Three-Dimensional Graphics

- A 3D point (x,y,z) – x,y, and Z coordinates
- We will still use column vectors to represent points
- Homogeneous coordinates of a 3D point: (x,y,z,1)
- Transformation will be performed using 4x4 matrix

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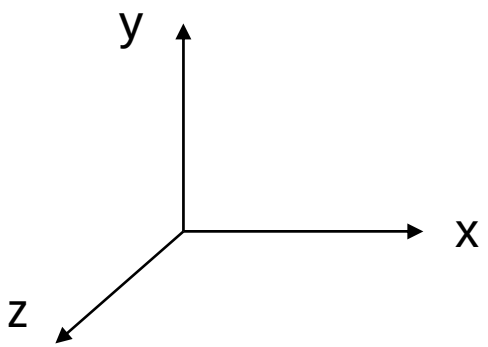
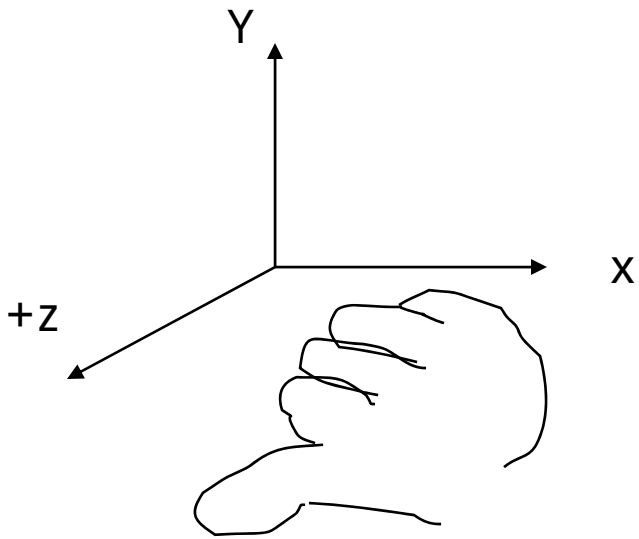
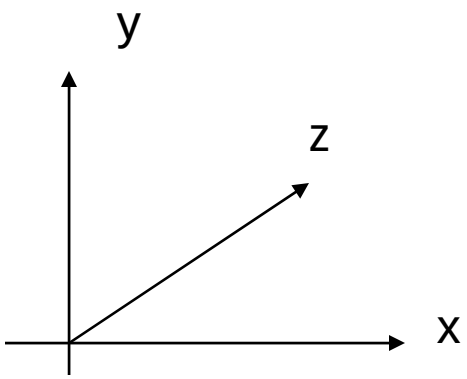


Right hand coordinate system

- $X \times Y = Z$; $Y \times Z = X$; $Z \times X = Y$;



Right hand coordinate system



Left hand coordinate system
Not used in this class and
Not in OpenGL

3D transformation

- Very similar to 2D transformation
- Translation

$x' = x + tx;\ y' = y + ty;\ z' = z + tz$

$$\begin{vmatrix} X' \\ Y' \\ Z' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \\ 1 \end{vmatrix}$$

homogeneous coordinates

3D transformation

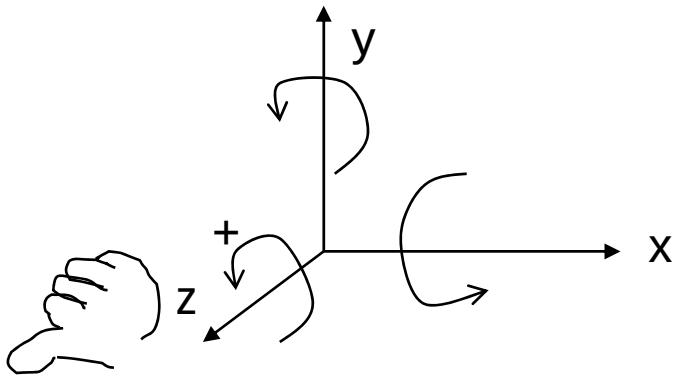
- Scaling

$X' = X * Sx;\ Y' = Y * Sy;\ Z' = Z * Sz$

$$\begin{vmatrix} X' \\ Y' \\ Z' \\ 1 \end{vmatrix} = \begin{vmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \\ 1 \end{vmatrix}$$

3D transformation

- 3D rotation is done around a rotation **axis**
- Fundamental rotations – rotate about x, y, or z axes
- Counter-clockwise rotation is referred to as positive rotation (when you look down negative axis)



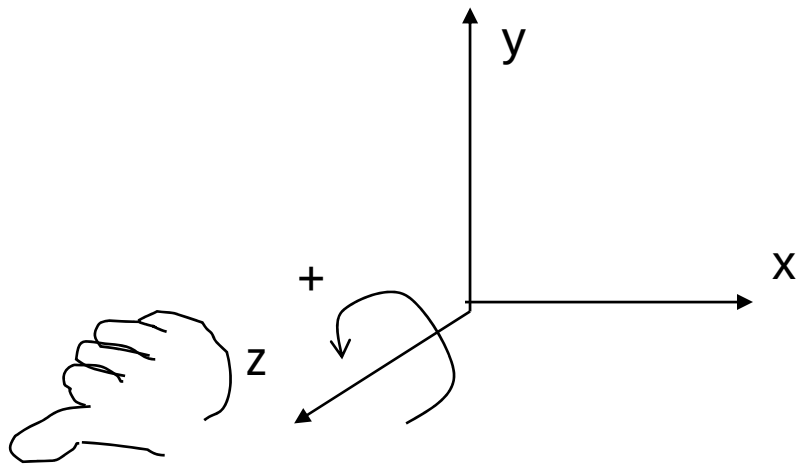
3D transformation

- Rotation about Z – similar to 2D rotation

$x' = x\cos(\theta) - y\sin(\theta)$

$y' = x\sin(\theta) + y\cos(\theta)$

$z' = z$



$$\begin{vmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

- OpenGL - glRotatef(θ, 0,0,1)

3D transformation

- Rotation about y

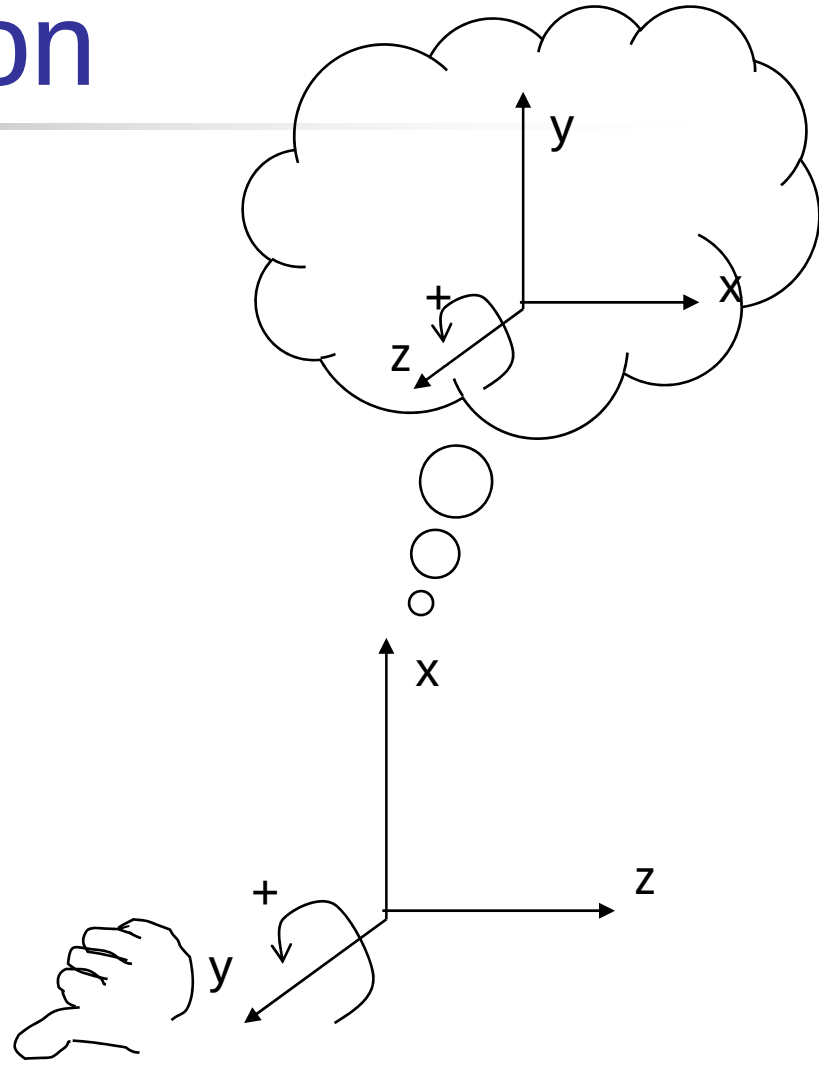
  $z' = z\cos(\theta) - x\sin(\theta)$

  $x' = z\sin(\theta) + x\cos(\theta)$

  $y' = y$

$$\begin{vmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

- OpenGL - glRotatef(θ, 0,1,0)



3D transformation

- Rotation about x

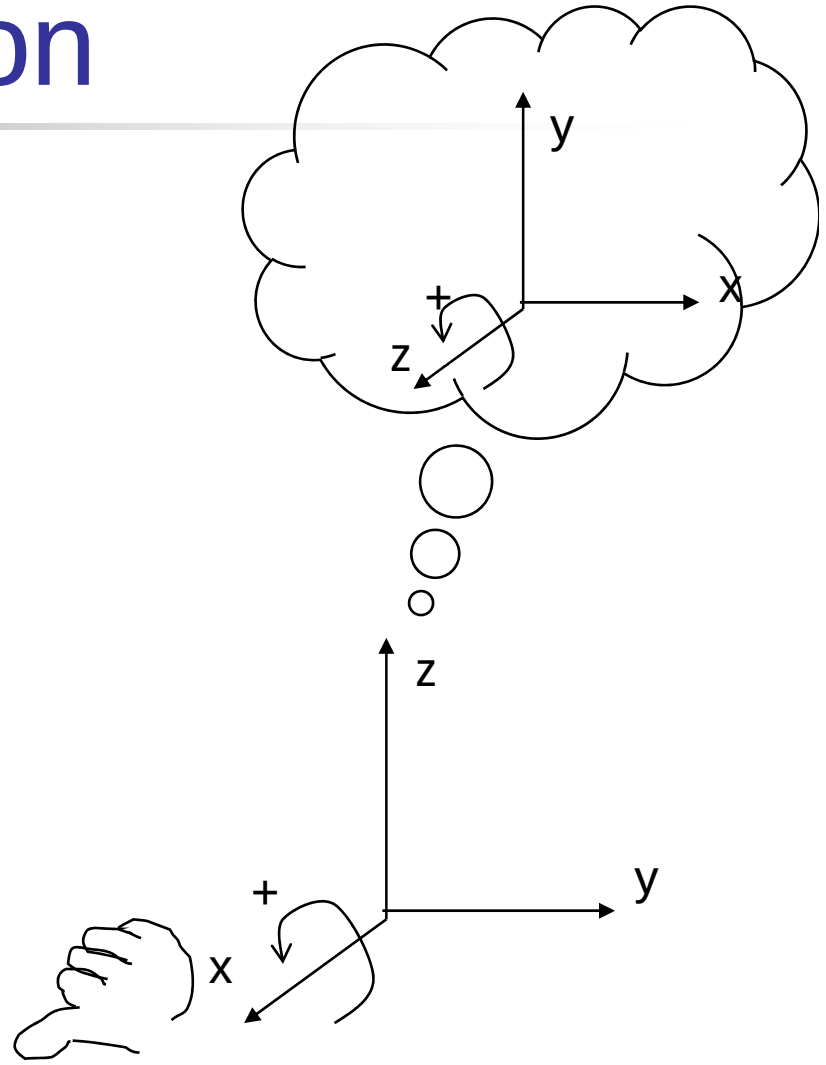
$y' = y\cos(\theta) - z\sin(\theta)$

$z' = y\sin(\theta) + z\cos(\theta)$

$x' = x$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

- OpenGL - glRotatef(θ, 1,0,0)



3D transformation

- Arbitrary rotation axis (rx,ry,rz)
- Text p. 212 explains how to do it
- We omit the detail here
- Use OpenGL:

  glRotatef(angle, rx, ry, rz)