Collision response: impulse

Once a collision is detected and the simulation has been backed up to the point of intersection, then the reaction to the collision can be calculated. By working back from the desired change in velocity due to a collision, an equation which computes the required change in momentum is formed. This equation uses a new term, called *impulse*, expressed in units of momentum. The impulse can be viewed as a large force acting over a short time interval (EQ 131). The change in momentum P, and therefore the change in velocity, can be computed once the impulse is known (EQ 132).

$$J = F \cdot \Delta t \qquad \textbf{(EQ 65)}$$

$$J = F \cdot \Delta t = M \cdot a \cdot \Delta t = M \cdot \Delta v = \Delta(M \cdot v) = \Delta P \qquad \textbf{(EQ 66)}$$

In order to characterize the elasticity of the collision response, the user selects the coefficient of restitution, $0 \le \varepsilon \le 1$. The coefficient of restitution relates the relative velocity before the collision to the relative velocity after the collision in the direction normal to the plane of intersection (). In order to compute j, the equations of how the velocities must change based on the equations of motion before and after the collision are used. These equations use the impulse, j, and can then be used to solve for its value.

$$v_{rel}^{+} = -\varepsilon \cdot v_{rel}^{-} \qquad \textbf{(EQ 67)}$$

Assume that the collision of two objects, A and B, has been detected at time t. Each object has a position for its center of mass ($x_A(t)$, $x_B(t)$), linear velocity ($v_A(t)$, $v_B(t)$) and angular velocity ($w_A(t)$, $w_B(t)$). The points of collision (p_A, p_B) have been identified on each object. See Figure 137.

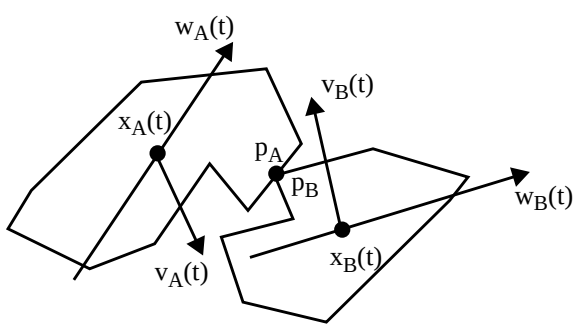

FIGURE 107. Configuration of colliding objects.

At the point of intersection, the normal to the surface of contact, n, is determined depending on whether it is a vertex-face contact or an edge-edge contact. The relative positions of the contact points with respect to the center of masses are r_A and r_B, respectively (EQ 134). The relative velocities of the contact points of the two objects in the direction of the normal to the surface is computed by EQ 135. The velocities of the points of contact are computed as in EQ 136.

$$r_A = p_A - x_A(t)$$
$$r_B = p_B - x_B(t)$$

(EQ 68)

$$v_{rel} = (\dot{p}_A(t) - \dot{p}_A(t)) \cdot n$$

(EQ 69)

$$\dot{p}_A(t) = v_A(t) + \omega_A(t) \times r_A$$
$$\dot{p}_B(t) = v_B(t) + \omega_B(t) \times r_B$$

(EQ 70)

The linear and angular velocities of the objects before the collision (v^-_A, v^-_B) are updated by the impulse to form the linear and angular velocities after the collision (v^+_A, v^+_B). Impulse is a vector quantity in the direction of the normal to the surface of contact, $J = j \cdot n$. The linear velocities are updated by adding in the effect of the impulse scaled by one over the mass (EQ 137). The angular velocities are updated by computing the affect of the impulse on the angular velocity of the objects (EQ 137).

$$v_A^+ = v_A^- + \frac{j \cdot n}{M_A}$$
$$v_B^+ = v_B^- + \frac{j \cdot n}{M_B}$$

(EQ 71)

$$\omega^+_A = \omega^-_A + I^{-1}_A(t) \cdot (r_A \times j \cdot n)$$
$$\omega^+_B = \omega^-_B + I^{-1}_B(t) \cdot (r_B \times j \cdot n)$$

(EQ 72)

To solve for the impulse, the difference between the velocities of the points after collision in the direction of the normal of the surface of collision is formed (EQ 139). The version of EQ 136 for velocities after collision are substituted into EQ 139. EQ 137 and EQ 138 are then substituted into that in order to produce EQ 140. Finally substituting in EQ 133 and solving for j produces EQ 141.

$$v_{rel}^+ = n \cdot (\dot{p}_A^+(t) - \dot{p}_B^+(t))$$

(EQ 73)

$$v^{+}_{rel} = n \cdot (v^{+}{}_{A}(t) + \omega^{+}{}_{A}(t) \times r_A - (v^{+}{}_{B}(t) + \omega^{+}{}_{B}(t) \times r_B)) \quad \textbf{(EQ 74)}$$

$$j = \frac{-((1+\varepsilon) \cdot v^{-}_{rel})}{\frac{1}{M_A} + \frac{1}{M_B} + n \cdot (I_A{}^{-1}(t)(r_A \times n)) \times r_A + (I_B{}^{-1}(t)(r_B \times n)) \times r_B} \quad \textbf{(EQ 75)}$$

Contact between two objects is defined by the point on each object involved in the contact and the normal to the surface of contact. A point of contact is tested to see if an actual collision is taking place. To test for collision, the velocity of the contact point on each object is calculated. A collision is detected if the component of the relative velocity of the two points in the direction of the normal indicates the contact points are approaching each other.

If there is a collision, then EQ 141 is used to compute the magnitude of the impulse. The impulse is used to scale the contact normal which can then be used to update the linear and angular momenta of each object.

```
Compute VA, VB                          ; EQ 121
Compute V-relative = dot(N,(VA-VB)      ; relative velocity of two contact points
if V-relative > threshold
        compute j                       ; EQ 126
        PA += jn                        ; update linear momentum
        PB -= jn                        ; update linear momentum
        LA += rA x jn                   ; update angular momentum
        LB -= rB x jn                   ;update angular momentum
else if V-relative < -theshold
        resting contact
else
        objects are moving away from each other
```

If there is more than one point of contact between two objects, then each must be tested for collision. Each time a collision point is identified, it is processed for updating the momentum as above. If any collision is processed in the list of contact points, then after the momenta have been updated, the contact list must be traversed again to see if there exist any collision points with the new object momenta. Each time one or more collisions are detected and processed in the list, the list must be traversed again until it can be traversed and no collisions detected.