Fundamentals of Rendering - Reflectance Functions

Chapter 9 of “Physically Based Rendering” by Pharr & Humphreys
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| 9.2 | Specular reflection and transmission  
Read about Snell’s law and Fresnel reflection; we’ll cover this after covering reflectance integrals |
| 9.3-9.6 | Specific models of reflection: Lambertian, microfacts, Lafortune, and Fresnel effects |
Surface Reflectance

– Measured data
  • Gonioreflectometer (See the Cornell Lab)

– Phenomenological models
  • Intuitive parameters
  • Most of graphics

– Simulation
  • Know composition of some materials
  • simulate complicated reflection from simple basis

– Physical (wave) optics
  • Using Maxwell’s equations
  • Computationally expensive

– Geometric optics
  • Use of geometric surface properties
Gonioreflectometer
Surface Reflectance

Diffuse
- Scatter light equally in all directions
- E.g. dull chalkboards, matte paint

Glossy specular
- Preferred set of direction around reflected direction
- E.g. plastic, high-gloss paint

Perfect specular
- E.g. mirror, glass

Retro-reflective
- E.g. velvet or earth’s moon
Surface Reflectance

• Isotropic vs. anisotropic
  – If you turn an object around a point -> does the shading change?
Surface Reflectance

Phong (isotropic)  
Banks (anisotropic)  
Banks (anisotropic)
Surface Properties

- Reflected radiance is proportional to incoming flux and to irradiance (incident power per unit area).

\[ dL_o(p, \omega_o) \propto dE(p, \omega_i) \]

Figure 2.9: Bidirectional reflection distribution function.
The BSDF

- **Bidirectional Scattering Distribution Function:** \( f(p, \omega_o, \omega_i) \)

- Measures portion of incident irradiance \( (E_i) \) that is reflected as radiance \( (L_o) \)
  \[
  f(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)}
  \]

- Or the ratio between incident radiance \( (L_i) \) and reflected radiance \( (L_o) \)
  \[
  f(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i) \cos \theta_i d\omega_i}
  \]
The BRDF and the BTDF

• Bidirectional Reflectance Distribution Function (BRDF)
  – Describes distribution of reflected light
• Bidirectional Transmittance Distribution Function (BTDF)
  – Describes distribution of transmitted light
• BSDF = BRDF + BTDF
Illumination via the BxDF

• The Reflectance Equation

\[ L_0(p, \omega_o) = \int_{S^2} f(p, \omega_o, \omega_i) L_i(p, \omega_i) |\cos \theta_i| d\omega_i \]

• The reflected radiance is
  
  – the sum of the incident radiance over the entire (hemi)sphere
  
  – foreshortened
  
  – scaled by the BxDF
Parameterizations

- **6-D BRDF** $f_r(p, \omega_o, \omega_i)$
  - Incident direction $L_i$
  - Reflected/Outgoing direction $L_o$
  - Surface position $p$: textured BxDF
- **4-D BRDF** $f_r(\omega_o, \omega_i)$
  - Homogeneous material
  - Anisotropic, depends on incoming azimuth
  - e.g. hair, brushed metal, ornaments
Parameterizations

- 3-D BRDF $f_r(\theta_o, \theta_i, \phi_o - \phi_i)$
  - Isotropic, independent of incoming azimuth
  - e.g. Phong highlight

- 1-D BRDF $f_r(\theta_i)$
  - Perfectly diffuse
  - e.g. Lambertian
BxDF Property 0

- Ranges from 0 to $\infty$ (strictly positive)
- Infinite when radiance distribution from single incident ray

\[
f_r(p,\omega_o,\omega_i) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i) \cos \theta_i \, d\omega_i}
\]
BRDF Property 1

• Linearity of functions

Sillion, Arvo, Westin, Greenberg
BRDF Property 2

Helmholtz Reciprocity

\[ f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o) \]

- Materials are not a one-way street
- Incoming to outgoing pathway same as outgoing to incoming pathway
BRDF Property 3

- Isotropic vs. anisotropic
  \[ f_r(\theta_i, \phi_i, \theta_o, \phi_o) = f_r(\theta_o, \theta_i, \phi_o - \phi_i) \]

- Reciprocity and isotropy
  \[ f_r(\theta_o, \theta_i, \phi_o - \phi_i) = f_r(\theta_i, \theta_o, \phi_i - \phi_o) = f_r(\theta_o, \theta_i, |\phi_o - \phi_i|) \]
  \[ f_r(\omega_o, \omega_i, \phi_o - \phi_i) = f_r(\omega_i, \omega_o, \phi_i - \phi_o) = f_r(\omega_o, \omega_i, |\phi_o - \phi_i|) \]
BRDF Property 4

• Conservation of Energy
  – Materials must not add energy (except for lights)
  – Materials must absorb some amount of energy
  – When integrated, must add to less than one
Reflectance

• Reflectance ratio of reflected to incident flux

$$\rho(p) = \frac{d\Phi_o(p)}{d\Phi_i(p)} = \frac{\int_{\Omega_o} L_o(p, \omega_o) \cos \theta_o d\omega_o}{\int_{\Omega_i} L_i(p, \omega_i) \cos \theta_i d\omega_i}$$

$$= \frac{\int_{\Omega_o} \int_{\Omega_i} f(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i \cos \theta_o d\omega_i d\omega_o}{\int_{\Omega_i} L_i(p, \omega_i) \cos \theta_i d\omega_i}$$

Reflectance between 0 and 1
Reflectance

If incident distribution is uniform and isotropic

$$\rho(p) = \frac{\int_{\Omega_o} \int_{\Omega_i} f(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i \cos \theta_o d\omega_i d\omega_o}{\int_{\Omega_i} L_i(p, \omega_i) \cos \theta_i d\omega_i}$$

$$= \frac{\int_{\Omega_o} \int_{\Omega_i} f(p, \omega_i, \omega_o) \cos \theta_i \cos \theta_o d\omega_i d\omega_o}{\int_{\Omega_i} \cos \theta_i d\omega_i}$$

Relates reflectance to the BRDF
Reflectance

- Hemispherical-directional reflectance
  - Reflection in a given direction due to constant illumination over a hemisphere
  - Total reflection over hemisphere due to light from a given direction (reciprocity)
  - Also called albedo - incoming photon is reflected with probability less than one

$$\rho_{hd}(p, \omega_o) = \int_{H^2(n)} f_r(p, \omega_o \omega_i) |\cos \theta_i| d\omega_i$$
Reflectance

- **Hemispherical-hemispherical reflectance**
  - Constant spectral value that gives the fraction of incident light reflected by a surface when the incident light is the same from all directions

\[
\rho_{hh}(p) = \frac{1}{\pi} \int_{H^2(n)} \int_{H^2(n)} f_r(p, \omega_o, \omega_i) |\cos \theta_o \cos \theta_i| \, d\omega_o \, d\omega_i
\]
Representations

• Tabulated BRDF’s
  – Require dense sampling and interpolation scheme

• Factorization
  – Into two 2D functions for data reduction (often after reparameterization)

• Basis Functions (Spherical Harmonics)
  – Loss of quality for high frequencies

• Analytical Models
  – Rough approximation only
  – Very compact
  – Most often represented as parametric equation (Phong, Cook-Torrance, etc.)
Law of Reflection

- Angle of reflectance = angle of incidence

\[ R = -I + (\cos \theta N + I) + (\cos \theta N + I) \]
\[ R = I - 2(I \cdot N)N \]
\[ \omega_r = R(\omega_i, N) \]

\[ \varphi_r = (\varphi_i + \pi ) \mod 2\pi \]
Polished Metal
Ideal Reflection (Mirror)

- BRDF cast as a delta function

\[
L_i(\theta_i, \varphi_i) \
\]

\[
L_r(\theta_r, \varphi_r) \
\]

\[
L_{r,m}(\theta_o, \varphi_o) = L_i(\theta_r, \varphi_r \pm \pi) 
\]

\[
f_{r,m}(\theta_i, \varphi_i, \theta_o, \varphi_o) = \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) 
\]

\[
L_{r,m}(\theta_o, \varphi_o) = \int f_{r,m}(\theta_i, \varphi_i, \theta_o, \varphi_o) L_i(\theta_i, \varphi_i) \cos \theta_i d \cos \theta_i d \varphi_i 
\]

\[
= \int \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i d \cos \theta_i d \varphi_i 
\]

\[
= L_i(\theta_r, \varphi_r \pm \pi) 
\]
Mirror Surface
Snell's Law

- $\eta_i, \eta_t$ indices of refraction (ratio of speed of light in vacuum to the speed of light in the medium)
  
  $\eta_i \sin \theta_i = \eta_t \sin \theta_t$
  
  $\eta_i N \times I = \eta_t N \times T$
  
  $\omega_t = T(\omega_i, N)$

\[ \varphi_r = (\varphi_i + \pi) \mod 2\pi \]
Law of Refraction

• Starting at Snell’s law:

\[
\begin{align*}
\frac{\eta_i}{\eta_t} N \times I &= N \times T \\
N \times (T - \mu I) &= 0
\end{align*}
\]

• We conclude that \( T = \mu I + \gamma N \)

• Assuming a normalized \( T \):

\[
T^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma(I \cdot N)
\]

• Solving this quadratic equation:

\[
\gamma = -\mu(I \cdot N) \pm \sqrt{1 - \mu^2(1 - (I \cdot N)^2)}
\]

• Leads to the total reflection condition:

\[
1 - \mu^2(1 - (I \cdot N)^2) \geq 0
\]
Optical Manhole

- Total Internal Reflection
- For water $n_w = 4/3$

Livingston and Lynch
Fresnel Reflection

• At top layer interface
  – Some light is reflected,
  – Remainder is transmitted through
• Simple ray-tracers: just given as a constant
• Physically based - depends on
  – incident angle
  – Polarization of light
  – wavelength
• Solution of Maxwell’s equations to smooth surfaces
• Dielectrics vs. conductors
Fresnel Reflection - Dielectrics

• Objects that don’t conduct electricity (e.g. glass)

• Fresnel term F for a dielectric is proportion of reflection (e.g. glass, plastic)
  – grazing angles: 100% reflected (see the material well!)
  – normal angles: 5% reflected (almost mirror-like)
Fresnel Reflection - Dielectrics

- Polarized light:
  
  \[ r_{\parallel} = \frac{\eta_t \cos \theta_i - \eta_i \cos \theta_t}{\eta_t \cos \theta_i + \eta_i \cos \theta_t} \]
  
  \[ r_{\perp} = \frac{\eta_i \cos \theta_i - \eta_t \cos \theta_t}{\eta_i \cos \theta_i + \eta_t \cos \theta_t} \]

- Where \( \omega_t \) is computed according to Snell’s law

- Unpolarized light:
  
  \[ F_r(\omega_i) = \frac{1}{2} \left( r_{\parallel}^2 + r_{\perp}^2 \right) \]
  
  \[ F_t(\omega_i) = (1 - F_r(\omega_i)) \]
Fresnel Reflection - Conductor

- Typically metals
- No transmission
- Absorption coefficient $k$
Fresnel Reflection - Conductor

- Polarized light:
  
  \[ r_{\parallel}^2 = \frac{\left( \eta^2 + k^2 \right) \cos^2 \theta_i - 2\eta \cos \theta_i + 1}{\left( \eta^2 + k^2 \right) \cos^2 \theta_i + 2\eta \cos \theta_i + 1} \]

  \[ r_{\perp}^2 = \frac{\left( \eta^2 + k^2 \right) - 2\eta \cos \theta_i + \cos^2 \theta_i}{\left( \eta^2 + k^2 \right) + 2\eta \cos \theta_i + \cos^2 \theta_i} \]

- Unpolarized light:

  \[ F_r(\omega_i) = \frac{1}{2} \left( r_{\parallel}^2 + r_{\perp}^2 \right) \]
Fresnel Reflection - Conductor

• How to determine $k$ or $\eta$?
• Measure $F_r$ for $\theta_i=0$ (normal angle)

1. Assume $k = 0$

\[ r_\perp^2 = r_\parallel^2 = \frac{(\eta - 1)^2}{(\eta + 1)^2} \quad \eta = \frac{1 + \sqrt{F_r(0)}}{1 - \sqrt{F_r(0)}} \]

2. Assume $\eta = 1$

\[ r_\perp^2 = r_\parallel^2 = \frac{k^2}{k^2 + 4} \quad k = 2\sqrt{\frac{F_r(0)}{1 - F_r(0)}} \]
Fresnel Normal (Dielectric)

10% reflected

90% transmitted

Air

Material
Fresnel Grazing (Dielectric)

90% reflected

10% transmitted
Fresnel Mid (Dielectric)

- 60% reflected
- 40% transmitted
Fresnel Reflection

Conductor (Aluminum)

Dielectric (N=1.5)

Schlick Approximation:

\[ F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5 \]
Fresnel Reflection

- Example - Copper
  - color shift as $\theta$ goes from 0 to $\pi/2$
  - at grazing, specular highlight is color of light

Measured Reflectance

Approximated Reflectance
Ideal Specular - Summary

- Reflection:

\[ f_r(p, \omega_i, \omega_o) = F_r(\omega_i) \frac{\delta(\omega_i - R(\omega_o, N))}{|\cos \theta_i|} \]

- Transmission:

\[ f_t(p, \omega_i, \omega_o) = \frac{\eta_o^2}{\eta_i^2} (1 - F_r(\omega_i)) \frac{\delta(\omega_o - T(\omega_i, N))}{|\cos \theta_i|} \]
Ideal Diffuse Reflection

• Uniform
  – Sends equal amounts of light in all directions
  – Amount depends on angle of incidence

• Perfect
  – all incoming light reflected
  – no absorption

\[ f_r(\omega_i, \omega_o) \propto k_d \]
Layered Surface

Varnish

Dye Layer
Layered Surface Larger

Varnish

Dye Particles
Ideal Diffuse Reflection

\[
L_{o,d}(\omega_o) = \int_{\Omega} f_{r,d}(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i \, d\omega_i \\
= f_{r,d} \int_{\Omega} L_i(\omega_i) \cos \theta_i \, d\omega_i \\
= f_{r,d} E \\
M = \int_{\Omega} L_{o,d}(\omega_o) \cos \theta_o \, d\omega_o \\
= L_{o,d} \int_{\Omega} \cos \theta_o \, d\omega_o \\
= L_{o,d} \pi \\
\rho_d = \frac{M}{E} = \frac{L_{o,d} \pi}{E} = \frac{f_{r,d} E \pi}{E} = f_{r,d} \pi \\
f_{r,d} = \frac{\rho_d}{\pi}
\]

Lamberts Cosine Law: \( M = \rho_d E = \rho_d E_s \cos \theta_s \)
Diffuse

- Helmholtz reciprocity?
- Energy preserving?

\[ \rho_d \leq 1 \]

\[ f_{r,d} = \frac{\rho_d}{\pi} \leq \frac{1}{\pi} \]
Reflectance Models

• Ideal
  – Diffuse
  – Specular

• Ad-hoc: Phong
  – Classical / Blinn
  – Modified
  – Ward
  – Lafortune

• Microfacets (Physically-based)
  – Torrance-Sparrow (Cook-Torrance)
  – Ashkhimin
Classical Phong Model

\[ L_o(p, \omega_o) = (k_d(N \cdot \omega_i) + k_d(R(\omega_o, N) \cdot \omega_i)^e) L_i(p, \omega_i) \]

- Where \(0 < k_d, k_s < 1\) and \(e > 0\)
- Cast as a BRDF:
  \[ f_r(p, \omega_i, \omega_o) = k_d + k_s \frac{(R(\omega_o, N) \cdot \omega_i)^e}{(N \cdot \omega_i)} \]
- Not reciprocal
- Not energy-preserving
- Specifically, too reflective at glancing angles, but not specular enough
- But cosine lobe itself symmetrical in \(\omega_i\) and \(\omega_o\)
Blinn-Phong

• Like classical Phong, but based on half-way vector

\[ f_r(p, \omega_i, \omega_o) = k_d + k_s \frac{(H(\omega_o, \omega_i) \cdot N)^e}{(N \cdot \omega_i)} \]

\[ \omega_h = H(\omega_o, \omega_i) = norm(\omega_o + \omega_i) \]

• Implemented in OpenGL
• Not reciprocal
• Not energy-preserving
• Specifically, too reflective at glancing angles, but not specular enough
• But cosine lobe itself symmetrical in \( \omega_i \) and \( \omega_o \)
Modified Phong

\[ f_r(p, \omega_i, \omega_o) = \frac{k_d}{\pi} + \frac{k_s(e + 2)}{2\pi} (R(\omega_o, N) \cdot \omega_i)^e \]

- For energy conservation: \( k_d + k_s < 1 \) (sufficient, not necessary)
- Peak gets higher as it gets sharper, but same total reflectivity
Ward-Phong

• Based on Gaussians

\[ f_r(p, \omega_i, \omega_o) = \frac{k_d}{\pi} + \frac{k_s}{\sqrt{\cos\theta_i \cos\theta_o}} \frac{\exp\left(-\frac{\tan^2 \omega_h}{\alpha^2}\right)}{4\pi\alpha^2} \]

• \( \alpha \): surface roughness, or blur in specular component.
Lafortune Model

- Phong cosine lobes symmetrical (reciprocal), easy to compute
- Add more lobes in order to match with measured BRDF
- How to generalize to anisotropic BRDFs?
- weight dot product:

\[
 f_r(p, \omega_i, \omega_o) = \frac{k_d}{\pi} + \sum_{i=1}^{nlobes} (\omega_o R_i \omega_i)^{e_i}
\]
Glossy
Physically-based Models

• Some basic principles common to many models:
  – Fresnel effect
  – Surface self-shadowing
  – Microfacets

• To really model well how surfaces reflect light, need to eventually move beyond BRDF

• Different physical models required for different kinds of materials

• Some kinds of materials don’t have good models

• Remember that BRDF makes approximation of completely local surface reflectance!
Cook-Torrance Model

- Based in part on the earlier Torrance-Sparrow model
- Neglects multiple scattering

\[ f_r(p, \omega_i, \omega_o) = \frac{F_r(\omega_h)D(\omega_h)G(\omega_o, \omega_i)}{4 \cos \theta_i \cos \theta_o} \]

- D - Microfacet Distribution Function
  - how many “cracks” do we have that point in our (viewing) direction?

- G - Geometrical Attenuation Factor
  - light gets obscured by other “bumps”

- F - Fresnel Term
Microfacet Models

- Microscopically rough surface
- Specular facets oriented randomly
- Measure of scattering due to variation in angle of microfacets
- A statistic approximation, i.e. need a statistic distribution function
Rough Surface

Diffuse Scattering

Reduced Specular
Microfacet Distribution Function $D$

- Blinn

$$D(\omega_h) = ce^{-(\frac{\omega_h \cdot N}{m})^2}$$

- where $m$ is the root mean square slope of the facets (as an angle)
- Blinn says $c$ is an arbitrary constant
- Really should be chosen to normalize BRDF...
Microfacet Distribution Function $D$

- Beckmann (most effective)
  
  $$D(\omega_h) = \frac{1}{m^2 \cos^4 \alpha} e^{-\left(\frac{\tan \alpha}{m}\right)^2}$$

- Represents a distribution of slopes

- But $\alpha = \tan \alpha$ for small $\alpha$

Fig. 3. (a) Beckmann distribution for $m = 0.2$, (b) Gaussian distribution for $m = 0.2$, (c) Beckmann distribution for $m = 0.6$, (d) Gaussian distribution for $m = 0.6$. 
Multiscale Distribution Function

- May want to model multiple scales of roughness:
  \[ D(\omega_h) = \sum_j w_j D_j(\omega_h) \]

  \[ \sum_j w_j = 1 \]

- Bumps on bumps …
Self-Shadowing (V-Groove Model)

• Geometrical Attenuation Factor $G$

  – how much are the “cracks” obstructing themselves?

$$G = \min[1, \frac{2(N \cdot \omega_h)(N \cdot \omega_o)}{\omega_o \cdot \omega_h}, \frac{2(N \cdot \omega_h)(N \cdot \omega_i)}{\omega_o \cdot \omega_h}]$$
Cook-Torrance - Summary

- Carbon
- Red Rubber
- Obsidian
- Lunar Dust
- Olive Drab
- Rust
- Bronze
- Tungsten
- Copper
- Tin
- Nickel
- Stainless Steel
Ashkhimin Model

- Modern Phong
- Phenomological, but:
  - Physically plausible
  - Anisotropic
- Good for both Monte-Carlo and HW implementation
Ashkhimin Model

- Weighted sum of diffuse and specular part:
  \[ f_r(p, \omega_i, \omega_o) = k_d (1 - k_s) f_d(p, \omega_i, \omega_o) + k_s f_s(p, \omega_i, \omega_o) \]

- Dependence of diffuse weight on \( k_s \) decreases diffuse reflectance when specular reflectance is large

- Specular part \( f_s \) not an impulse, really just glossy

- Diffuse part \( f_d \) not constant: energy specularly reflected cannot be diffusely reflected

- For metals, \( f_d = 0 \)
Ashkhimin Model

- $k_s$: Spectrum or color of specular reflectance at normal incidence.
- $k_d$: Spectrum or color of diffuse reflectance (away from the specular peak).
- $q_u$, $q_v$: Exponents to control shape of specular peak.
  - Similar effects to Blinn-Phong model
  - If an isotropic model is desired, use single value $q$
  - A larger value gives a sharper peak
  - Anisotropic model requires two tangent vectors $u$ and $v$
  - The value $q_u$ controls sharpness in the direction of $u$
  - The value $q_v$ controls sharpness in the direction of $v$
Ashkhiminin Model

- $\phi$ is the angle between $u$ and $\omega h$

$$D(\omega_h) = \sqrt{(q_u + 1)(q_v + 1)(\omega_h \cdot N)(q_u \cos^2 \phi + q_v \cos^2 \phi)}$$
Ashkhimin Model

- Diffuse term given by:

\[ f_d(p, \omega_i, \omega_o) = \frac{28}{23\pi} (1 - (1 - (\omega_o \cdot N))^5)(1 - (1 - (\omega_i \cdot N))^5) \]

- Leading constant chosen to ensure energy conservation
- Form comes from Schlick approximation to Fresnel factor
- Diffuse reflection due to subsurface scattering: once in, once out
Complex BRDF

- Combination of the three.
BRDF illustrations

Phong Illumination

Oren-Nayar
BRDF illustrations

Cook-Torrance-Sparrow BRDF

Hapke BRDF
BRDF illustrations

lumber
cement
BRDF illustrations

Measured BRDF
(scaled by cubic root function)

White paint
Blue paint
Commercial aluminum
Blue plastic

Surface microstructure
$bv = \text{Brdf Viewer}$

Diffuse

Torrance-Sparrow

Anisotropic

Szymon Rusinkiewicz
Princeton U.
BRDF cannot

Spatial variation of reflectance
BRDF cannot

Transparency and Translucency (depth)

Glass: transparent
Wax: translucent
BTDF

Opaque milk (rendered)
Translucent milk (rendered)

BSSRDF