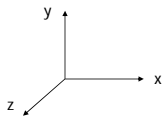


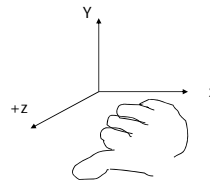
Three-Dimensional Graphics

- A 3D point (x,y,z) – x,y, and Z coordinates
- We will still use column vectors to represent points
- Homogeneous coordinates of a 3D point: $(x,y,z,1)$
- Transformation will be performed using 4x4 matrix

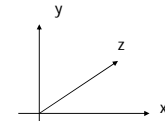


Right hand coordinate system

- $X \times Y = Z$; $Y \times Z = X$; $Z \times X = Y$;



Right hand coordinate system



Left hand coordinate system
Not used in this class and
Not in OpenGL

3D transformation

- Very similar to 2D transformation
- Translation

$$x' = x + tx; \quad y' = y + ty; \quad z' = z + tz$$

$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

homogeneous coordinates

3D transformation

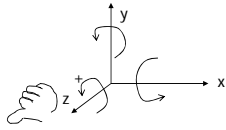
- Scaling

$$X' = X * Sx; \quad Y' = Y * Sy; \quad Z' = Z * Sz$$

$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

3D transformation

- 3D rotation is done around a rotation **axis**
- Fundamental rotations – rotate about x, y, or z axes
- Counter-clockwise rotation is referred to as positive rotation (when you look down negative axis)

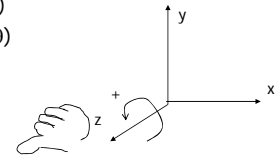


3D transformation

- Rotation about Z – similar to 2D rotation

$$\begin{aligned}x' &= x \cos(\theta) - y \sin(\theta) \\y' &= x \sin(\theta) + y \cos(\theta) \\z' &= z\end{aligned}$$

$$\begin{vmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$



- OpenGL - `glRotatef(θ, 0,0,1)`

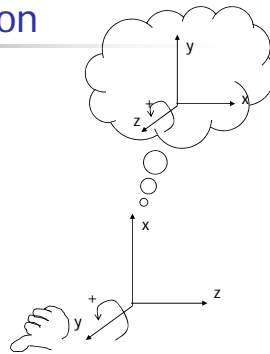
3D transformation

- Rotation about y

$$\begin{aligned}z' &= z \cos(\theta) - x \sin(\theta) \\x' &= z \sin(\theta) + x \cos(\theta) \\y' &= y\end{aligned}$$

$$\begin{vmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

- OpenGL - `glRotatef(θ, 0,1,0)`



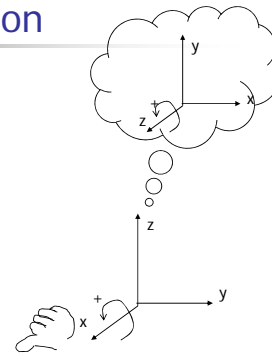
3D transformation

- Rotation about x

$$\begin{aligned}y' &= y \cos(\theta) - z \sin(\theta) \\z' &= y \sin(\theta) + z \cos(\theta) \\x' &= x\end{aligned}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

- OpenGL - `glRotatef(θ, 1,0,0)`





3D transformation

- Arbitrary rotation axis (rx, ry, rz)
- Text p. 212 explains how to do it
- We omit the detail here
- Use OpenGL:
`glRotatef(angle, rx, ry, rz)`

