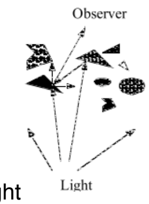
Fundametals of Rendering - Radiometry / Photometry

"Physically Based Rendering"
by Pharr & Humphreys

- Chapter 5: Color and Radiometry
- Chapter 6: Camera Models - we won't cover this in class

Realistic Rendering

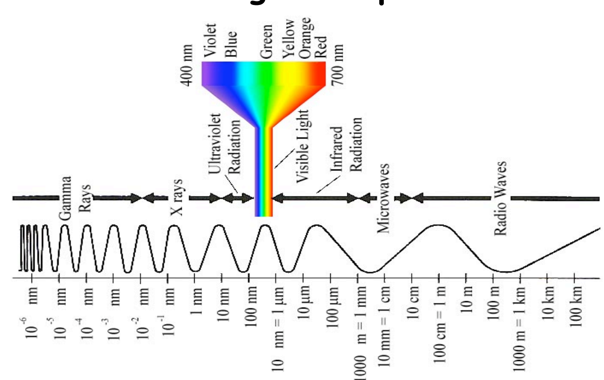
- Determination of Intensity
- Mechanisms
  - Emittance (+)
  - Absorption (-)
  - Scattering (+) (single vs. multiple)
- Cameras or retinas record quantity of light



Pertinent Questions

- Nature of light and how it is:
  - Measured
  - Characterized / recorded
- (local) reflection of light
- (global) spatial distribution of light

Electromagnetic spectrum



Spectral Power Distributions

e.g., Fluorescent Lamps

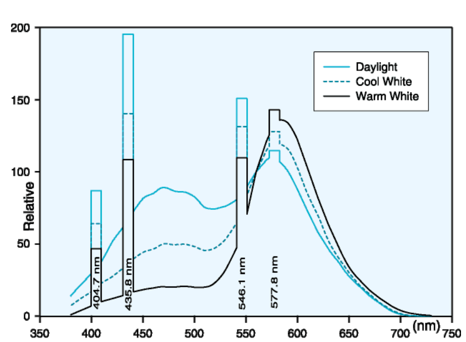
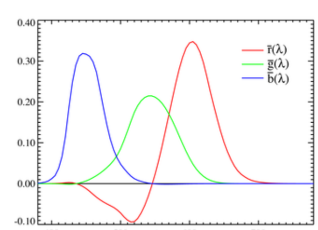


Fig. 5.6 Typical fluorescent sources.

Tristimulus Theory of Color

Metamers: SPDs that appear the same visually



Color matching functions of standard human observer International Commision on Illumination, or CIE, of 1931

"These color matching functions are the amounts of three standard monochromatic primaries needed to match the monochromatic test primary at the wavelength shown on the horizontal scale." from Wikipedia "CIE 1931 Color Space"

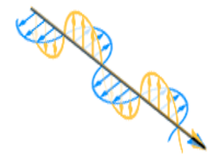
Optics

Three views

- **Geometrical or ray**
  - Traditional graphics
  - Reflection, refraction
  - Optical system design
- **Physical or wave**
  - Dispersion, interference
  - Interaction of objects of size comparable to wavelength
- **Quantum or photon optics**
  - Interaction of light with atoms and molecules

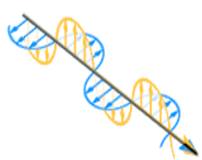
What Is Light ?

- Light - particle model (Newton)
  - Light travels in straight lines
  - Light can travel through a vacuum
    (waves need a medium to travel in)
  - Quantum amount of energy
- Light – wave model (Huygens): electromagnetic radiation: sinusiodal wave formed coupled electric (E) and magnetic (H) fields



Nature of Light

- **Wave-particle duality**
  - Light has some wave properties: frequency, phase, orientation
  - Light has some quantum particle properties: quantum packets (photons).
- **Dimensions of light**
  - Amplitude or Intensity
  - Frequency
  - Phase
  - Polarization



Nature of Light

- **Coherence** - Refers to frequencies of waves
- Laser light waves have uniform frequency
- Natural light is incoherent- waves are multiple frequencies, and random in phase.

- **Polarization** - Refers to orientation of waves.
  - Polarized light waves have uniform orientation
  - Natural light is unpolarized - it has many waves summed all with random orientation
  - Focused feflected light tends to be parallel to surface of reflection

Radiometry

- Science of measuring energy (light in our case)
- Analogous science called **photometry** is based on human perception.

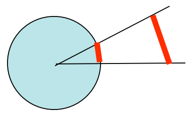
Radiometry Questions

- Measure energy leaving a light source, as a function of direction
- Measure energy hitting a surface, in a particular direction
- Measure energy leaving a surface, in a particular direction

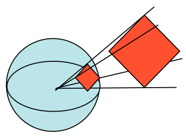
The energy is light, photons in this case

Solid Angle

First, need to define 3D angular units



2D
full circle - 4π radians



3D
full sphere - 4πsteradians

Terminology

Energy	Radiant energy
Power	Energy per unit of time

Power on a surface

Flux	Radiant power passing through a surface
Flux density	Radiant power per unit area on a surface
Incident flux density	Flux density arriving at a surface in all directions
Exitant flux density (aka Radiosity)	Flux density leaving from a surface in all directions

Power in a direction

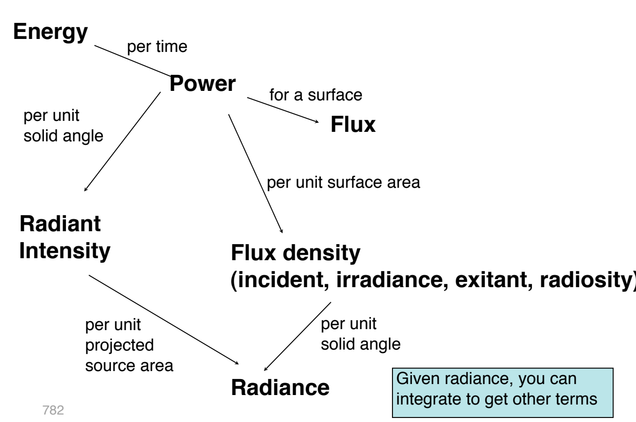
Radiant intensity	Power per unit solid angle
Radiance	Power radiated per unit solid angle per unit projected source area

for rendering:
important when considering the direction toward the camera

Radiometry - Quantities

- **Energy** Q
- **Power** Φ
  - Energy per time
- **Irradiance** E and **Radiosity** B
  - Power per area
- **Intensity** I
  - Power per solid angle
- **Radiance** L
  - Power per projected area and solid angle

Terms & Units

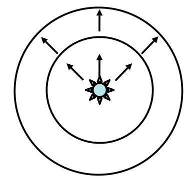


Radiant Energy - Q

- Think of photon as carrying quantum of energy
- Wave packets
- Total energy, Q, is then energy of the total number of photons
- Units - joules or eV

Power - Φ

- Flow of energy (important for transport)
- Also - radiant power or flux.
- Energy per unit time (joules/s = eV/s)
- Unit: W - watts
- $\Phi = dQ/dt$
- Falls off with square of distance

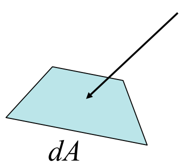


Radiant Flux Area Density

or simply flux density

- Area density of flux (W/m^2)
- u = Energy arriving/leaving a surface per unit time interval
- dA can be any 2D surface in space
- E.g. sphere: $u = \dfrac{\Phi}{4\pi r^2}$

$$u = \frac{d\Phi}{dA}$$



Irradiance E

- Power per unit area incident on a surface

$$E = \frac{d\Phi}{dA}$$

Radiosity or Radiant Exitance B

- Power per unit area leaving surface
- Also known as Radiosity
- Same unit as irradiance, just direction changes

$$B = \frac{d\Phi}{dA}$$

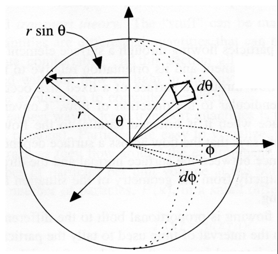
Intensity I

- Flux density per unit solid angle $I = \dfrac{d\Phi}{d\omega}$
- Units – watts per steradian
- Radiant intensity
- “intensity” is heavily overloaded. Why?
  - Power of light source
  - Perceived brightness

Solid Angle

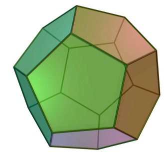
- Size of a patch, dA, in terms of its angular direction, is $dA = (r \sin\theta d\phi)(r d\theta)$
- Solid angle is

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$



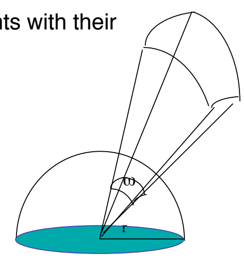
Solid Angle (contd.)

- Solid angle generalizes angle!
- Steradian
- Sphere has 4π steradians! Why?
- Dodecahedron – 12 faces, each pentagon.
- One steradian approx equal to solid angle subtended by a single face of dodecahedron



Hemispherical Projection

- Use a hemisphere H over surface to measure incoming/outgoing flux
- Replace objects and points with their hemispherical projection



ω

Isotropic Point Source

$$I = \frac{d\Phi}{d\omega} = \frac{\Phi}{4\pi}$$

- Even distribution over sphere

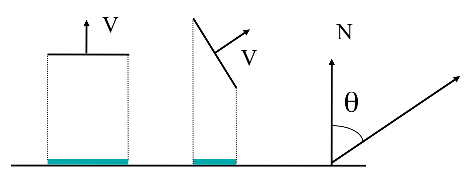
Radiance L

- Power per unit projected area per unit solid angle.
- Units – watts per (steradian m^2)
- We have now introduced projected area, a cosine term.

$$L = \frac{d^2\Phi}{dA_p d\omega} \qquad L = \frac{d^2\Phi}{dA\cos\theta d\omega}$$

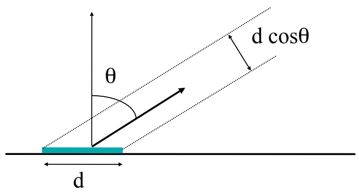
Projected Area

$$A_p = A(N \cdot V) = A\cos\theta$$



Why the Cosine Term?

- Foreshortening is by cosine of angle.
- Radiance gives energy by effective surface area.

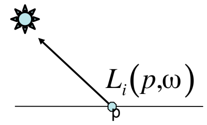


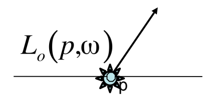
Incident and Exitant Radiance

- Incident Radiance: $L_i(p, \omega)$
- Exitant Radiance: $L_o(p, \omega)$
- In general: $L_i(p,\omega) \neq L_o(p,\omega)$
- p - no surface, no participating media

Note that direction is always **away** from point

$$L_i(p,\omega) = L_o(p,-\omega)$$

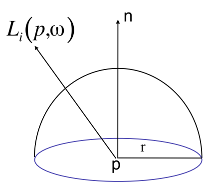




Irradiance from Radiance

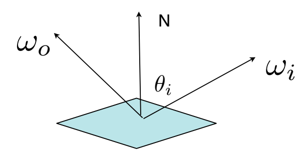
$$E(p,n) = \int_{\Omega} L_i(p,\omega)|\cos\theta|d\omega$$

- $|\cos\theta|d\omega$ is projection of a differential area
- We take $|\cos\theta|$ in order to integrate over the whole sphere



Reflected Radiance & BRDFs

$$dL_o(p,\omega_o) \propto dE(p,\omega_i)$$



$$f_r(p,\omega_o,\omega_i) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i)\cos\theta_i d\omega_i}$$

Bidirectional Reflection Distribution Functions

Reciprocity: $f_r(p,\omega_i,\omega_o) = f_r(p,\omega_o,\omega_i)$

Energy Conservation:

$$\int_{H^2(n)} f_r(p,\omega_o,\omega\prime)\cos\theta\prime d\omega\prime \leq 1$$

Bidirectional Scattering Distribution Functions

Bidirectional Reflection Distribution Function (BRDF)

$$f_r(p,\omega_o,\omega_i)$$

Bidirectional Transmittance Distribution Function (BTDF)

$$f_t(p,\omega_o,\omega_i)$$

Bidirectional Scattering Distribution Function (BSDF)

$$f(p,\omega_o,\omega_i)$$

Bidirectional Scattering Distribution Functions

$$dL_o(p, \omega_o) = f(p, \omega_o, \omega_i) L_i(p, \omega_i) |\cos\theta_i| d\omega_i$$

$$L_o(p, \omega_o) = \int_{S^2(n)} f(p, \omega_o, \omega_i) L_i(p, \omega_i) |\cos\theta_i| d\omega_i$$