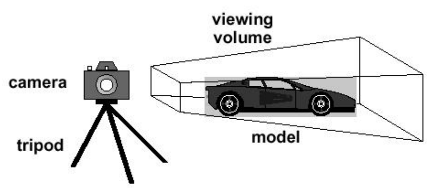
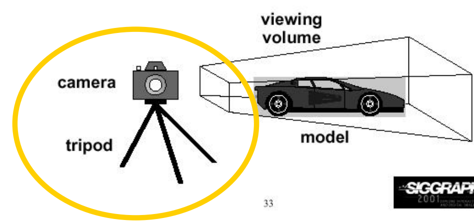
Introduction to 3D viewing

- 3D is just like taking a photograph!



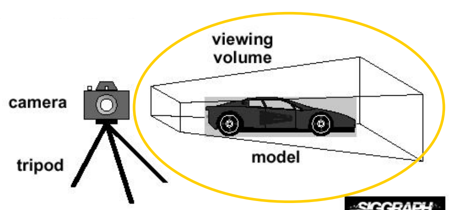
Viewing Transformation

- Position and orient your camera



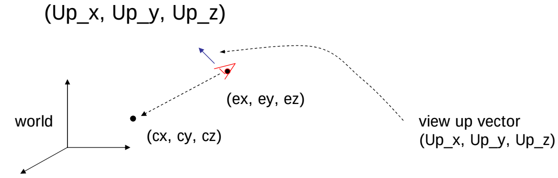
Projection Transformation

- Control the “lens” of the camera
- Project the object from 3D world to 2D screen



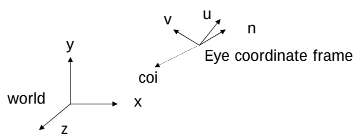
Viewing Transformation (2)

- Important camera parameters to specify
  - Camera (eye) position (Ex,Ey,Ez) in world coordinate system
  - Center of interest (coi) (cx, cy, cz)
  - Orientation (which way is up?) View-up vector (Up_x, Up_y, Up_z)



Viewing Transformation (3)

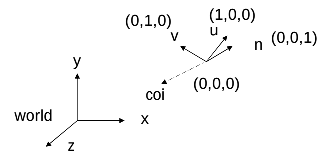
- Transformation?
  - Form a camera (eye) coordinate frame



  - Transform objects from world to eye space

Viewing Transformation (4)

- Eye space?



- Transform to eye space can simplify many downstream operations (such as projection) in the pipeline

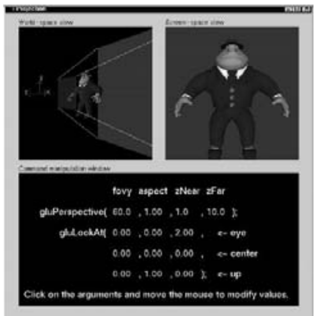
Viewing Transformation (5)

- In OpenGL:
  - gluLookAt (Ex, Ey, Ez, cx, cy, cz,
    Up_x, Up_y, Up_z)
  - The view up vector is usually (0,1,0)
  - Remember to set the OpenGL matrix mode to GL_MODELVIEW first

Viewing Transformation (6)

```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(0,0,1,0,0,0,0,1,0);
    display_all();   // your display routine
}
```

Demo

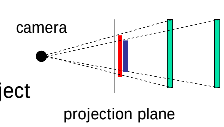


Projection Transformation

- Important things to control
  - Perspective or Orthographic
  - Field of view and image aspect ratio
  - Near and far clipping planes

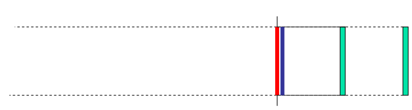
Perspective Projection

- Characterized by object foreshortening
  - Objects appear to be larger if they are closer to the camera
  - This is what happens in the real world
- Need:
  - Projection center
  - Projection plane
- Projection: Connecting the object to the projection center



Orthographic Projection

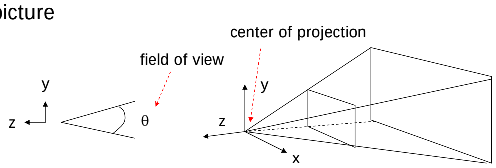
- No foreshortening effect – distance from camera does not matter
- The projection center is at infinite



- Projection calculation – just drop z coordinates

Field of View

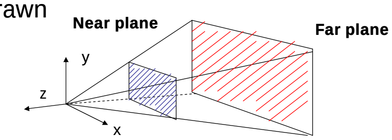
- Determine how much of the world is taken into the picture



- The larger is the field view, the smaller is the object projection size

Near and Far Clipping Planes

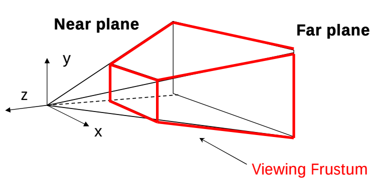
- Only objects between near and far planes are drawn



- Near plane + far plane + field of view = Viewing Frustum

Viewing Frustum

- 3D counterpart of 2D world clip window



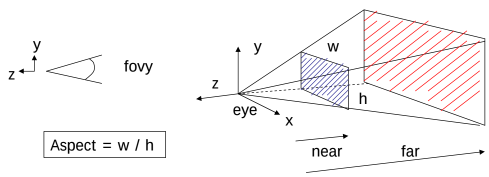
- Objects outside the frustum are clipped

Projection Transformation

- In OpenGL:
  - Set the matrix mode to GL_PROJECTION
  - Perspective projection: use
    - gluPerspective(fovy, aspect, near, far) **or**
    - glFrustum(left, right, bottom, top, near, far)
  - Orthographic:
    - glOrtho(left, right, bottom, top, near, far)

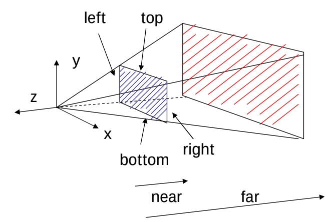
gluPerspective(fovy, aspect, near, far)

- Aspect ratio is used to calculate the window width



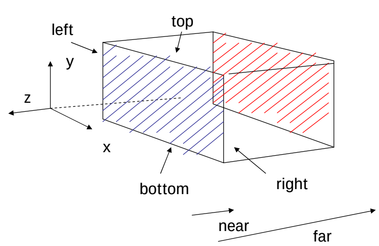
glFrustum(left, right, bottom, top, near, far)

- Or You can use this function in place of gluPerspective()



glOrtho(left, right, bottom, top, near, far)

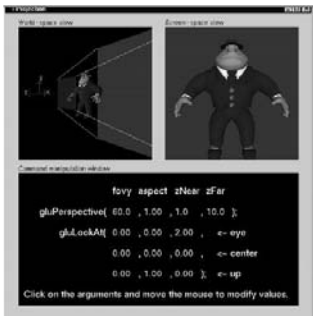
- For orthographic projection



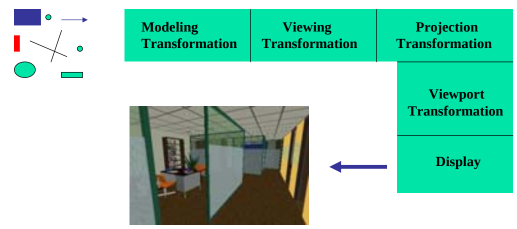
Projection Transformation

```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT);
    glMatrixMode(GL_PROJETION);
    glLoadIdentity();
    gluPerspective(fove, aspect, near, far);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(0,0,1,0,0,0,0,1,0);
    display_all();    // your display routine
}
```

Demo



3D viewing under the hood



3D viewing under the hood

Topics of Interest:

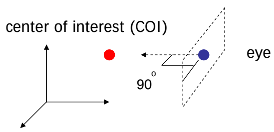
- Viewing transformation
- Projection transformation

Viewing Transformation

- Transform the object from world to eye space
  - Construct an eye space coordinate frame
  - Construct a matrix to perform the coordinate transformation
  - Flexible Camera Control

Eye Coordinate Frame

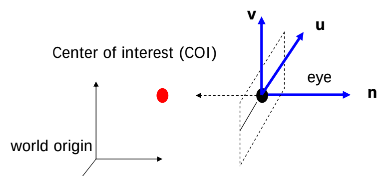
- Known: eye position, center of interest, view-up vector
- To find out: new origin and three basis vectors



Assumption: the direction of view is orthogonal to the view plane (the plane that objects will be projected onto)

Eye Coordinate Frame (2)

- Origin: eye position (that was easy)
- Three basis vectors: one is the normal vector (**n**) of the viewing plane, the other two are the ones (**u** and **v**) that span the viewing plane



(u,v,n should be orthogonal to each other)

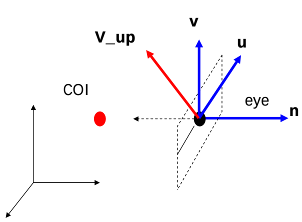
n is pointing away from the world because we use right hand coordinate system

N = eye – COI
n = N / | N |

Remember **u,v,n** should be all unit vectors

Eye Coordinate Frame (3)

- How about u and v?



We can get u first -

u is a vector that is perpendicular to the plane spanned by N and view up vector (V_up)

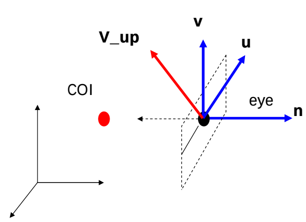
U = V_up x **n**

u = U / | U |

Eye Coordinate Frame (4)

- How about v?

Knowing n and u, getting v is easy



v = n x u

v is already normalized

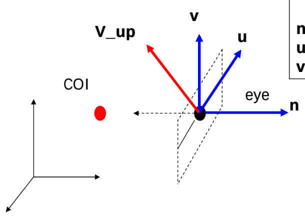
Eye Coordinate Frame (5)

- Put it all together

Eye space **origin: (Eye.x , Eye.y, Eye.z)**

Basis vectors:

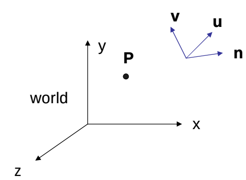
n = (eye – COI) / | eye – COI|
u = (V_up x **n**) / | V_up x **n** |
v = **n** x **u**



World to Eye Transformation

- Transformation matrix (M_{w2e}) ?

$P' = M_{w2e} \times P$

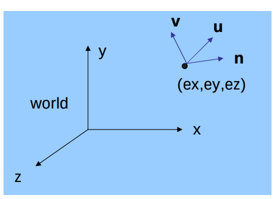


1. Come up with the transformation sequence to move eye coordinate frame to the world

2. And then apply this sequence to the point P in a reverse order

World to Eye Transformation

- Rotate the eye frame so that it will be "aligned" with the world frame
- Translate (-ex, -ey, -ez)



Rotation: $\begin{vmatrix} ux & uy & uz & 0 \\ vx & vy & vz & 0 \\ nx & ny & nz & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

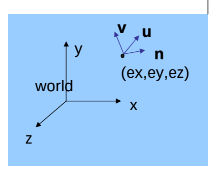
How to verify the rotation matrix?

Translation: $\begin{vmatrix} 1 & 0 & 0 & -ex \\ 0 & 1 & 0 & -ey \\ 0 & 0 & 1 & -ez \\ 0 & 0 & 0 & 1 \end{vmatrix}$

World to Eye Transformation (2)

- Transformation order: apply the transformation to the object in a reverse order - translation first, and then rotate

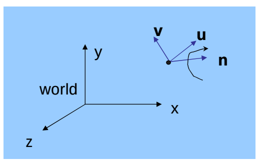
$$Mw2e = \begin{vmatrix} ux & uy & ux & 0 \\ vx & vy & vz & 0 \\ nx & ny & nz & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & -ex \\ 0 & 1 & 0 & -ey \\ 0 & 0 & 1 & -ez \\ 0 & 0 & 0 & 1 \end{vmatrix}$$



World to Eye Transformation (3)

- Head tilt: Rotate your head by δ
- Just rotate the object about the eye space z axis - δ
- Mw2e =

$$M_{w2e} = \begin{vmatrix} \cos(-\delta) & -\sin(-\delta) & 0 & 0 \\ \sin(-\delta) & \cos(-\delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} ux & uy & ux & 0 \\ vx & vy & vz & 0 \\ nx & ny & nz & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & -ex \\ 0 & 1 & 0 & -ey \\ 0 & 0 & 1 & -ez \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

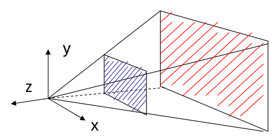


Why $-\delta$?

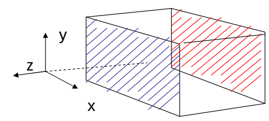
When you rotate your head by δ, it is like rotate the object by $-\delta$

Projection Transformation

- Projection – map the object from 3D space to 2D screen



Perspective: **gluPerspective()**



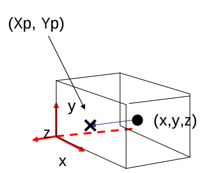
Parallel: **glOrtho()**

Parallel Projection

- After transforming the object to the eye space, parallel projection is relative easy – we could just drop the Z

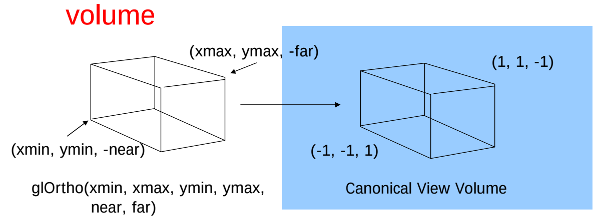
  Xp = x
  Yp = y
  Zp = -d

- We actually want to keep Z – why?



Parallel Projection (2)

- OpenGL maps (projects) everything in the visible volume into a canonical view volume



Parallel Projection (3)

- Transformation sequence:
  1. Translation (M1): (-near = zmax, -far = zmin)

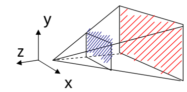
     -(xmax+xmin)/2, -(ymax+ymin)/2, -(zmax+zmin)/2
  2. Scaling (M2):

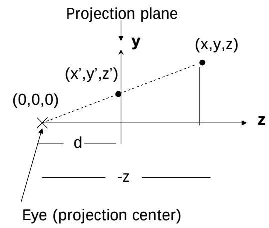
     2/(xmax-xmin), 2/(ymax-ymin), 2/(zmax-zmin)

$$M2 \times M1 = \begin{vmatrix} 2/(xmax-xmin) & 0 & 0 & -(xmax+xmin)/(xmax-xmin) \\ 0 & 2/(ymax-ymin) & 0 & -(ymax+ymin)/(ymax-ymin) \\ 0 & 0 & 2/(zmax-zmin) & -(zmax+zmin)/(zmax-zmin) \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Perspective Projection

- Side view:





Based on similar triangle:

$$\frac{y}{y'} = \frac{-z}{d}$$

$$\Rightarrow \quad Y' = y \times \frac{d}{-z}$$

Perspective Projection (2)

- Same for x. So we have:

  x' = x × d / -z

  y' = y × d / - z

  z' = -d
- Put in a matrix form:

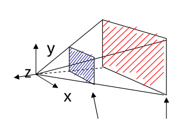
$$\begin{matrix} x' \\ y' \\ z' \\ w \end{matrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & (1/-d) & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix}$$

- OpenGL assume d = 1, i.e. the image plane is at z = -1

Perspective Projection (3)

- We are not done yet. We want to somewhat keep the z information so that we can perform depth comparison
- Use pseudo depth – OpenGL maps the near plane to 1, and far plane to -1
- Need to modify the projection matrix: solve a and b

$$\begin{vmatrix} x' \\ y' \\ z' \\ w \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & (1/-d) & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix}$$



How to solve a and b?

Perspective Projection (4)

- Solve a and b

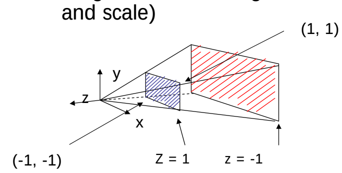
$$\begin{vmatrix} x' \\ y' \\ z' \\ w \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & (1/-d) & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix}$$

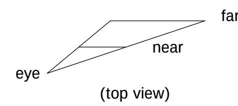
- $(0,0,1)^T = M \times (0,0,-near)^T$
  $(0,0,-1)^T = M \times (0,0,-far)^T$ M

- a = -(far+near)/(far-near) Verify this!
  b = (-2 x far x near) / (far-near)

Perspective Projection (5)

- Not done yet. OpenGL also normalizes the x and y ranges of the viewing frustum to [-1, 1] (translate and scale)





- And takes care the case that eye is not at the center of the view volume (shear)

Perspective Projection (6)

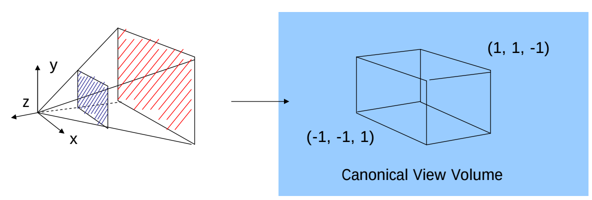
- Final Projection Matrix:

$$\begin{array}{c} x' \\ y' \\ z' \\ w' \end{array} = \begin{vmatrix} 2N/(xmax-xmin) & 0 & (xmax+xmin)/(xmax-xmin) & 0 \\ 0 & 2N/(ymax-ymin) & (ymax+ymin)/(ymax-ymin) & 0 \\ 0 & 0 & -(F+N)/(F-N) & -2F*N/(F-N) \\ 0 & 0 & -1 & 0 \end{vmatrix} \begin{array}{c} x \\ y \\ z \\ 1 \end{array}$$

glFrustum(xmin, xmax, ymin, ymax, N, F) N = near plane, F = far plane

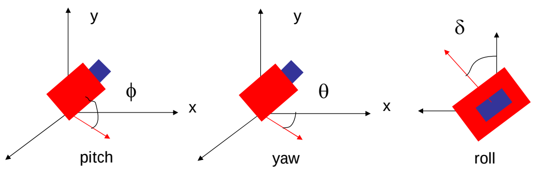
Perspective Projection (7)

- After perspective projection, the viewing frustum is also projected into a canonical view volume (like in parallel projection)



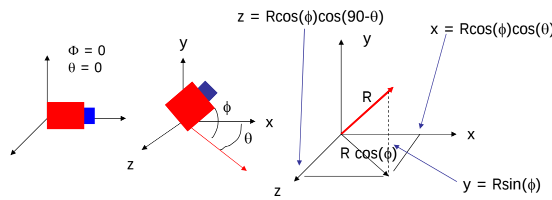
Flexible Camera Control

- Instead of provide COI, it is possible to just give camera orientation
- Just like control a airplane's orientation



Flexible Camera Control

- How to compute the viewing vector (x,y,z) from pitch(ϕ) and yaw(θ) ?



Flexible Camera Control

- gluLookAt() does not let you control pitch and yaw
- you need to compute/maintain the vector by yourself
- And then calculate COI = Eye + (x,y,z) before you can call gluLookAt().