Brief Review: Vectors

Vectors

- Basics
- Normalizing a vector => unit vector
- Dot product
- Cross product
- Reflection vector
- Parametric form of a line

Basics

Vectors

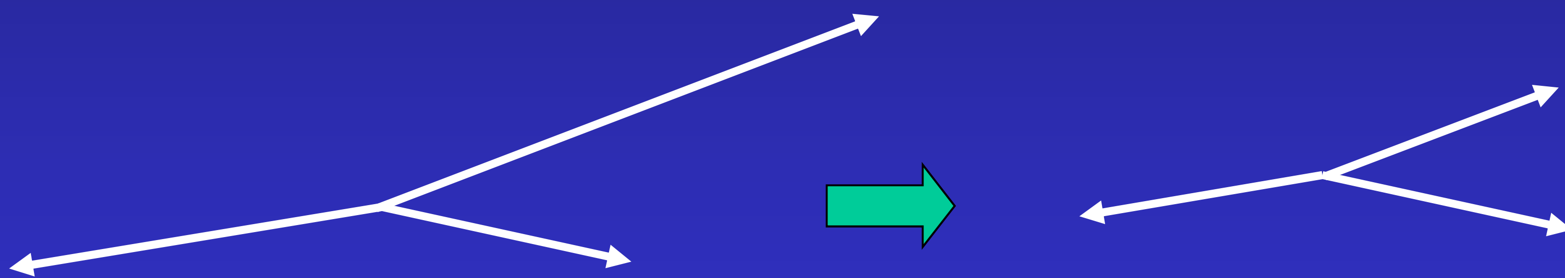
- Have a direction and a length
- Do not have a position in space

Normal vector

- Is ‘normal’, or perpendicular, to a surface
- Are usually unit-length, also called ‘normalized’

Normalizing a Vector

- Compute the magnitude and divide through
- Produces a UNIT VECTOR
- Aka NORMALIZED VECTOR



To normalize (x,y,z) :

$$len = \sqrt{x^2 + y^2 + z^2}$$

$$\left(\frac{x}{len} \quad \frac{y}{len} \quad \frac{z}{len}\right)$$

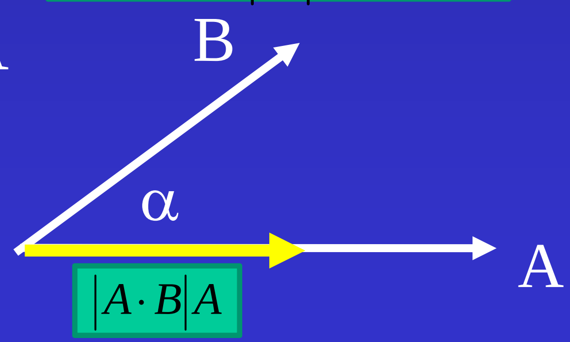
Dot Product

Scalar Product $\boxed{A \cdot B = A_x B_x + A_y B_y + A_z B_z}$

$$\boxed{A \cdot B = B \cdot A}$$

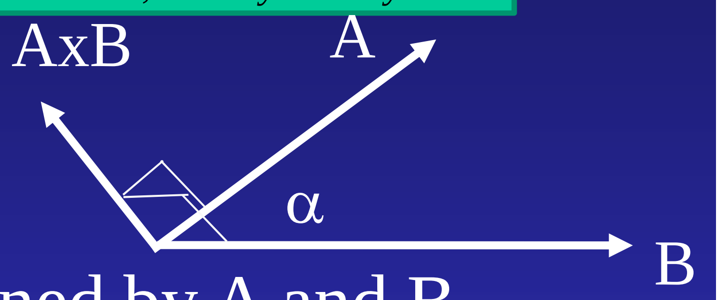
$$\boxed{A \cdot B = |A||B|\cos(\alpha)}$$

- If A and B are unit vectors, $\boxed{A \cdot B = \cos(\alpha)}$
- If A is unit vector, $\boxed{A \cdot B = |B|\cos(\alpha)}$ is the length of B projected onto A



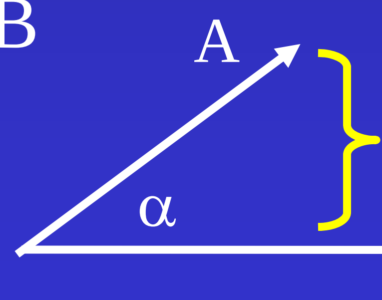
Cross Product

Vector Product $A \times B = (A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x)$



$A \times B = -B \times A$

- $A \times B$ is orthogonal to plane defined by A and B
- With length $|A \times B| = |A||B|\sin(\alpha)$
- If A and B are unit vectors, $|A \times B| = \sin(\alpha)$
- If **B** is unit vector, $|A \times B|$ is perpendicular distance from A to B



$|A \times B| = |A|\sin(\alpha)$

Reflection Vector

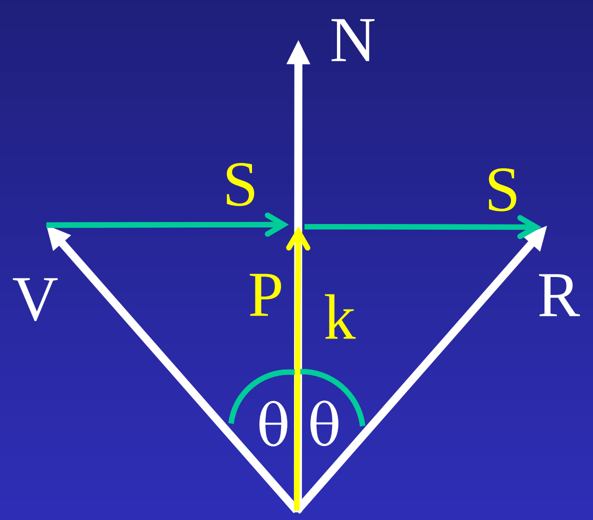
In 3D, Reflect V about N to make R
Assume N is normalized

$$R = V + 2S$$

$$S = P - V$$

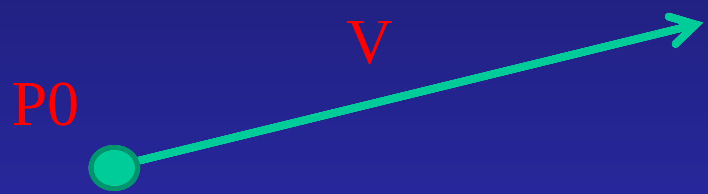
$$k = |P| = N \cdot V$$

$$P = kN = (V \cdot N)N$$



$$R = V + 2(P - V) = V + 2((V \cdot N)N - V) = 2(V \cdot N)N - V$$

Parametric Equation of Line



$$\boxed{P(u) = P_0 + uV}$$

- P_0 is point on line
- V is direction of line
- Generalizes to any dimension (2D, 3D, etc)
- As $0 < u < 1.0$, $P(u)$ goes from P_0 to P_0+V