 2D plane

2D point-inside-a-polygon test

(can also be used for convex polygons)

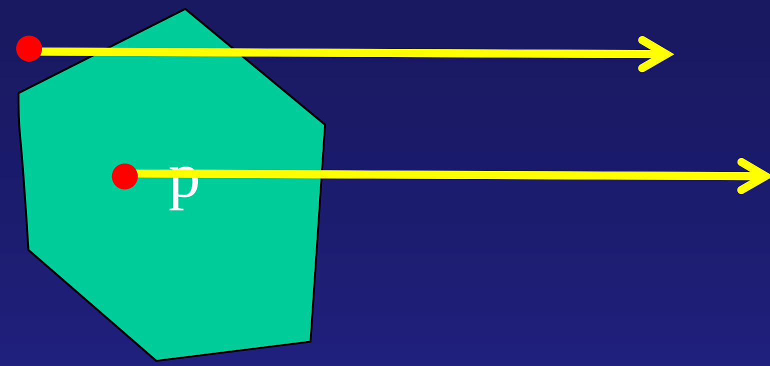


Project to plane of 2 smallest coordinates of normal vector

Form semi-infinite ray and count ray-edge intersections

2D Point Inside a Convex Polygon

Semi-infinite ray test

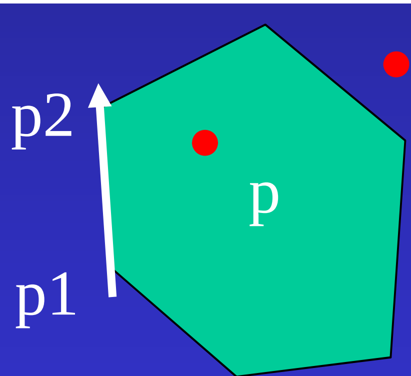


$$\left(p_y < p1_y\right) \&\& \left(p_y > p2_y\right) \,||\, \left(p_y > p1_y\right) \&\& \left(p_y < p1_y\right)$$

$$p_x < p1_x + (p2_x - p1_x)(p_y - p1_y)/(p2_y - p1_y)$$

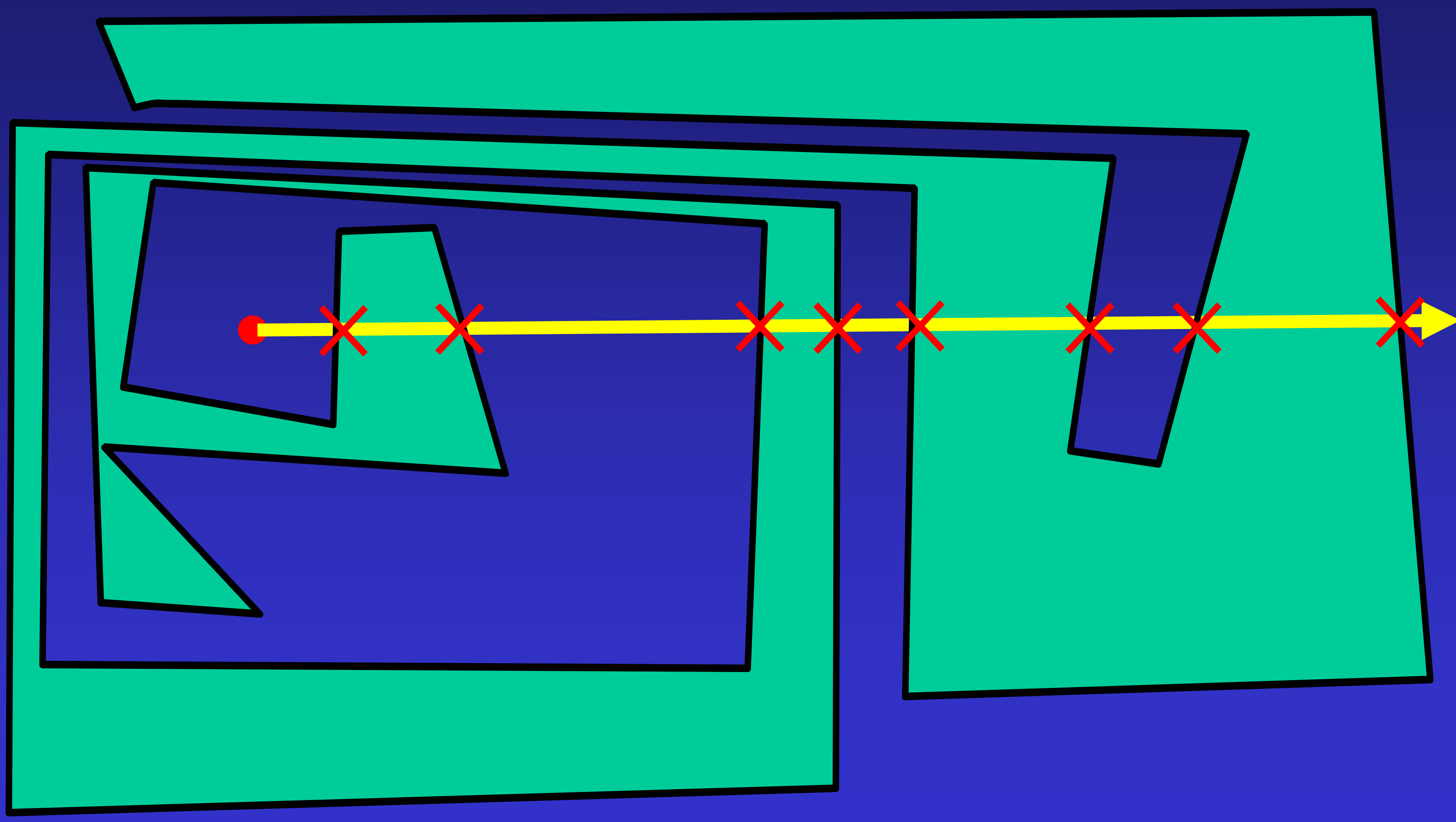
Test to see if point is on
'inside' side of each edge

Need to know order (CW or CCW)
of 2D vertices



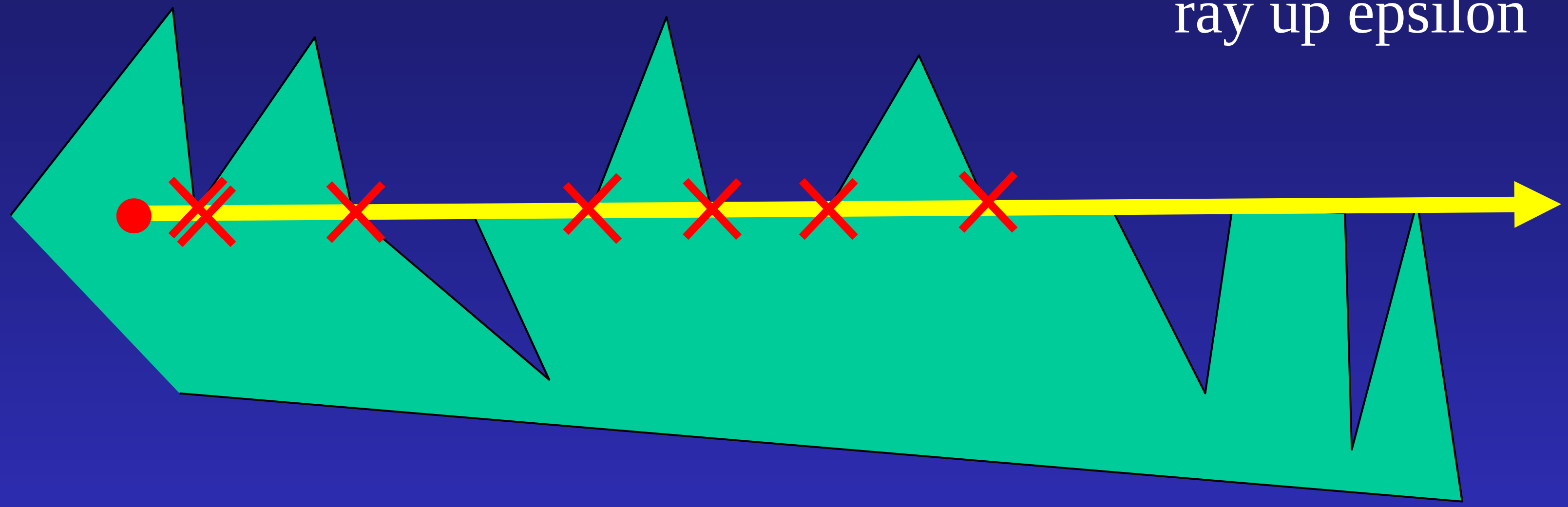
$$\left(p_x - p1_x\right)\left(p2_y - p1_y\right) - \left(p_y - p1_y\right)\left(p2_x - p1_x\right)$$

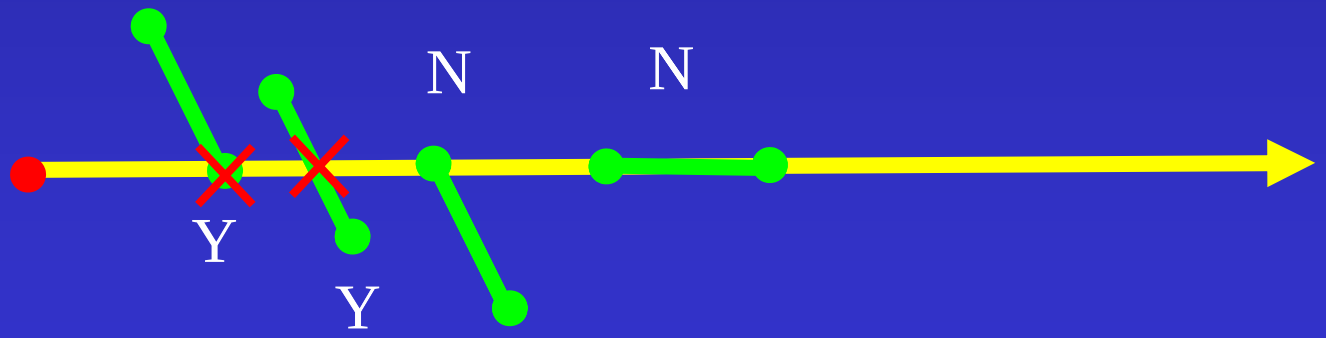
2D point inside a polygon test



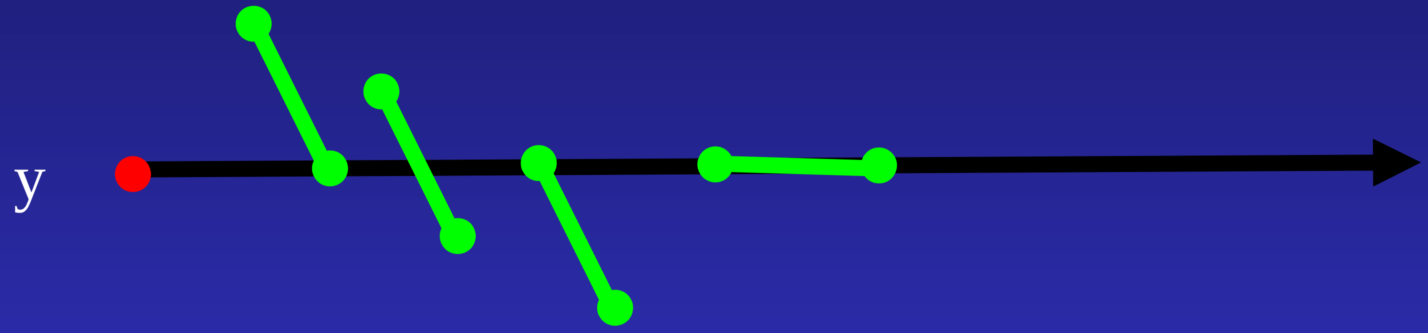
Special Cases

Logically move ray up epsilon





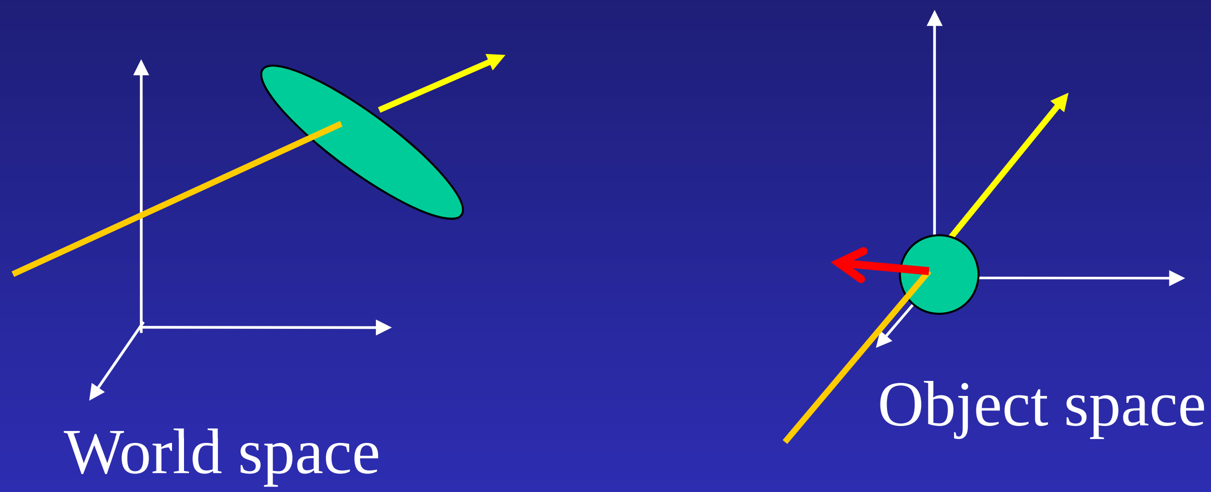
2D point inside a polygon test



$$if\,(((y < y_2)\,\&\&\,(y >= y_1))\,||\,((y < y_1)\,\&\&\,(y >= y_2)))$$

Transformed objects

e.g., Ellipse is transformed sphere



Intersect ray with transformed object

Use inverse of object transformation to transform ray
Intersect transformed ray with untransformed object

Transformed objects

$$r(t) = s + tv$$

World space ray

$$s = [s_x, s_y, s_z, 1]$$

$$v = [v_x, v_y, v_z, 0]$$

$$M$$

Object to world transform matrix

$$R(t)^T = M^{-1}s^T + M^{-1}v^T$$

Object space ray

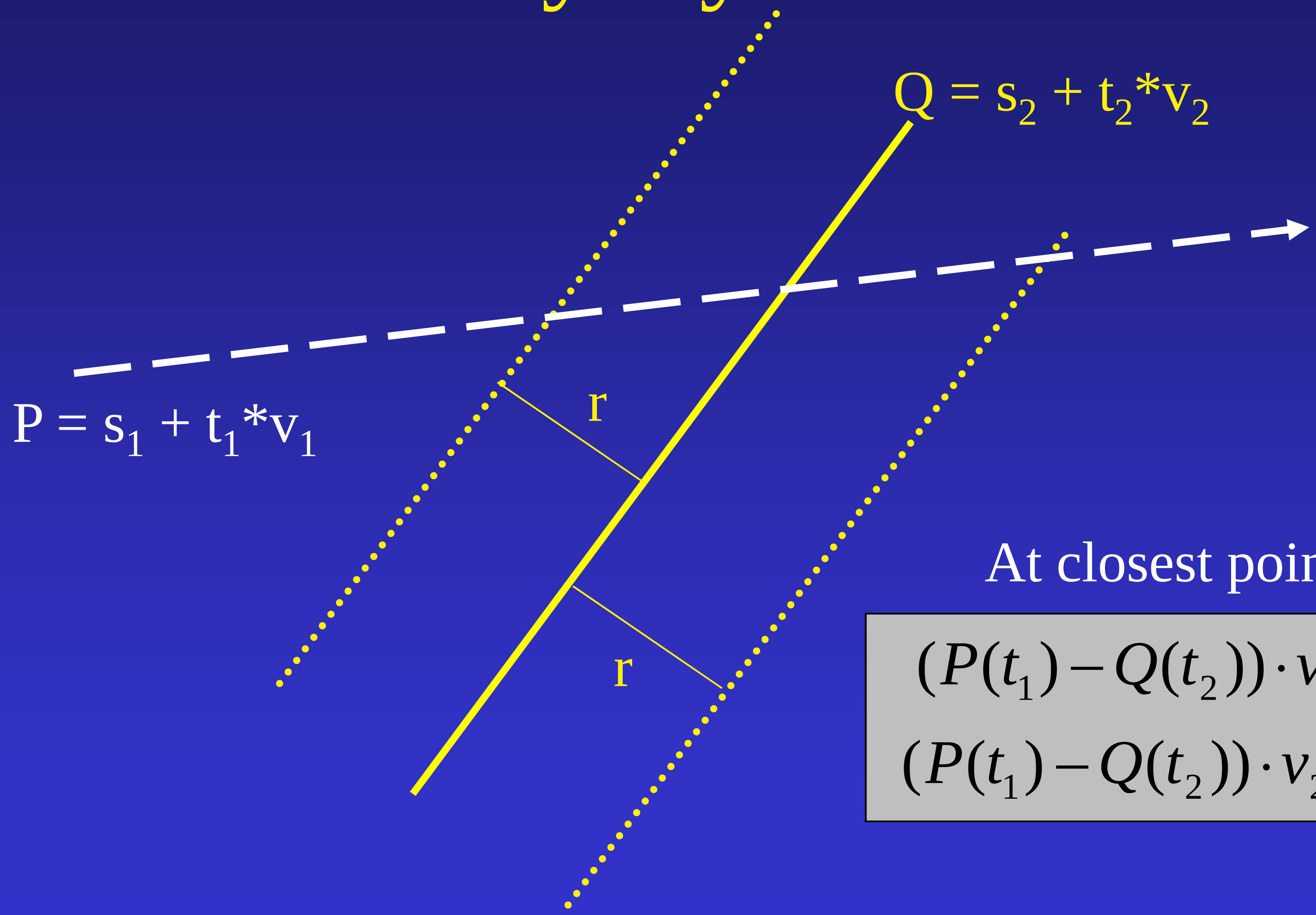
Intersect ray with object in object space

Transform intersection point and normal back to world space

$$P_{world}^T = MP_{object}^T$$

$$N_{world}^T = (M^{-1})^T N_{object}^T$$

Ray-Cylinder



At closest points

$$(P(t_1) - Q(t_2)) \cdot v_1 = 0$$
$$(P(t_1) - Q(t_2)) \cdot v_2 = 0$$

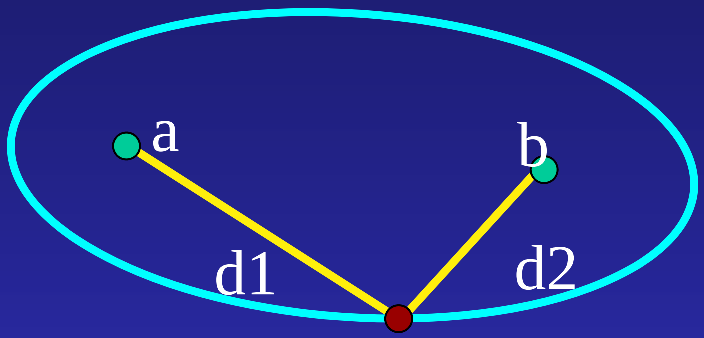
Ray-Cylinder

$$(P(t_1) - Q(t_2)) \cdot v_1 = 0$$
$$(P(t_1) - Q(t_2)) \cdot v_2 = 0$$

$$(s_1 + t_1 v_1 - (s_2 + t_2 v_2)) \cdot v_1 = 0$$
$$(s_1 + t_1 v_1 - (s_2 + t_2 v_2)) \cdot v_2 = 0$$

Ray-Ellipsoid

Geometric construction: all points p such that
|p-a| + |p-b| = r



Algebraic equation – axis aligned, origin centered

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

Ray-Quadric

$$P(t) = (x, y, z) = s + tv = (s_x + tv_x, s_y + tv_y, s_z + tv_z)$$

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

http://en.wikipedia.org/wiki/Quadric