

# Fundamentals of Rendering - Reflectance Functions

Chapter 9 of “Physically Based Rendering”  
by Pharr&Humphreys

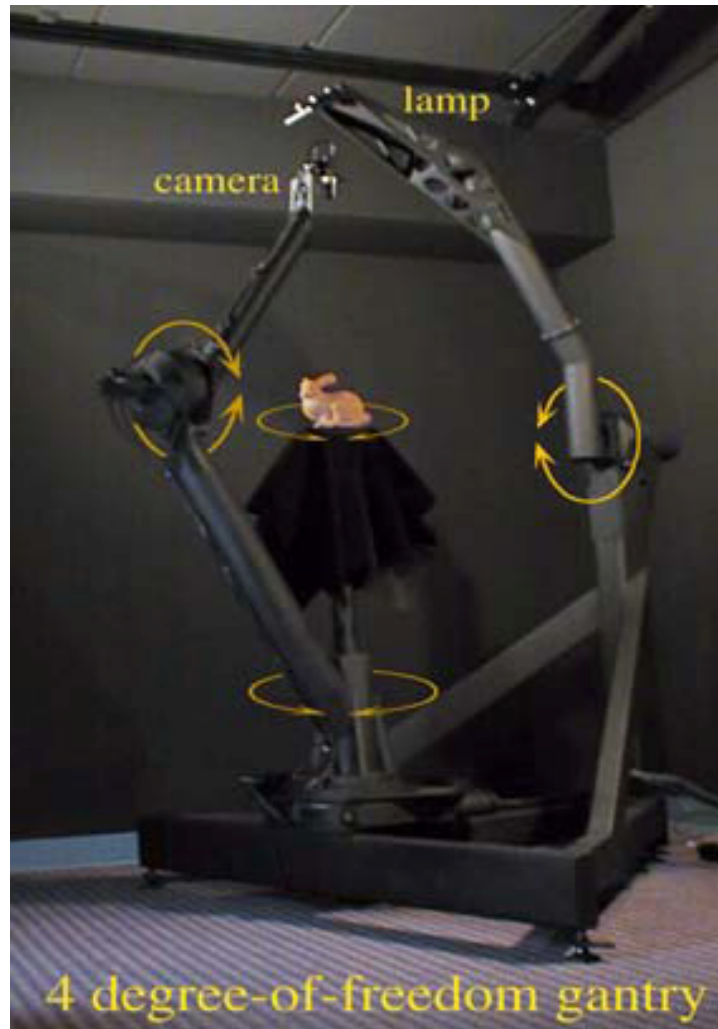
# Chapter 9

9.0	Terms, etc.
9.1	PBRT Interface
9.2	Specular reflection and transmission Read about Snell's law and Fresnel reflection; we'll cover this after covering reflectance integrals
9.3-9.6	Specific models of reflection: Lambertian, microfacts, Lafortune, and Fresnel effects

# Surface Reflectance

- Measured data
  - Gonioreflectometer (See the Cornell Lab)
- Phenomenological models
  - Intuitive parameters
  - Most of graphics
- Simulation
  - Know composition of some materials
  - simulate complicated reflection from simple basis
- Physical (wave) optics
  - Using Maxwell's equations
  - Computationally expensive
- Geometric optics
  - Use of geometric surface properties

# Gonioreflectometer



# Surface Reflectance



## Diffuse

- Scatter light equally in all directions
- E.g. dull chalkboards, matte paint



## Glossy specular

- Preferred set of direction around reflected direction
- E.g. plastic, high-gloss paint

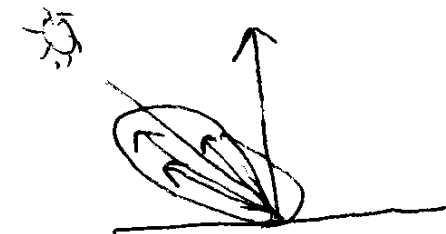
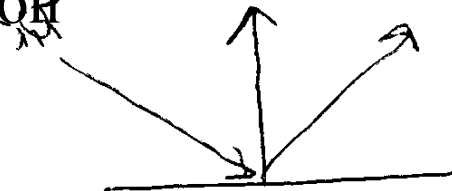
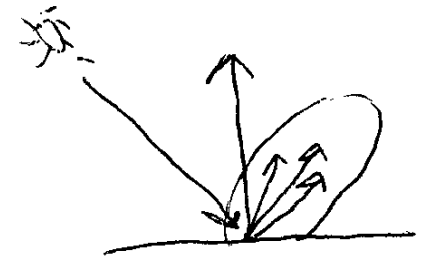
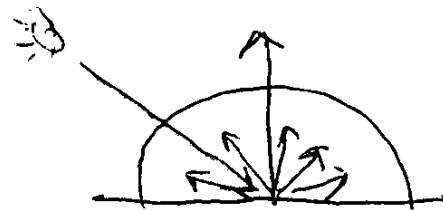


## Perfect specular

- E.g. mirror, glass

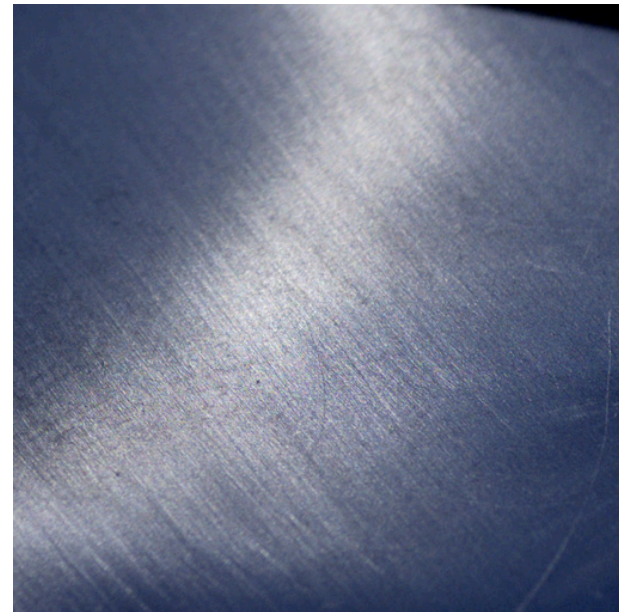
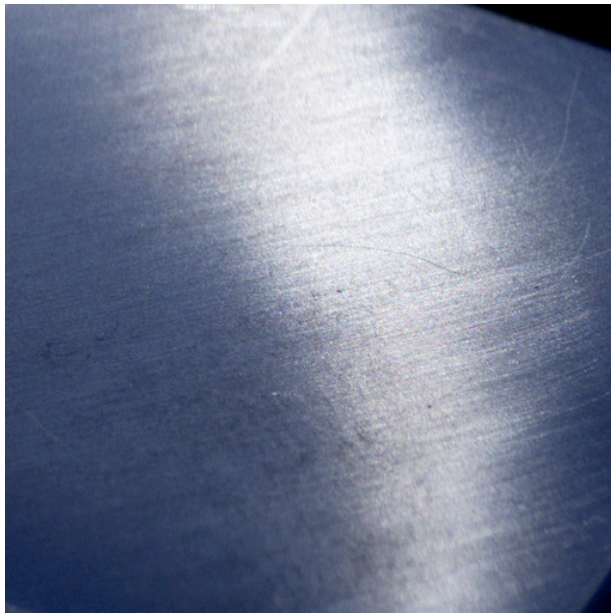
## Retro-reflective

- E.g. velvet or earth's moon



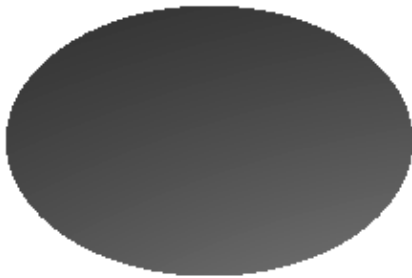
# Surface Reflectance

- Isotropic vs. anisotropic
  - If you turn an object around a point -> does the shading change?

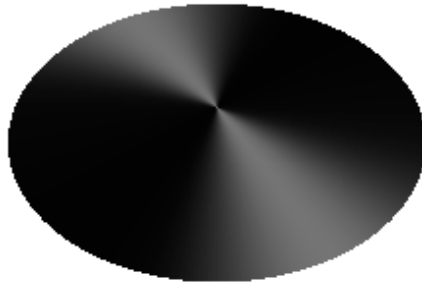


# Surface Reflectance

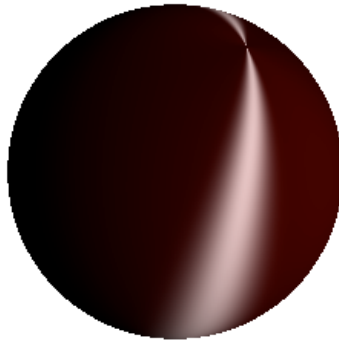
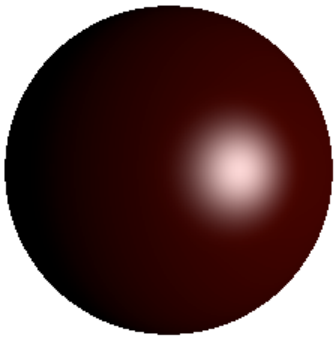
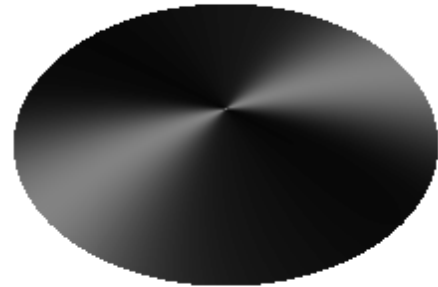
Phong (isotropic)



Banks (anisotropic)



Banks (anisotropic)



# Surface Properties

- Reflected radiance is proportional to incoming flux and to irradiance (incident power per unit area).

$$dL_o(p, \omega_o) \propto dE(p, \omega_i)$$

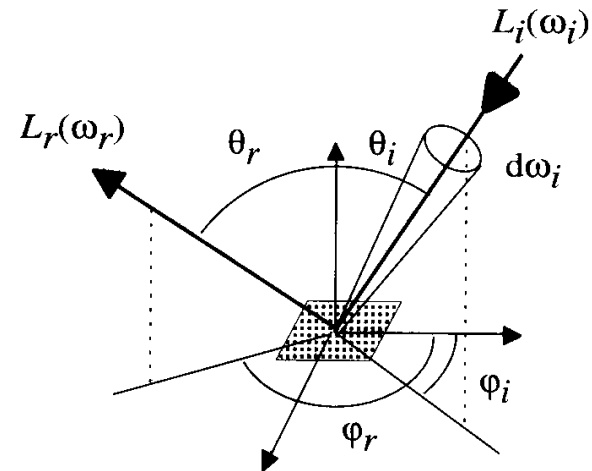


Figure 2.9: Bidirectional reflection distribution function.



# The BSDF

- Bidirectional Scattering Distribution

Function:  $f(p, \omega_o, \omega_i)$

- Measures portion of incident irradiance ( $E_i$ ) that is reflected as radiance ( $L_o$ )

$$f(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)}$$

- Or the ratio between incident radiance ( $L_i$ ) and reflected radiance ( $L_o$ )

$$f(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i) \cos \theta_i d\omega_i}$$

# The BRDF and the BTDF

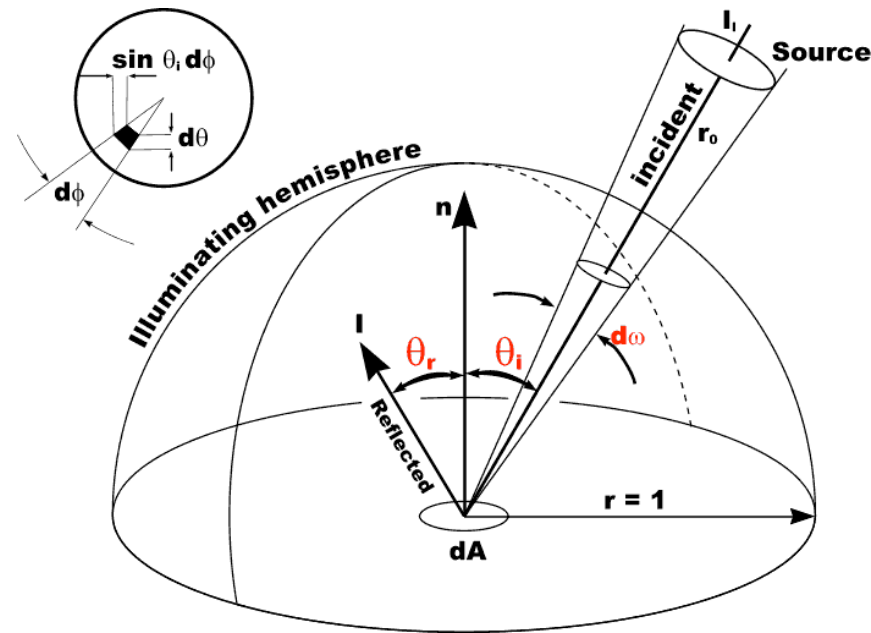
- Bidirectional Reflectance Distribution Function (BRDF)
  - Describes distribution of reflected light
- Bidirectional Transmittance Distribution Function (BTDF)
  - Describes distribution of transmitted light
- $BSDF = BRDF + BTDF$

# Illumination via the BxDF

- The Reflectance Equation

$$L_o(p, \omega_o) = \int_{S^2} f(p, \omega_o, \omega_i) L_i(p, \omega_i) |\cos \theta_i| d\omega_i$$

- The reflected radiance is
  - the sum of the incident radiance over the entire (hemi)sphere
  - foreshortened
  - scaled by the BxDF



# Parameterizations

- 6-D BRDF  $f_r(p, \omega_o, \omega_i)$ 
  - Incident direction  $L_i$
  - Reflected/Outgoing direction  $L_o$
  - Surface position  $p$ : textured BxDF
- 4-D BRDF  $f_r(\omega_o, \omega_i)$ 
  - Homogeneous material
  - Anisotropic, depends on incoming azimuth
  - e.g. hair, brushed metal, ornaments

# Parameterizations

- 3-D BRDF  $f_r(\theta_o, \theta_i, \phi_o - \phi_i)$ 
  - Isotropic, independent of incoming azimuth
  - e.g. Phong highlight
- 1-D BRDF  $f_r(\theta_i)$ 
  - Perfectly diffuse
  - e.g. Lambertian

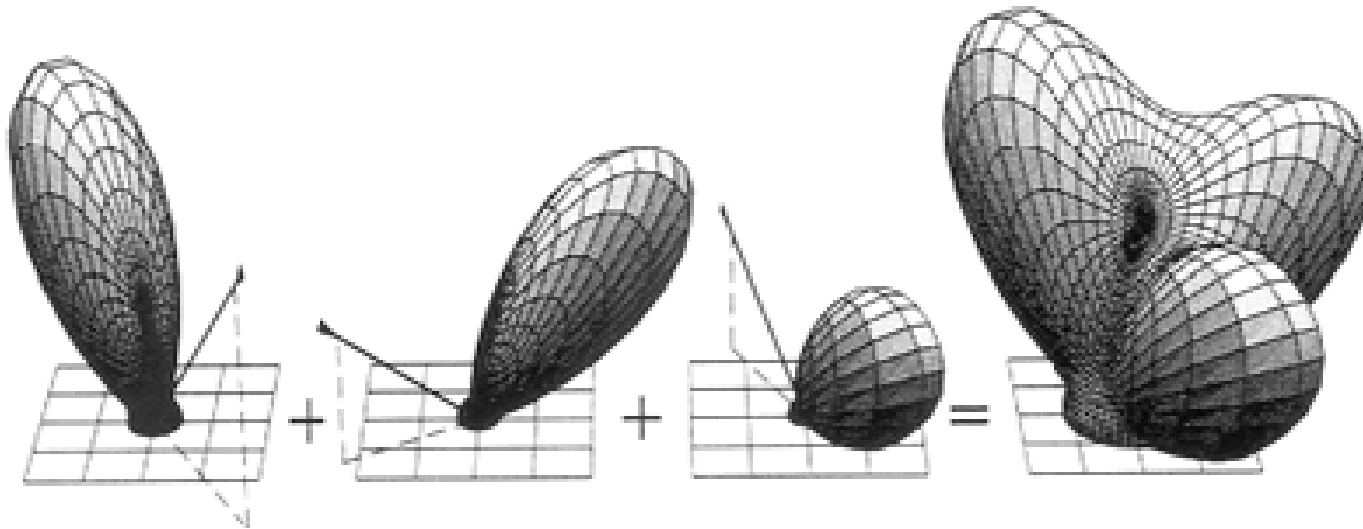
# BxDF Property 0

- Ranges from 0 to  $\infty$  (strictly positive)
- Infinite when radiance distribution from single incident ray

$$f_r(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i) \cos \theta_i d\omega_i}$$

# BRDF Property 1

- Linearity of functions



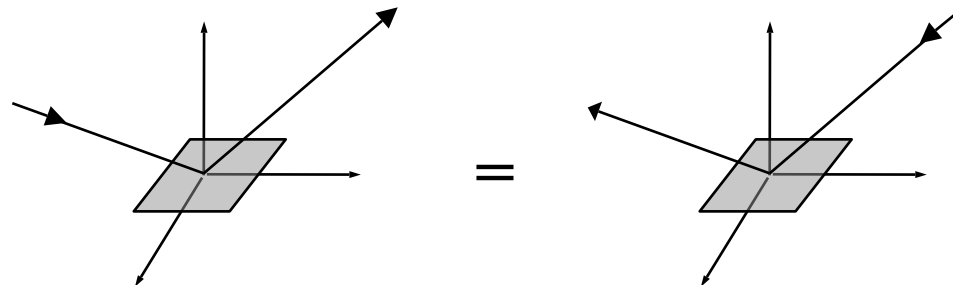
Sillion, Arvo, Westin, Greenberg

# BRDF Property 2

## Helmholtz Reciprocity

$$f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o)$$

- Materials are not a one-way street
- Incoming to outgoing pathway same as outgoing to incoming pathway

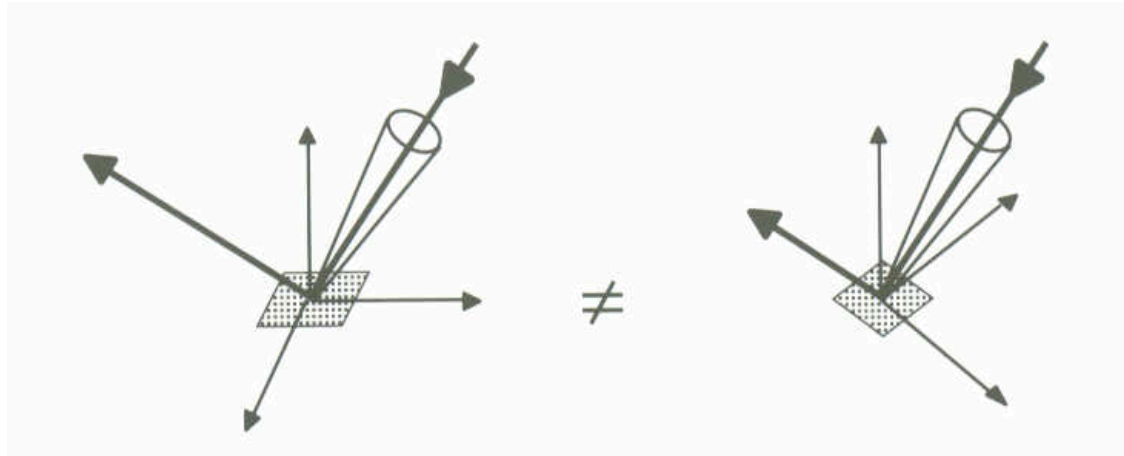




# BRDF Property 3

- Isotropic vs. anisotropic

$$f_r(\theta_i, \phi_i, \theta_o, \phi_o) = f_r(\theta_o, \theta_i, \phi_o - \phi_i)$$



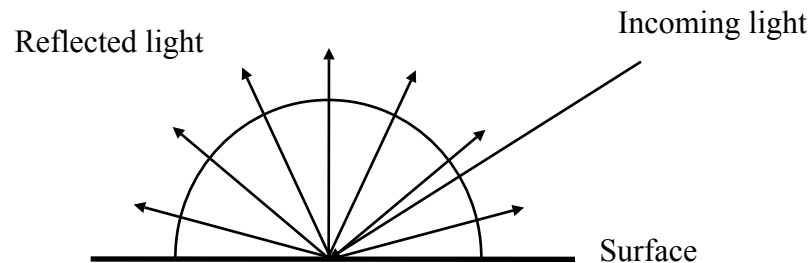
- Reciprocity and isotropy

$$f_r(\theta_o, \theta_i, \phi_o - \phi_i) = f_r(\theta_i, \theta_o, \phi_i - \phi_o) = f_r(\theta_o, \theta_i, |\phi_o - \phi_i|)$$

$$f_r(\omega_o, \omega_i, \phi_o - \phi_i) = f_r(\omega_i, \omega_o, \phi_i - \phi_o) = f_r(\omega_o, \omega_i, |\phi_o - \phi_i|)$$

# BRDF Property 4

- Conservation of Energy
  - Materials must not add energy (except for lights)
  - Materials must absorb some amount of energy
  - When integrated, must add to less than one



# Reflectance

- Reflectance ratio of reflected to incident flux

$$\begin{aligned}\rho(p) &= \frac{d\Phi_o(p)}{d\Phi_i(p)} = \frac{\int_{\Omega_o} L_o(p, \omega_o) \cos \theta_o d\omega_o}{\int_{\Omega_i} L_i(p, \omega_i) \cos \theta_i d\omega_i} \\ &= \frac{\int_{\Omega_o} \int_{\Omega_i} f(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i \cos \theta_o d\omega_i d\omega_o}{\int_{\Omega_i} L_i(p, \omega_i) \cos \theta_i d\omega_i}\end{aligned}$$

Reflectance between 0 and 1

# Reflectance

- If incident distribution is uniform and isotropic

$$\begin{aligned}\rho(p) &= \frac{\int_{\Omega_o} \int_{\Omega_i} f(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i \cos \theta_o d\omega_i d\omega_o}{\int_{\Omega_i} L_i(p, \omega_i) \cos \theta_i d\omega_i} \\ &= \frac{\int_{\Omega_o} \int_{\Omega_i} f(p, \omega_i, \omega_o) \cos \theta_i \cos \theta_o d\omega_i d\omega_o}{\int_{\Omega_i} \cos \theta_i d\omega_i}\end{aligned}$$

Relates reflectance to the BRDF

# Reflectance

- Hemispherical-directional reflectance
  - Reflection in a given direction due to constant illumination over a hemisphere
  - Total reflection over hemisphere due to light from a given direction (reciprocity)
  - Also called albedo - incoming photon is reflected with probability less than one

$$\rho_{hd}(p, \omega_o) = \int_{H^2(n)} f_r(p, \omega_o, \omega_i) |\cos \theta_i| d\omega_i$$

# Reflectance

- Hemispherical-hemispherical reflectance
  - Constant spectral value that gives the fraction of incident light reflected by a surface when the incident light is the same from all directions

$$\rho_{hh}(p) = \frac{1}{\pi} \int_{H^2(n)} \int_{H^2(n)} f_r(p, \omega_o \omega_i) |\cos \theta_o \cos \theta_i| d\omega_o d\omega_i$$

# Representations

- Tabulated BRDF's
  - Require dense sampling and interpolation scheme
- Factorization
  - Into two 2D functions for data reduction (often after reparameterization)
- Basis Functions (Spherical Harmonics)
  - Loss of quality for high frequencies
- Analytical Models
  - Rough approximation only
  - Very compact
  - Most often represented as parametric equation (Phong, Cook-Torrance, etc.)

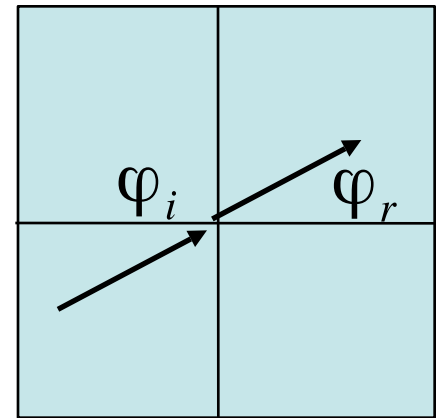
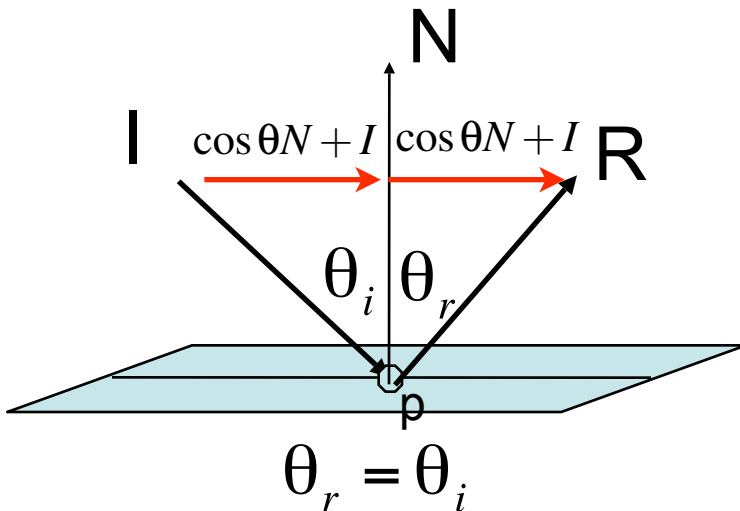
# Law of Reflection

- Angle of reflectance = angle of incidence

$$R = -I + (\cos \theta N + I) + (\cos \theta N + I)$$

$$R = I - 2(I \cdot N)N$$

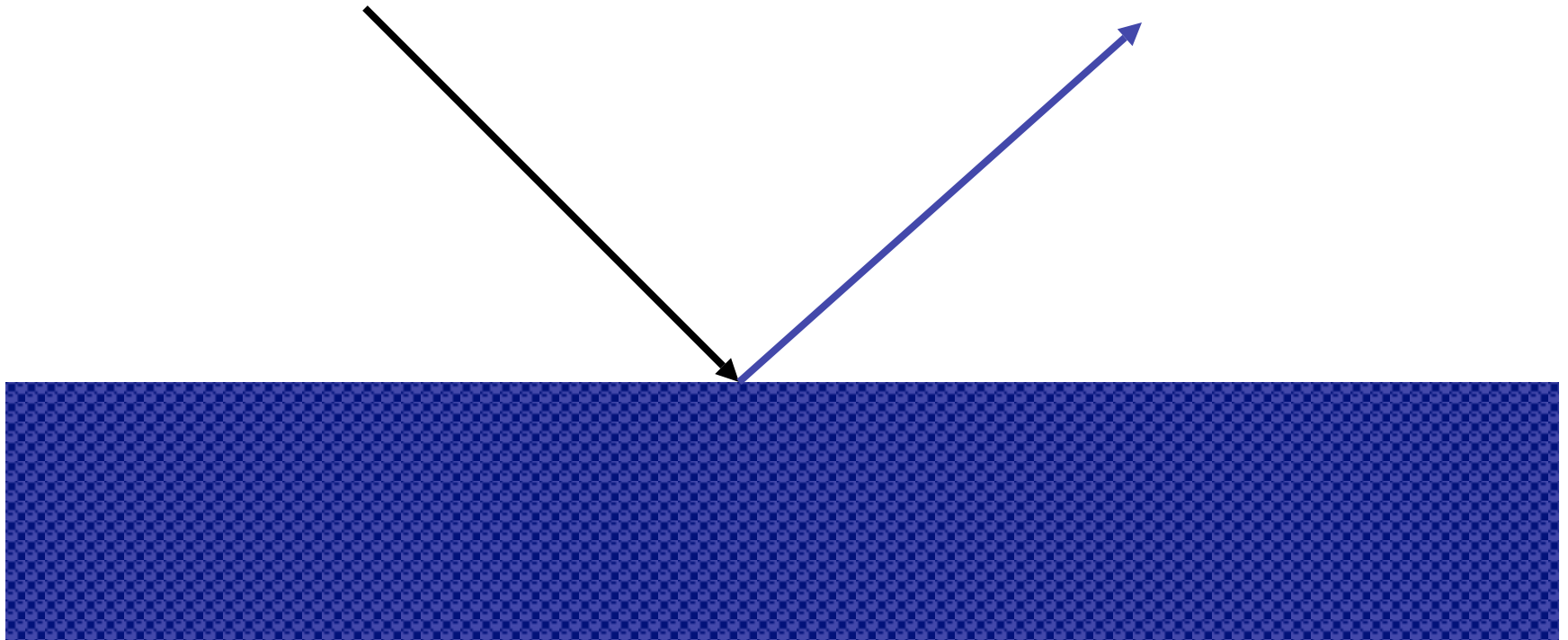
$$\omega_r = R(\omega_i, N)$$



$$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$$

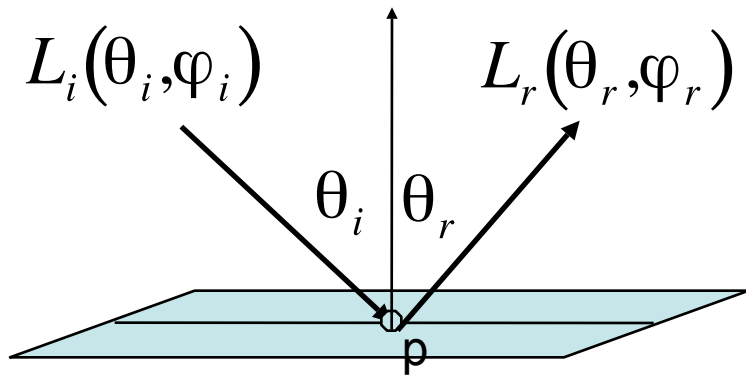


# Polished Metal



# Ideal Reflection (Mirror)

- BRDF cast as a delta function

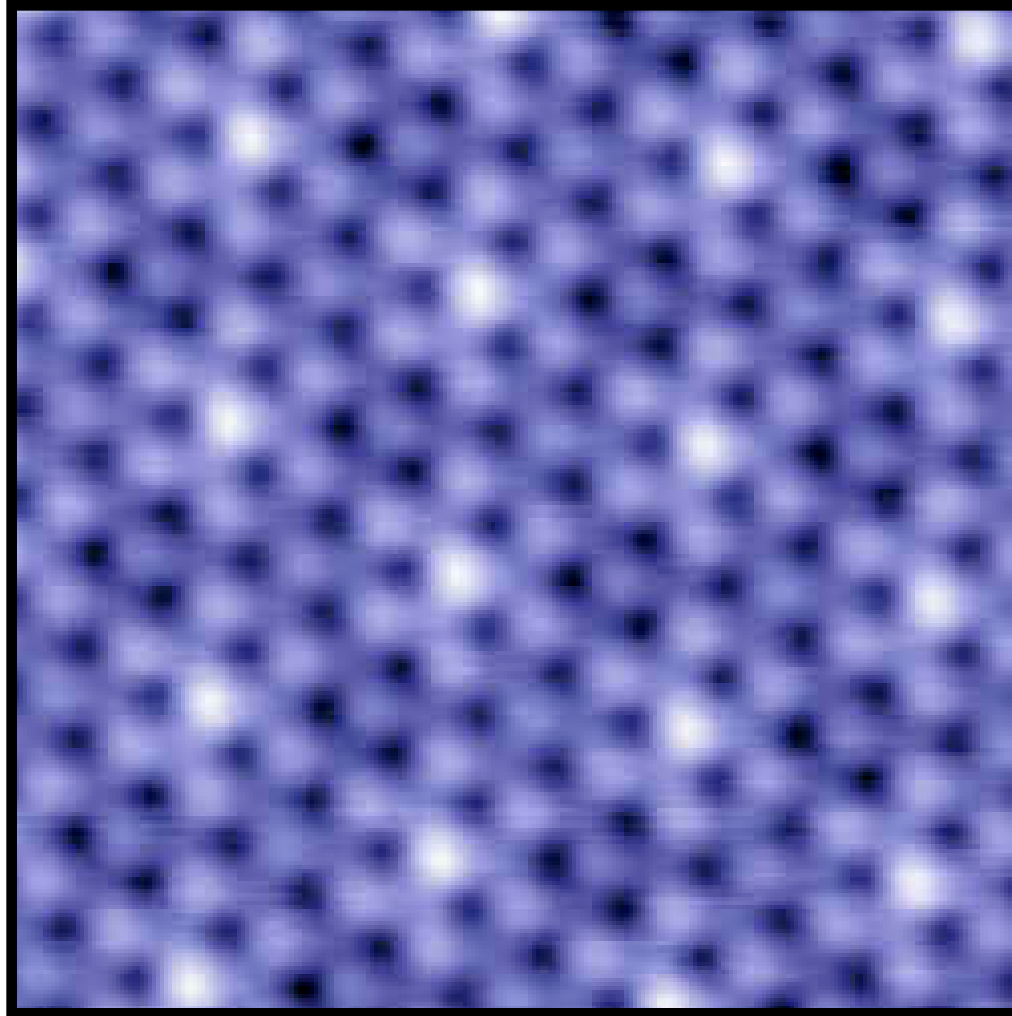


$$L_{r,m}(\theta_o, \varphi_o) = L_i(\theta_r, \varphi_r \pm \pi)$$

$$f_{r,m}(\theta_i, \varphi_i, \theta_o, \varphi_o) = \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i} \delta(\varphi_i - \varphi_r \pm \pi)$$

$$\begin{aligned} L_{r,m}(\theta_o, \varphi_o) &= \int f_{r,m}(\theta_i, \varphi_i, \theta_o, \varphi_o) L_i(\theta_i, \varphi_i) \cos\theta_i d\cos\theta_i d\varphi_i \\ &= \int \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i} \delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos\theta_i d\cos\theta_i d\varphi_i \\ &= L_i(\theta_r, \varphi_r \pm \pi) \end{aligned}$$

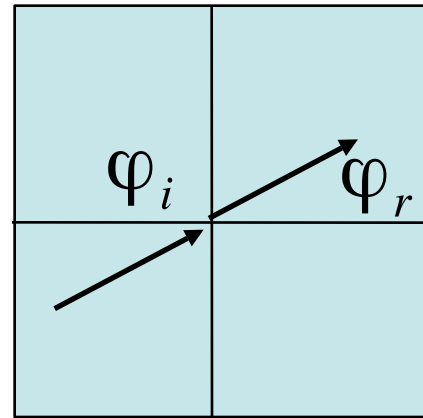
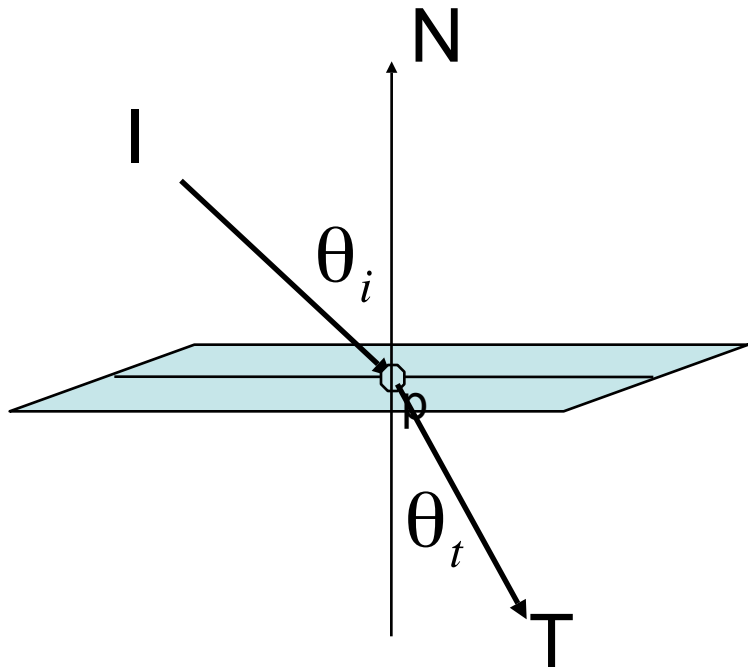
# Mirror Surface



# Snell's Law

- $\eta_i, \eta_t$  indices of refraction (ratio of speed of light in vacuum to the speed of light in the medium)

$$\eta_i \sin \theta_i = \eta_t \sin \theta_t \quad \omega_t = T(\omega_i, N)$$
$$\eta_i N \times I = \eta_t N \times T$$



$$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$$

# Law of Refraction

- Starting at Snell's law:

$$\frac{\eta_i}{\eta_t} N \times I = N \times T$$
$$N \times (T - \mu I) = 0$$

- We conclude that  $T = \mu I + \gamma N$

- Assuming a normalized T:  $T^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma(I \cdot N)$

- Solving this quadratic

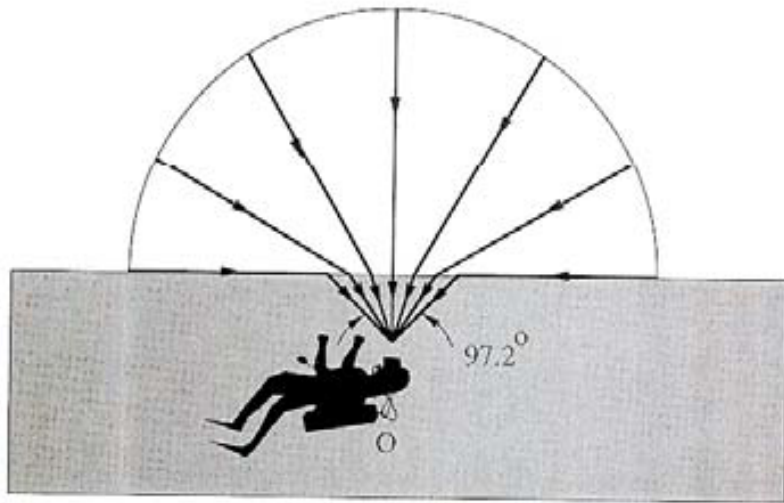
equation:  $\gamma = -\mu(I \cdot N) \pm \sqrt{1 - \mu^2(1 - (I \cdot N)^2)}$

- Leads to the total

reflection condition:  $1 - \mu^2(1 - (I \cdot N)^2) \geq 0$

# Optical Manhole

- Total Internal Reflection
- For water  $n_w = 4/3$



Livingston and Lynch



# Fresnel Reflection

- At top layer interface
  - Some light is reflected,
  - Remainder is transmitted through
- Simple ray-tracers: just given as a constant
- Physically based - depends on
  - incident angle
  - Polarization of light
  - wavelength
- Solution of Maxwell's equations to smooth surfaces
- Dielectrics vs. conductors

# Fresnel Reflection - Dielectrics

- Objects that don't conduct electricity (e.g. glass)
- Fresnel term  $F$  for a dielectric is proportion of reflection (e.g. glass, plastic)
  - grazing angles: 100% reflected (see the material well!)
  - normal angles: 5% reflected (almost mirror-like)



# Fresnel Reflection - Dielectrics

- Polarized light:

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t}$$

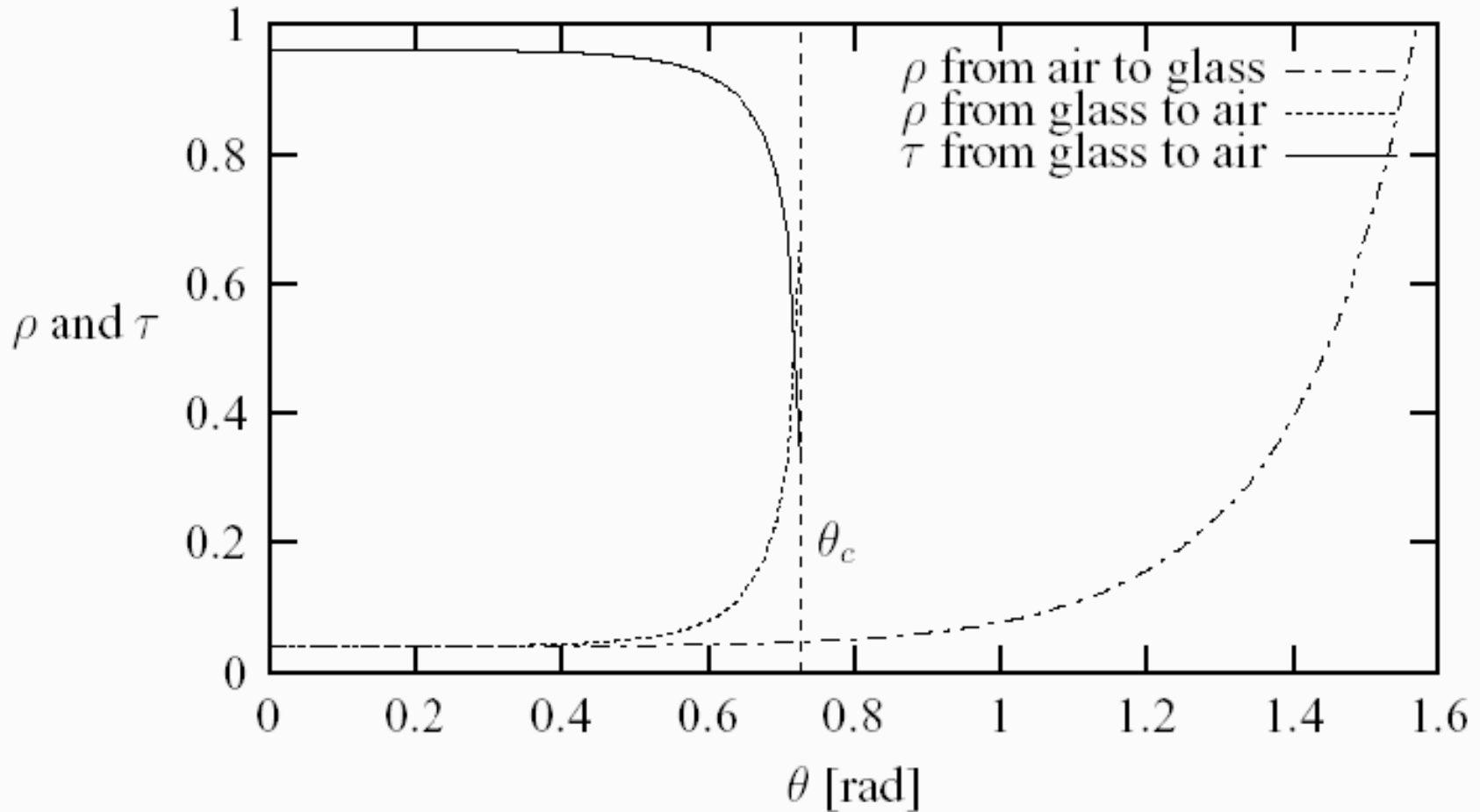
- Where  $\omega_t$  is computed according to Snell's law

- Unpolarized light:

$$F_r(\omega_i) = \frac{1}{2} (r_{\parallel}^2 + r_{\perp}^2)$$

$$F_t(\omega_i) = (1 - F_r(\omega_i))$$

# Fresnel Reflection - Dielectrics



# Fresnel Reflection - Conductor

- Typically metals
- No transmission
- Absorption coefficient  $k$

# Fresnel Reflection - Conductor

- Polarized light:

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2)\cos^2 \theta_i - 2\eta \cos \theta_i + 1}{(\eta^2 + k^2)\cos^2 \theta_i + 2\eta \cos \theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta \cos \theta_i + \cos^2 \theta_i}{(\eta^2 + k^2) + 2\eta \cos \theta_i + \cos^2 \theta_i}$$

- Unpolarized light:

$$F_r(\omega_i) = \frac{1}{2} (r_{\parallel}^2 + r_{\perp}^2)$$

# Fresnel Reflection - Conductor

- How to determine  $k$  or  $\eta$ ?
- Measure  $F_r$  for  $\theta_i=0$  (normal angle)

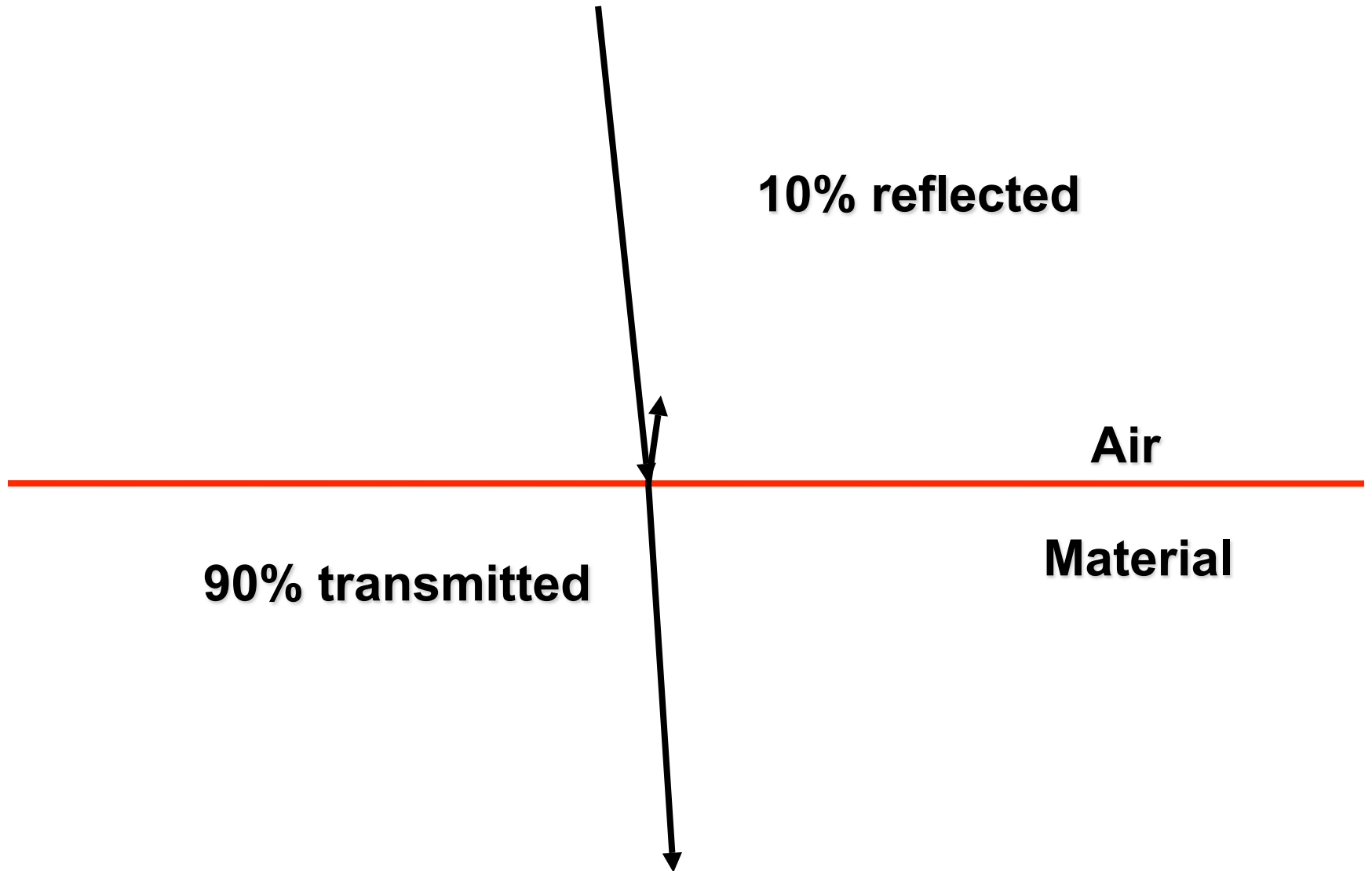
- 1. Assume  $k = 0$

$$r_{\perp}^2 = r_{\parallel}^2 = \frac{(\eta - 1)^2}{(\eta + 1)^2} \quad \eta = \frac{1 + \sqrt{F_r(0)}}{1 - \sqrt{F_r(0)}}$$

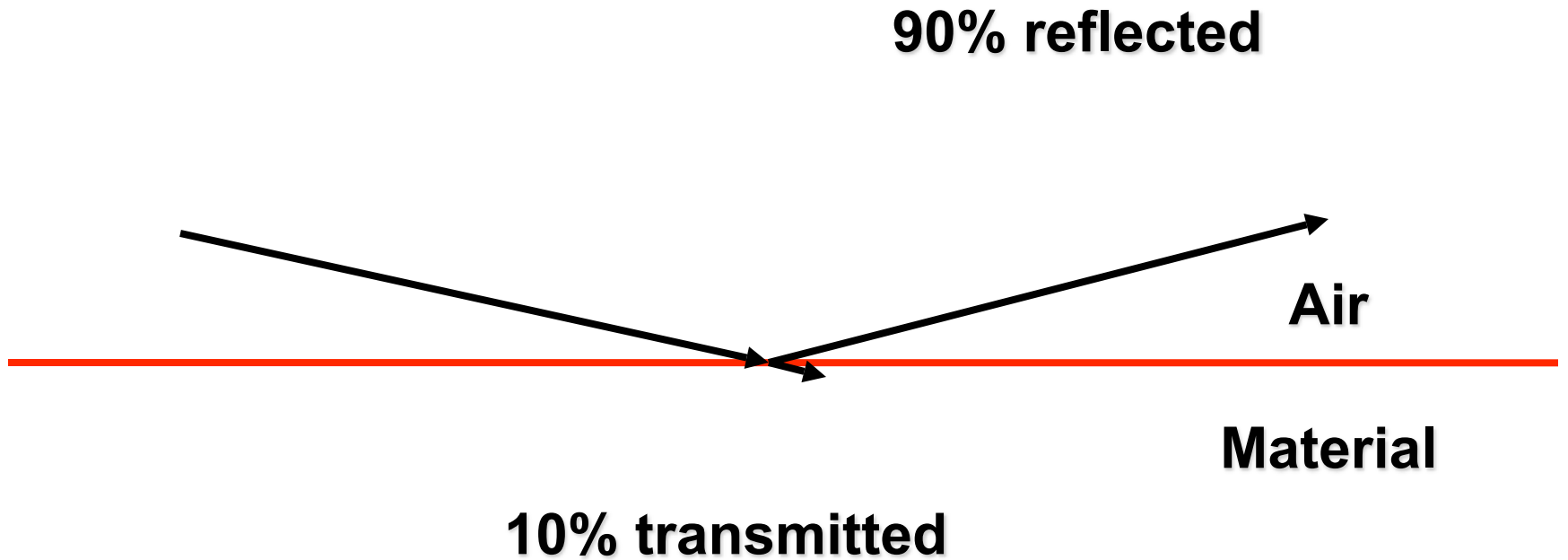
- 2. Assume  $\eta = 1$

$$r_{\perp}^2 = r_{\parallel}^2 = \frac{k^2}{k^2 + 4} \quad k = 2\sqrt{\frac{F_r(0)}{1 - F_r(0)}}$$

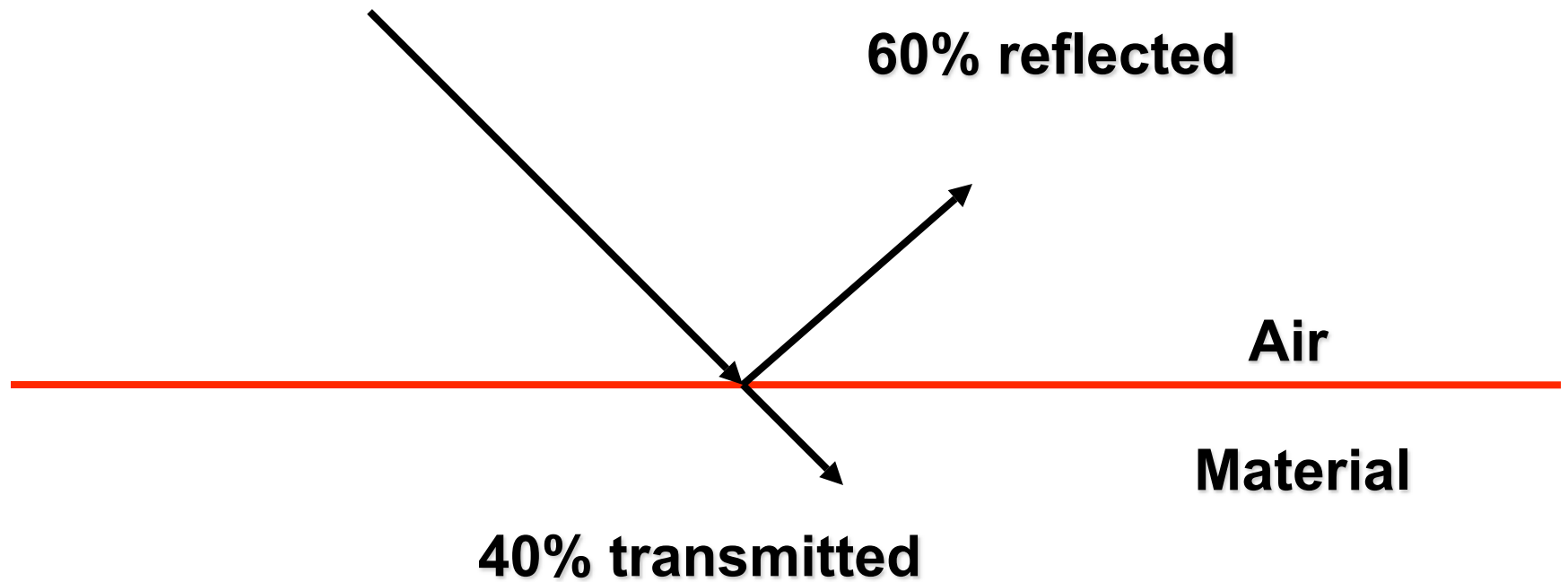
# Fresnel Normal (Dielectric)



# Fresnel Grazing (Dielectric)



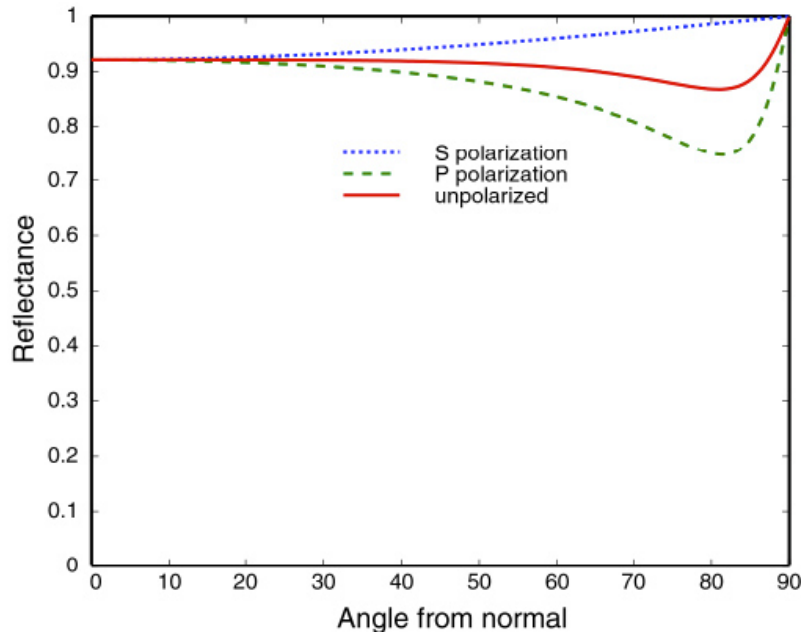
# Fresnel Mid (Dielectric)



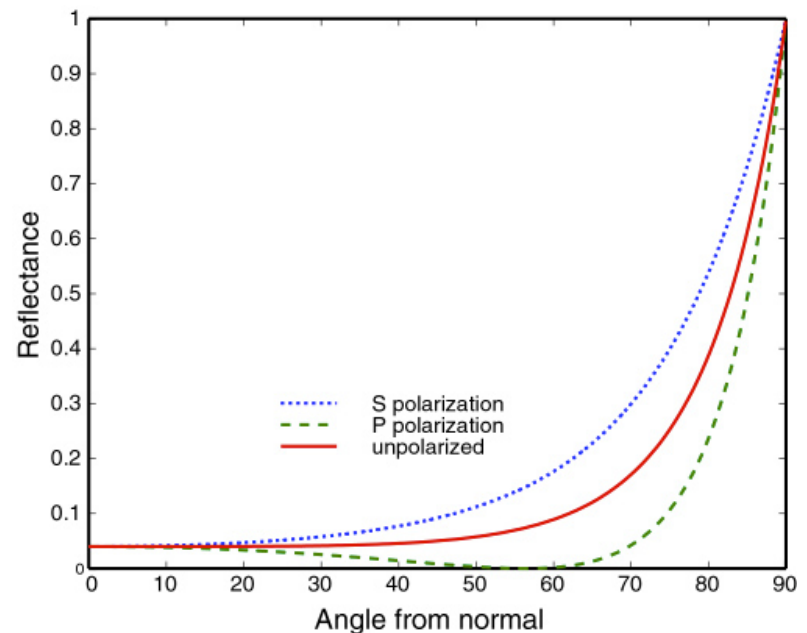


# Fresnel Reflection

## Conductor (Aluminum)



## Dielectric (N=1.5)

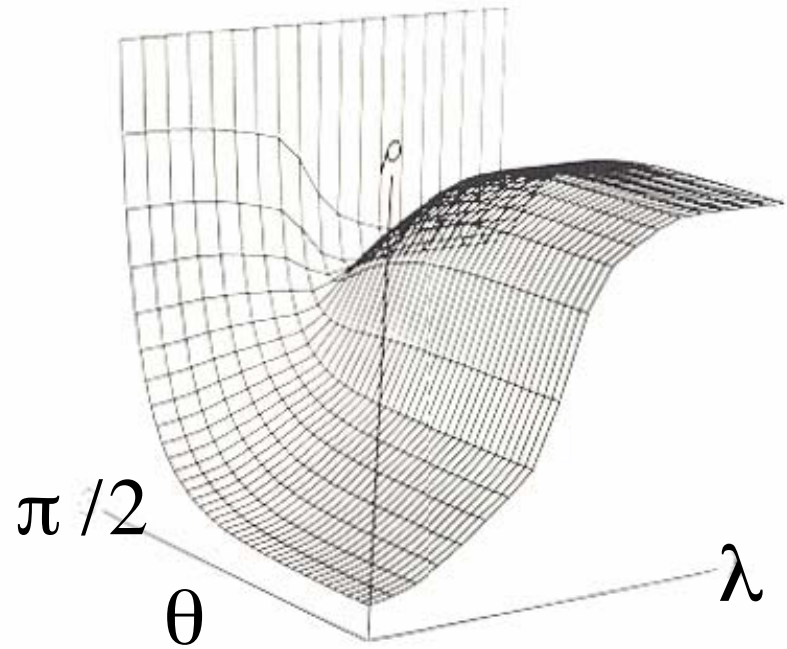


Schlick Approximation:

$$F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$$

# Fresnel Reflection

- Example - Copper
  - color shift as  $\theta$  goes from 0 to  $\pi/2$
  - at grazing, specular highlight is color of light



Measured  
Reflectance



Approximated Reflectance

# Ideal Specular - Summary

- Reflection:

$$f_r(p, \omega_i, \omega_o) = F_r(\omega_i) \frac{\delta(\omega_i - R(\omega_o, N))}{|\cos\theta_i|}$$

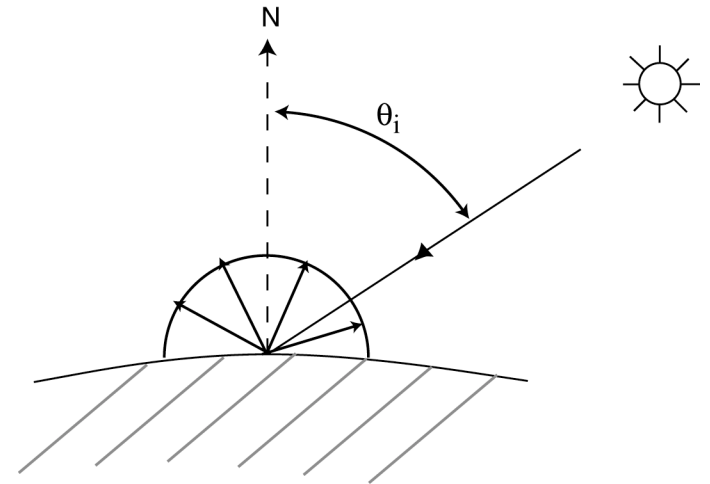
- Transmission:

$$f_t(p, \omega_i, \omega_o) = \frac{\eta_o^2}{\eta_i^2} (1 - F_r(\omega_i)) \frac{\delta(\omega_o - T(\omega_i, N))}{|\cos\theta_i|}$$

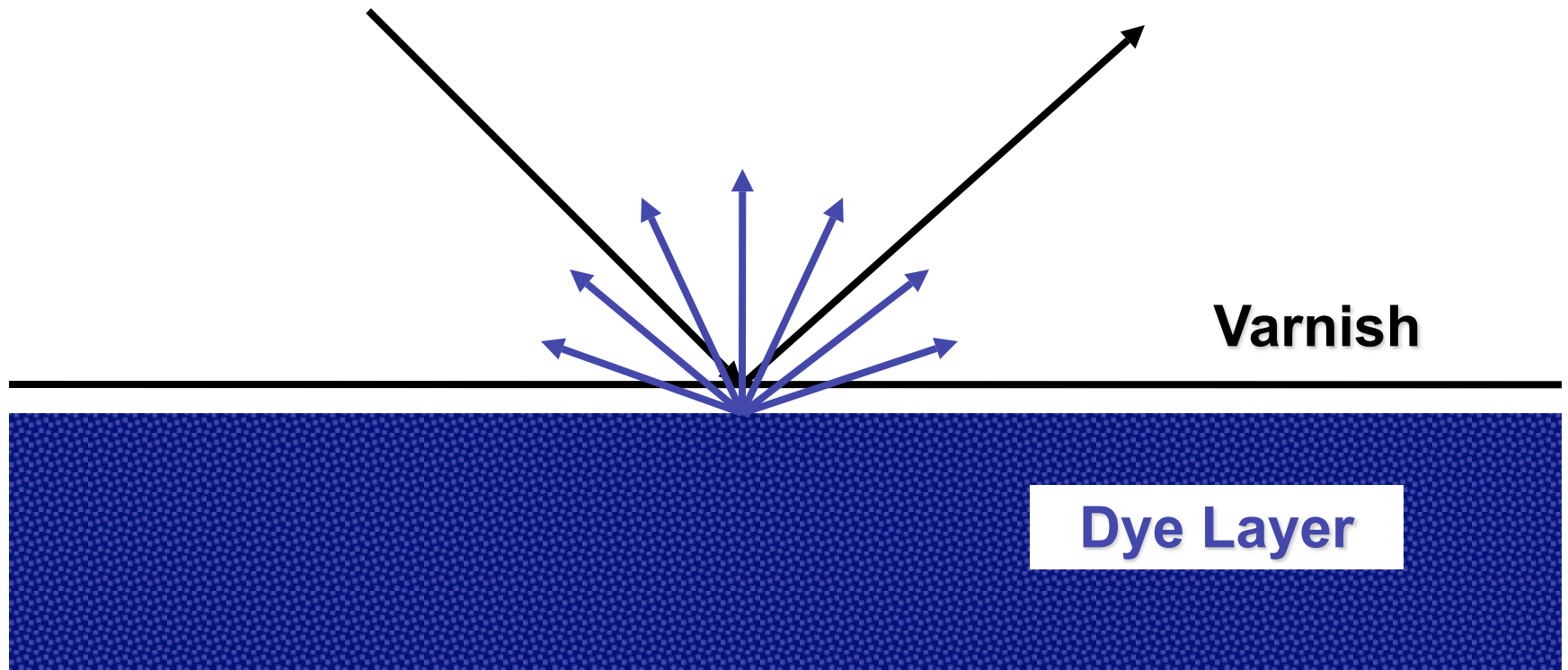
# Ideal Diffuse Reflection

- Uniform
  - Sends equal amounts of light in all directions
  - Amount depends on angle of incidence
- Perfect
  - all incoming light reflected
  - no absorption

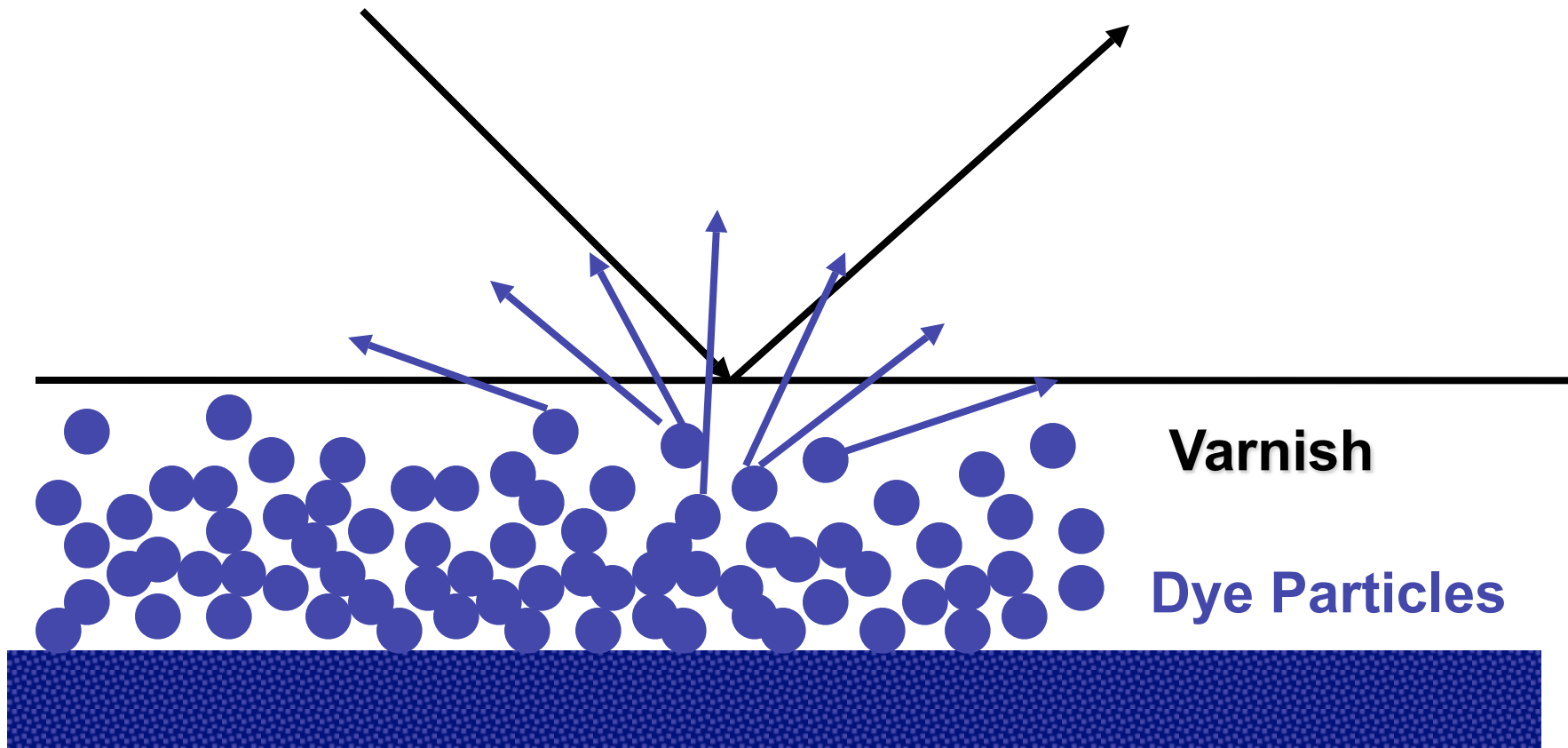
$$f_r(\omega_i, \omega_o) \propto k_d$$



# Layered Surface



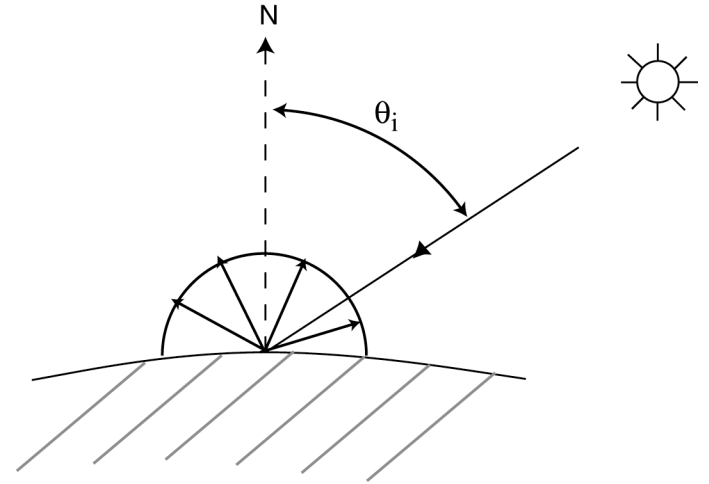
# Layered Surface Larger



# Ideal Diffuse Reflection

$$\begin{aligned}
 L_{o,d}(\omega_o) &= \int_{\Omega} f_{r,d}(\omega_i, \omega_r) L_i(\omega_i) \cos\theta_i d\omega_i \\
 &= f_{r,d} \int_{\Omega} L_i(\omega_i) \cos\theta_i d\omega_i \\
 &= f_{r,d} E
 \end{aligned}$$

$$\begin{aligned}
 M &= \int_{\Omega} L_{o,d}(\omega_o) \cos\theta_o d\omega_o \\
 &= L_{o,d} \int_{\Omega} \cos\theta_o d\omega_o \\
 &= L_{o,d} \pi
 \end{aligned}$$



$$\rho_d = \frac{M}{E} = \frac{L_{o,d} \pi}{E} = \frac{f_{r,d} E \pi}{E} = f_{r,d} \pi$$

$$f_{r,d} = \frac{\rho_d}{\pi}$$

Lamberts Cosine Law:  $M = \rho_d E = \rho_d E_s \cos\theta_s$

# Diffuse

- Helmholtz reciprocity?
- Energy preserving?

$$\rho_d \leq 1$$

$$f_{r,d} = \frac{\rho_d}{\pi} \leq \frac{1}{\pi}$$



# Reflectance Models

- Ideal
  - Diffuse
  - Specular
- Ad-hoc: Phong
  - Classical / Blinn
  - Modified
  - Ward
  - Lafortune
- Microfacets (Physically-based)
  - Torrance-Sparrow (Cook-Torrance)
  - Ashkhimin

# Classical Phong Model

$$L_o(p, \omega_o) = (k_d(N \cdot \omega_i) + k_s(R(\omega_o, N) \cdot \omega_i)^e)L_i(p, \omega_i)$$

- Where  $0 < k_d, k_s < 1$  and  $e > 0$

- Cast as a BRDF:

$$f_r(p, \omega_i, \omega_o) = k_d + k_s \frac{(R(\omega_o, N) \cdot \omega_i)^e}{(N \cdot \omega_i)}$$

- Not reciprocal
- Not energy-preserving
- Specifically, too reflective at glancing angles, but not specular enough
- But cosine lobe itself symmetrical in  $\omega_i$  and  $\omega_o$

# Blinn-Phong

- Like classical Phong, but based on half-way vector

$$f_r(p, \omega_i, \omega_o) = k_d + k_s \frac{(H(\omega_o, \omega_i) \cdot N)^e}{(N \cdot \omega_i)}$$

$$\omega_h = H(\omega_o, \omega_i) = \text{norm}(\omega_o + \omega_i)$$

- Implemented in OpenGL
- Not reciprocal
- Not energy-preserving
- Specifically, too reflective at glancing angles, but not specular enough
- But cosine lobe itself symmetrical in  $\omega_i$  and  $\omega_o$

# Modified Phong

$$f_r(p, \omega_i, \omega_o) = \frac{k_d}{\pi} + \frac{k_s(e+2)}{2\pi} (R(\omega_o, N) \cdot \omega_i)^e$$

- For energy conservation:  $k_d + k_s < 1$   
(sufficient, not necessary)
- Peak gets higher as it gets sharper, but same total reflectivity

# Ward-Phong

- Based on Gaussians

$$f_r(p, \omega_i, \omega_o) = \frac{k_d}{\pi} + \frac{k_s}{\sqrt{\cos\theta_i \cos\theta_o}} \frac{\exp\left(-\frac{\tan^2 \omega_h}{\alpha^2}\right)}{4\pi\alpha^2}$$

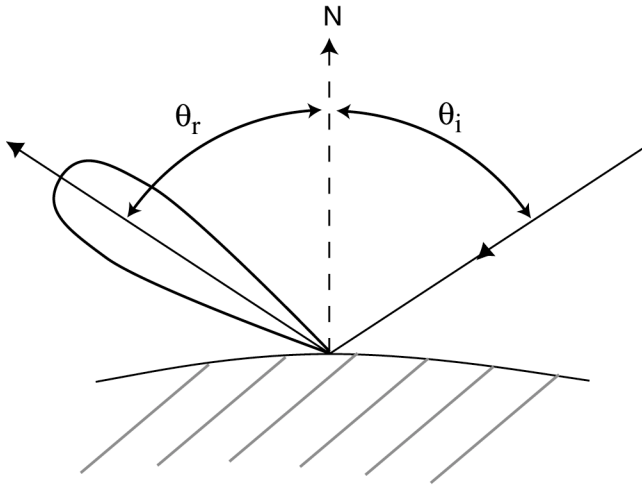
- $\alpha$ : surface roughness, or blur in specular component.

# Lafortune Model

- Phong cosine lobes symmetrical (reciprocal), easy to compute
- Add more lobes in order to match with measured BRDF
- How to generalize to anisotropic BRDFs?
- weight dot product:

$$f_r(p, \omega_i, \omega_o) = \frac{k_d}{\pi} + \sum_{i=1}^{nlobes} (\omega_o R_i \omega_i)^{e_i}$$

# Glossy



# Physically-based Models

- Some basic principles common to many models:
  - Fresnel effect
  - Surface self-shadowing
  - Microfacets
- To really model well how surfaces reflect light, need to eventually move beyond BRDF
- Different physical models required for different kinds of materials
- Some kinds of materials don't have good models
- Remember that BRDF makes approximation of completely local surface reflectance!



# Cook-Torrance Model

- Based in part on the earlier Torrance-Sparrow model
- Neglects multiple scattering

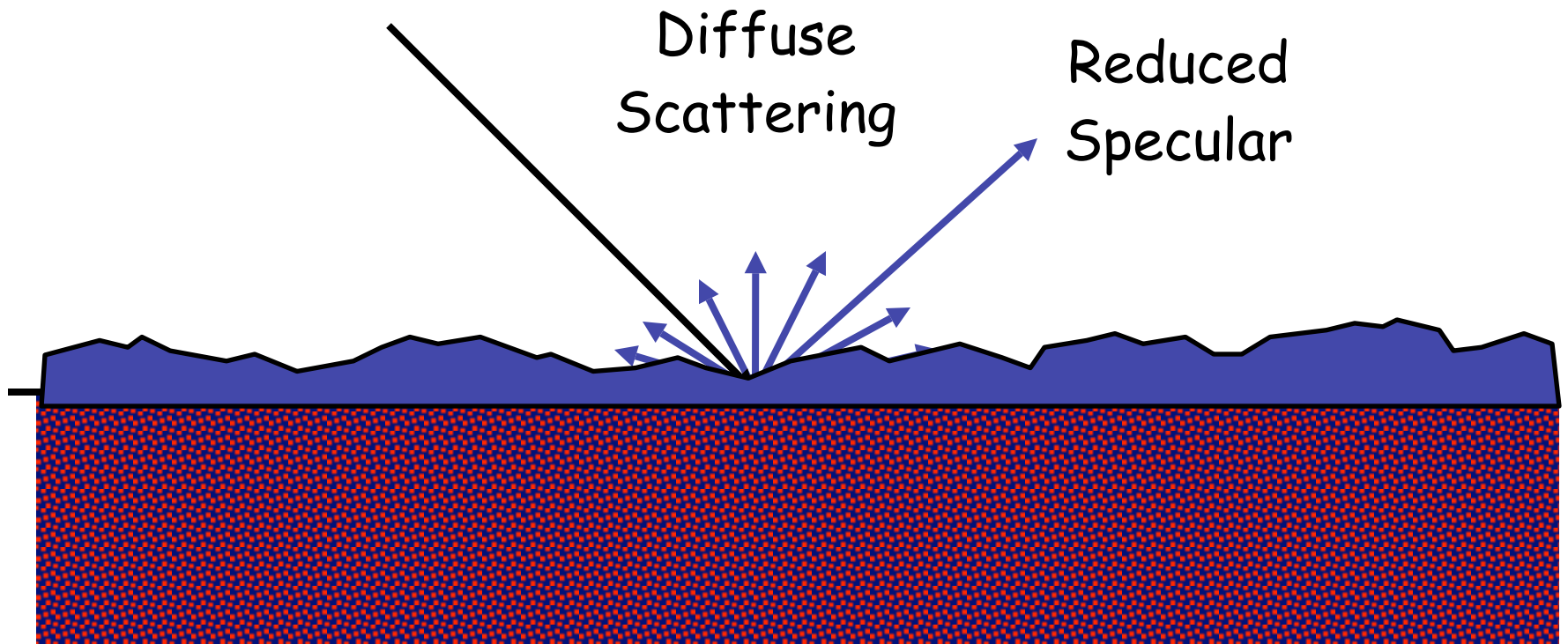
$$f_r(p, \omega_i, \omega_o) = \frac{F_r(\omega_h) D(\omega_h) G(\omega_o, \omega_i)}{4 \cos \theta_i \cos \theta_o}$$

- D - Microfacet Distribution Function
  - how many “cracks” do we have that point in our (viewing) direction?
- G - Geometrical Attenuation Factor
  - light gets obscured by other “bumps”
- F - Fresnel Term

# Microfacet Models

- Microscopically rough surface
- Specular facets oriented randomly
- measure of scattering due to variation in angle of microfacets
- a statistic approximation, I.e. need a statistic distribution function

# Rough Surface



# Microfacet Distribution Function $D$

- Blinn

$$D(\omega_h) = ce^{-\left(\frac{\omega_h \cdot N}{m}\right)^2}$$

- where  $m$  is the root mean square slope of the facets (as an angle)
- Blinn says  $c$  is a arbitrary constant
- Really should be chosen to normalize BRDF. . .

# Microfacet Distribution Function D

- Beckmann (most effective)

$$D(\omega_h) = \frac{1}{m^2 \cos^4 \alpha} e^{-\left(\frac{\tan \alpha}{m}\right)^2}$$

- Represents a distribution of slopes
- But  $\alpha \approx \tan \alpha$  for small  $\alpha$

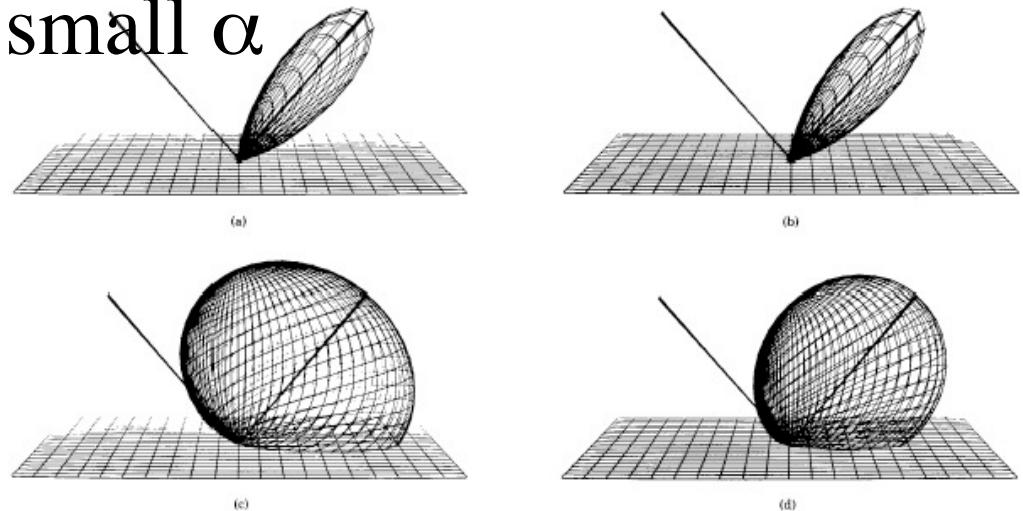


Fig. 3. (a) Beckmann distribution for  $m = 0.2$ , (b) Gaussian distribution for  $m = 0.2$ , (c) Beckmann distribution for  $m = 0.6$ , (d) Gaussian distribution for  $m = 0.6$ .

# Multiscale Distribution Function

- May want to model multiple scales of

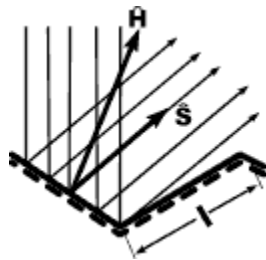
roughness: 
$$D(\omega_h) = \sum_j w_j D_j(\omega_h)$$

$$\sum_j w_j = 1$$

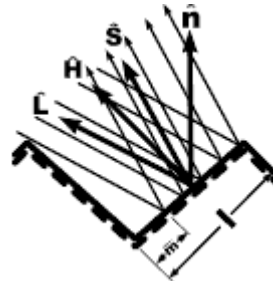
- Bumps on bumps ...

# Self-Shadowing (V-Groove Model)

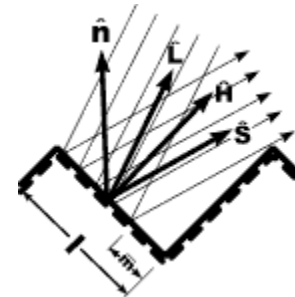
- Geometrical Attenuation Factor  $G$ 
  - how much are the “cracks” obstructing themselves?



No interference



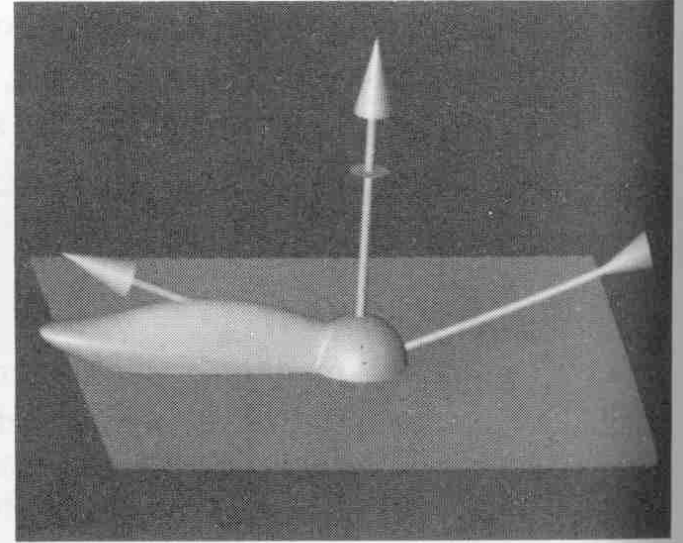
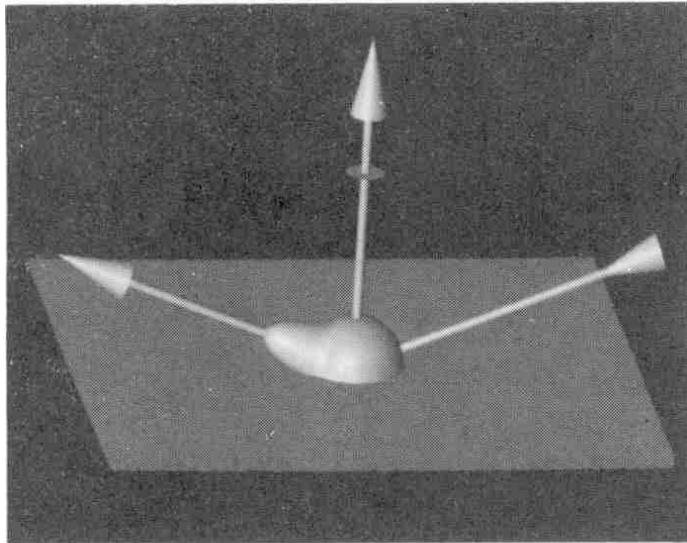
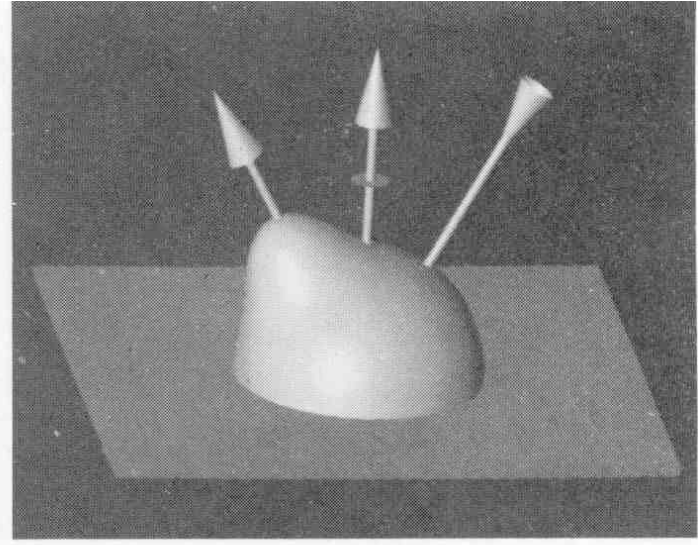
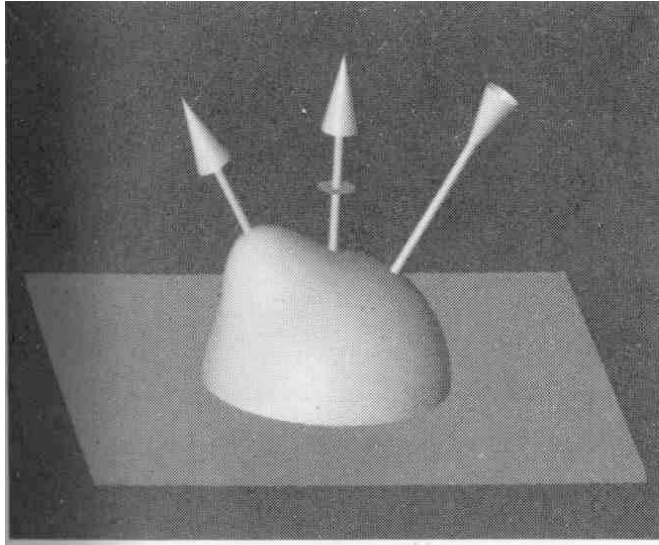
shadowing



masking

$$G = \min \left[ 1, \frac{2(N \cdot \omega_h)(N \cdot \omega_o)}{(\omega_o \cdot \omega_h)}, \frac{2(N \cdot \omega_h)(N \cdot \omega_i)}{(\omega_o \cdot \omega_h)} \right]$$

# Cook-Torrance - Summary



(a) Phong model

(b) Torrance-Sparrow model



# Cook-Torrance - Summary



**Carbon**



**Red  
Rubber**



**Obsidian**



**Lunar  
Dust**



**Olive  
Drab**



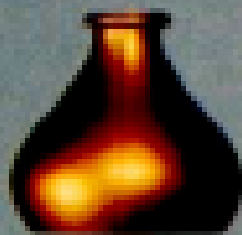
**Rust**



**Bronze**



**Tungsten**



**Copper**



**Tin**



**Nickel**



**Stainless  
Steel**

# Ashkhimin Model

- Modern Phong
- Phenomological, but:
  - Physically plausible
  - Anisotropic
- Good for both Monte-Carlo and HW implementation

# Ashkhimin Model

- Weighted sum of diffuse and specular part:

$$f_r(p, \omega_i, \omega_o) = k_d (1 - k_s) f_d(p, \omega_i, \omega_o) + k_s f_s(p, \omega_i, \omega_o)$$

- Dependence of diffuse weight on  $k_s$  decreases diffuse reflectance when specular reflectance is large
- Specular part  $f_s$  not an impulse, really just glossy
- Diffuse part  $f_d$  not constant: energy specularly reflected cannot be diffusely reflected
- For metals,  $f_d = 0$

# Ashkhimin Model

- $k_s$ : Spectrum or color of specular reflectance at normal incidence.
- $k_d$ : Spectrum or color of diffuse reflectance (away from the specular peak).
- $q_u, q_v$ : Exponents to control shape of specular peak.
  - Similar effects to Blinn-Phong model
  - If an isotropic model is desired, use single value  $q$
  - A larger value gives a sharper peak
  - Anisotropic model requires two tangent vectors  $u$  and  $v$
  - The value  $q_u$  controls sharpness in the direction of  $u$
  - The value  $q_v$  controls sharpness in the direction of  $v$

# Ashkhimin Model

- $\phi$  is the angle between  $u$  and  $\omega_h$

$$D(\omega_h) = \sqrt{(q_u + 1)(q_v + 1)} (\omega_h \cdot N)^{(q_u \cos^2 \phi + q_v \cos^2 \phi)}$$

# Ashkhimin Model

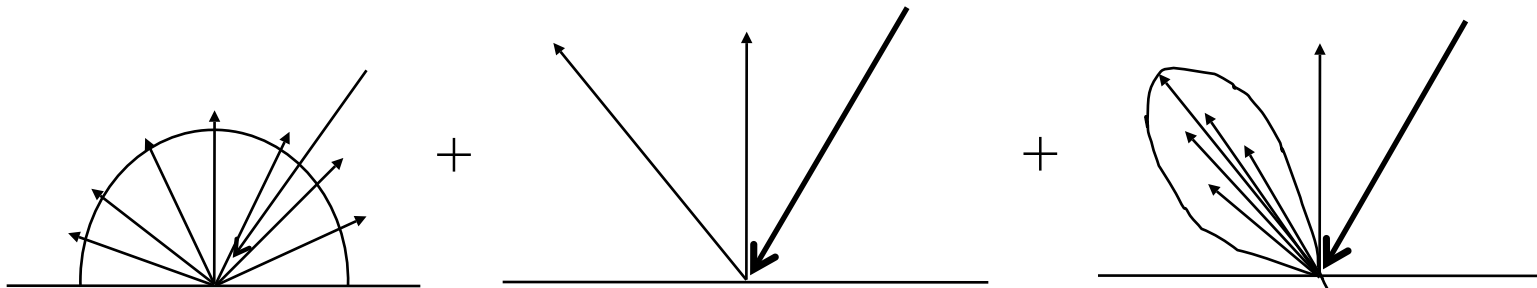
- Diffuse term given by:

$$f_d(p, \omega_i, \omega_o) = \frac{28}{23\pi} (1 - (1 - (\omega_o \cdot N))^5) (1 - (1 - (\omega_i \cdot N))^5)$$

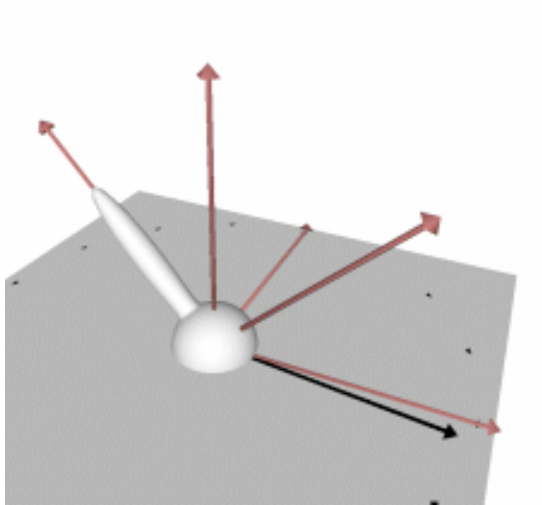
- Leading constant chosen to ensure energy conservation
- Form comes from Schlick approximation to Fresnel factor
- Diffuse reflection due to subsurface scattering: once in, once out

# Complex BRDF

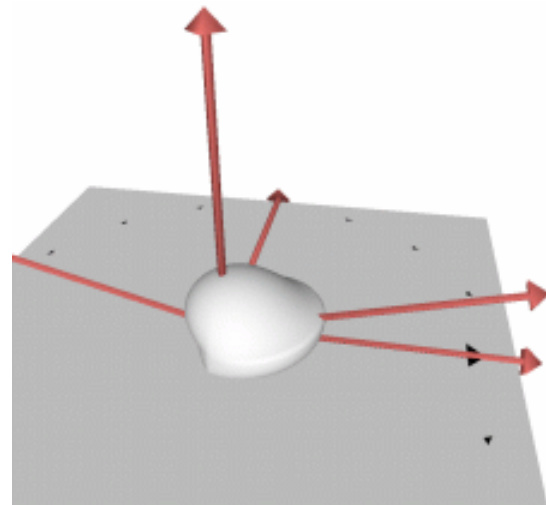
- Combination of the three.



# BRDF illustrations



**Phong  
Illumination**

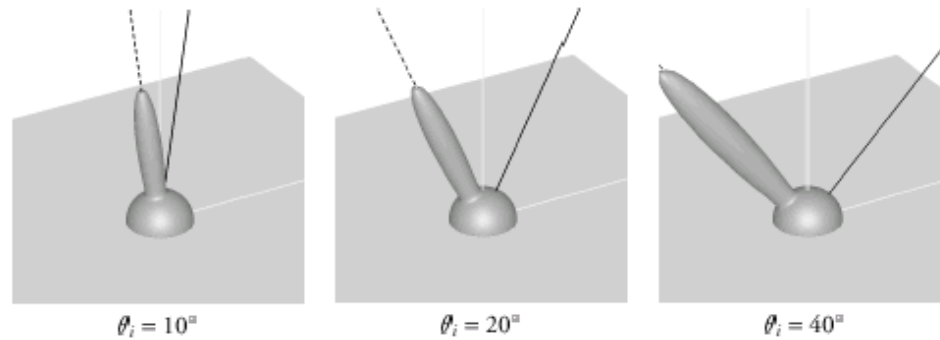


**Oren-Nayar**

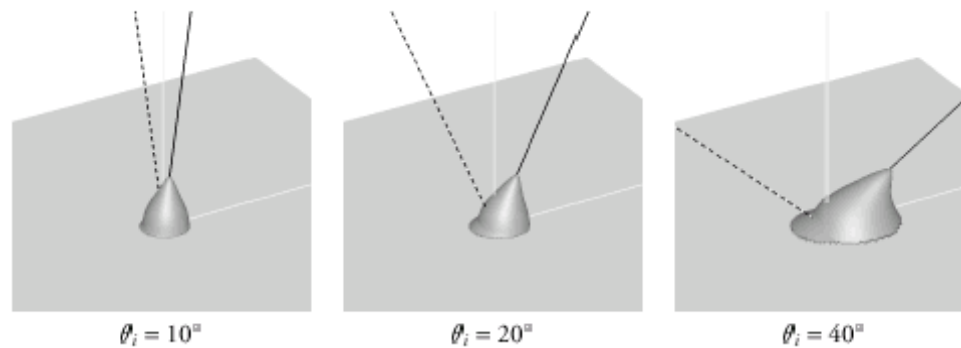


# BRDF illustrations

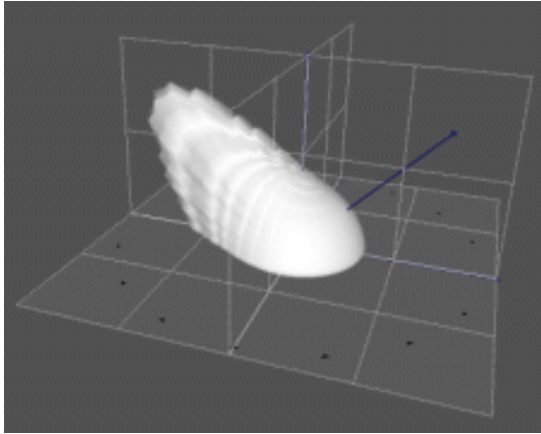
## Cook-Torrance-Sparrow BRDF



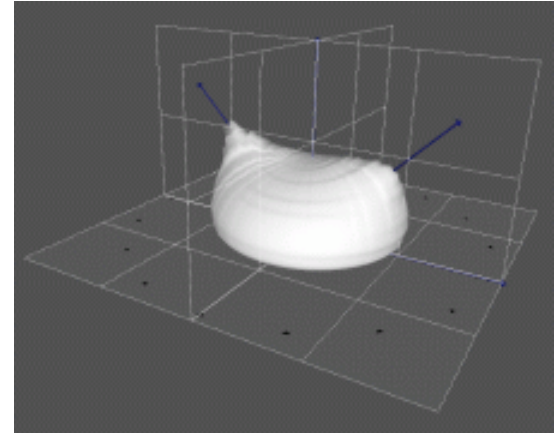
## Hapke BRDF



# BRDF illustrations

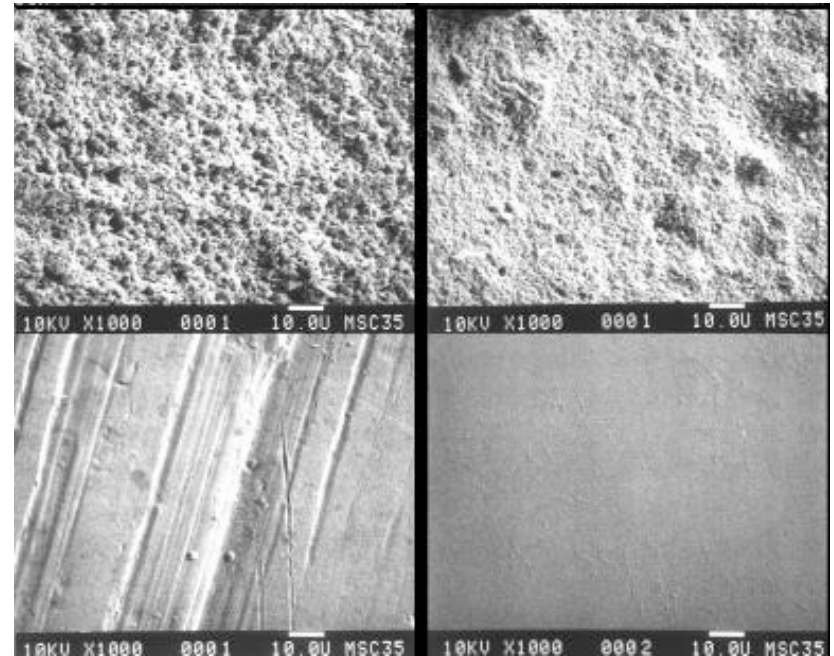
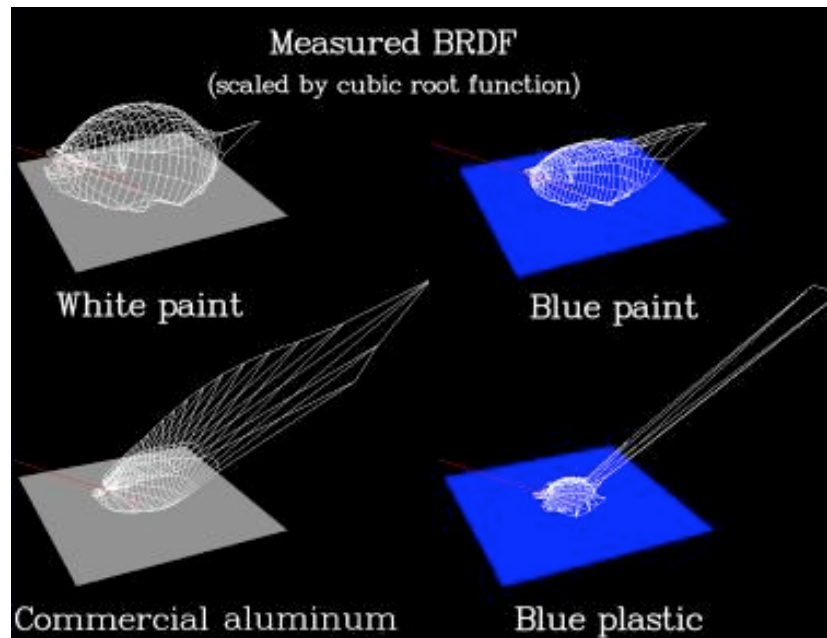


**lumber**



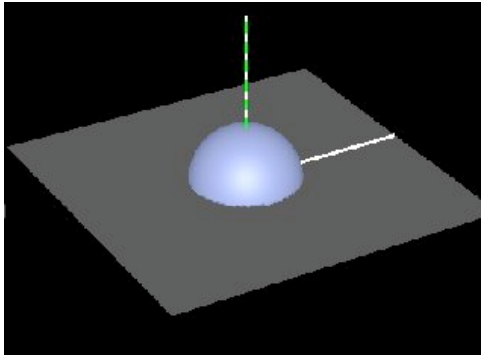
**cement**

# BRDF illustrations

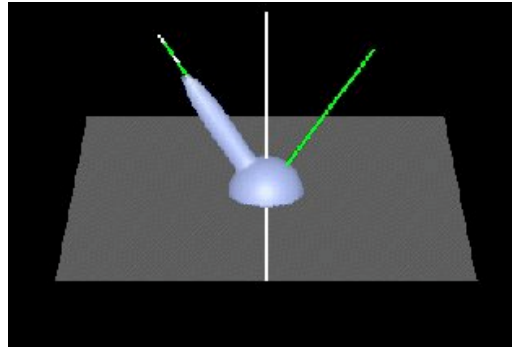


Surface microstructure

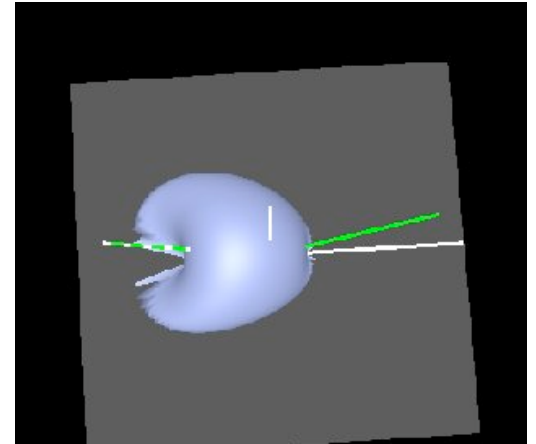
# bv = Brdf Viewer



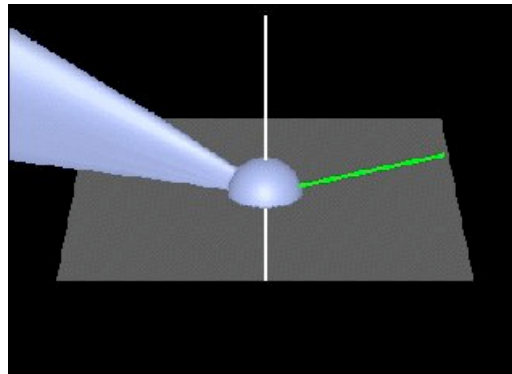
Diffuse



Torrance-Sparrow



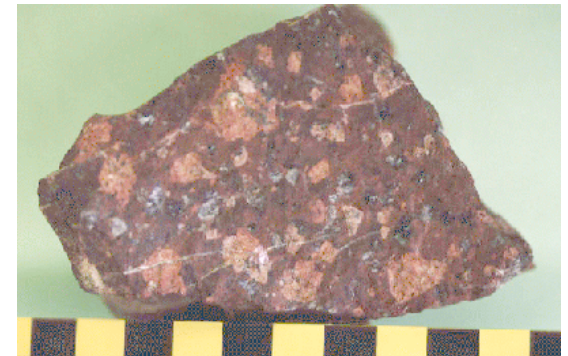
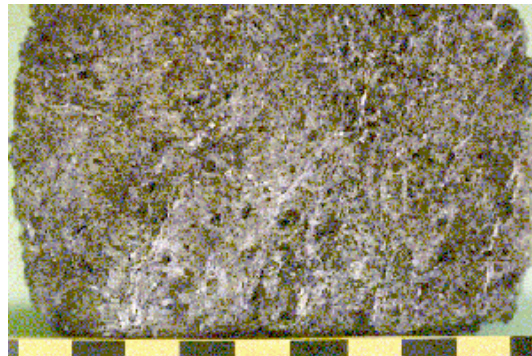
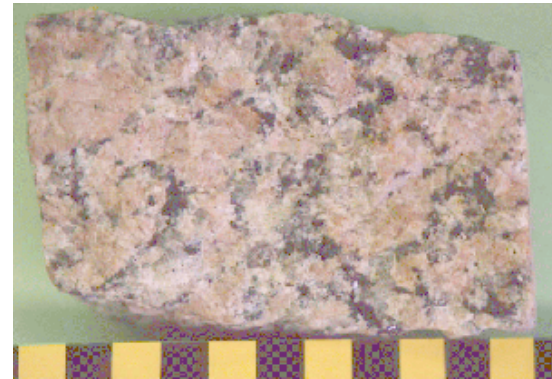
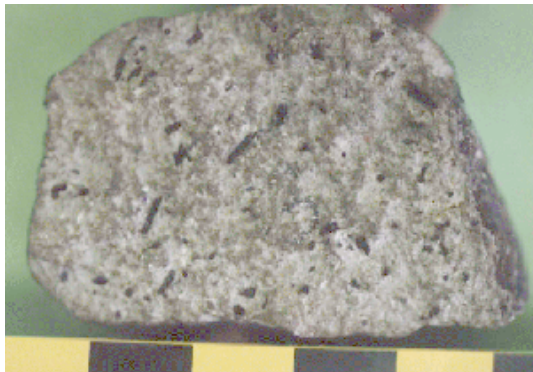
Anisotropic



**Szymon Rusinkiewicz  
Princeton U.**

# BRDF cannot

Spatial variation of reflectance



# BRDF cannot

## Transparency and Translucency (depth)



Glass: transparent  
Wax: translucent  
BTDF



Opaque milk  
(rendered)



Translucent milk  
(rendered)

BSSRDF