Fundamentals of Rendering - Radiometry / Photometry

“Physically Based Rendering”
by Pharr & Humphreys

- Chapter 5: Color and Radiometry
- Chapter 6: Camera Models - we won’t cover this in class
Realistic Rendering

- Determination of Intensity
- Mechanisms
  - Emittance (+)
  - Absorption (-)
  - Scattering (+) (single vs. multiple)
- Cameras or retinas record quantity of light
Pertinent Questions

• Nature of light and how it is:
  – Measured
  – Characterized / recorded

• (local) reflection of light

• (global) spatial distribution of light
Electromagnetic spectrum
Spectral Power Distributions

* e.g., Fluorescent Lamps

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**Fig. 5.6** Typical fluorescent sources.
Tristimulus Theory of Color

Metamers: SPDs that appear the same visually

Color matching functions of standard human observer International Commission on Illumination, or CIE, of 1931

“These color matching functions are the amounts of three standard monochromatic primaries needed to match the monochromatic test primary at the wavelength shown on the horizontal scale.” from Wikipedia “CIE 1931 Color Space”
Optics

Three views

• **Geometrical or ray**
  - Traditional graphics
  - Reflection, refraction
  - Optical system design

• **Physical or wave**
  - Dispersion, interference
  - Interaction of objects of size comparable to wavelength

• **Quantum or photon optics**
  - Interaction of light with atoms and molecules
What Is Light?

- Light - particle model (Newton)
  - Light travels in straight lines
  - Light can travel through a vacuum
    (waves need a medium to travel in)
  - Quantum amount of energy

- Light – wave model (Huygens): electromagnetic radiation: sinusiodal wave formed coupled electric (E) and magnetic (H) fields
Nature of Light

• **Wave-particle duality**
  - Light has some wave properties: frequency, phase, orientation
  - Light has some quantum particle properties: quantum packets (photons).

• **Dimensions of light**
  - Amplitude or Intensity
  - Frequency
  - Phase
  - Polarization
Nature of Light

• **Coherence** - Refers to frequencies of waves
  - Laser light waves have uniform frequency
  - Natural light is incoherent - waves are multiple frequencies, and random in phase.

• **Polarization** - Refers to orientation of waves.
  - Polarized light waves have uniform orientation
  - Natural light is unpolarized - it has many waves summed all with random orientation
  - Focused reflected light tends to be parallel to surface of reflection
Radiometry

- Science of measuring energy (light in our case)
- Analogous science called photometry is based on human perception.
Radiometry Questions

- Measure energy leaving a light source, as a function of direction
- Measure energy hitting a surface, in a particular direction
- Measure energy leaving a surface, in a particular direction

The energy is light, photons in this case
Solid Angle

First, need to define 3D angular units

2D
full circle - $4\pi$ radians

3D
full sphere - $4\pi$ steradians
# Terminology

<table>
<thead>
<tr>
<th>Energy</th>
<th>Radiant energy</th>
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<tbody>
<tr>
<td>Power</td>
<td>Energy per unit of time</td>
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Power on a surface

**Flux**
Radiant power passing through a surface

**Flux density**
Radiant power per unit area on a surface

**Incident flux density**
Flux density arriving at a surface in all directions

**Exitant flux density** (aka Radiosity)
Flux density leaving from a surface in all directions
Power in a direction

Radiant intensity

Power per unit solid angle

Radiance

Power radiated per unit solid angle per unit projected source area

for rendering:

important when considering the direction toward the camera
Radiometry - Quantities

- **Energy** $Q$
- **Power** $\Phi$
  - Energy per time
- **Irradiance** $E$ and **Radiosity** $B$
  - Power per area
- **Intensity** $I$
  - Power per solid angle
- **Radiance** $L$
  - Power per projected area and solid angle
Terms & Units

Energy
- per time

Power
- per unit solid angle
- for a surface

Flux
- per unit surface area

Radiant Intensity
- per unit projected source area

Flux density
- (incident, irradiance, exitant, radiosity)

Radiance
- per unit solid angle

Given radiance, you can integrate to get other terms
Radiant Energy - Q

- Think of photon as carrying quantum of energy
- Wave packets
- Total energy, Q, is then energy of the total number of photons
- Units - joules or eV
**Power - \( \Phi \)**

- Flow of energy (important for transport)
- Also - radiant power or flux.
- Energy per unit time (joules/s = eV/s)
- Unit: W - watts
- \( \Phi = \frac{dQ}{dt} \)
- Falls off with square of distance
Radiant Flux Area Density
or simply flux density

- Area density of flux (W/m$^2$)
- $u = \text{Energy arriving/leaving a surface per unit time interval}$

- $dA$ can be any 2D surface in space

- E.g. sphere:

$$u = \frac{\Phi}{4\pi r^2}$$

$$u = \frac{d\Phi}{dA}$$
Irradiance $E$

- Power per unit area incident on a surface

$$E = \frac{d\Phi}{dA}$$
Radiosity or Radiant Exitance $B$

- Power per unit area leaving surface
- Also known as Radiosity
- Same unit as irradiance, just direction changes

$$B = \frac{d\Phi}{dA}$$
Intensity $I$

- Flux density per unit solid angle \[ I = \frac{d\Phi}{d\omega} \]
- Units – watts per steradian
- Radiant intensity
- “intensity” is heavily overloaded. Why?
  - Power of light source
  - Perceived brightness
Solid Angle

- Size of a patch, dA, in terms of its angular direction, is \( dA = (r \sin \theta d\phi)(rd\theta) \)

- Solid angle is

\[
d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi
\]
Solid Angle (contd.)

- Solid angle generalizes angle!
- Steradian
- Sphere has $4\pi$ steradians! Why?
- Dodecahedron – 12 faces, each pentagon.
- One steradian approx equal to solid angle subtended by a single face of dodecahedron
Hemispherical Projection

- Use a hemisphere H over surface to measure incoming/outgoing flux

- Replace objects and points with their hemispherical projection
Isotropic Point Source

\[ I = \frac{d\Phi}{d\omega} = \frac{\Phi}{4\pi} \]

- Even distribution over sphere
Radiance $L$

- Power per unit projected area per unit solid angle.
- Units – watts per (steradian m\(^2\))
- We have now introduced projected area, a cosine term.

\[
L = \frac{d^2 \Phi}{dA_p \, d\omega}
\]

\[
L = \frac{d^2 \Phi}{dA \, \cos \theta \, d\omega}
\]
Projected Area

\[ A_p = A(N \cdot V) = A \cos \theta \]
Why the Cosine Term?

- Foreshortening is by cosine of angle.
- Radiance gives energy by effective surface area.

![Diagram showing cosine relationship and foreshortening]
Incident and Exitant Radiance

- Incident Radiance: $L_i(p, \omega)$
- Exitant Radiance: $L_o(p, \omega)$

- In general: $L_i(p, \omega) \neq L_o(p, \omega)$
- $p$ - no surface, no participating media

\[ L_i(p, \omega) = L_o(p, -\omega) \]

Note that direction is always away from point.
Irradiance from Radiance

\[ E(p, n) = \int_{\Omega} L_{i}(p, \omega) |\cos \theta| d\omega \]

- $|\cos \theta| d\omega$ is projection of a differential area
- We take $|\cos \theta|$ in order to integrate over the whole sphere
Reflected Radiance & BRDFs

\[ dL_o(p, \omega_o) \propto dE(p, \omega_i) \]

\[ f_r(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i) \cos \theta_i d\omega_i} \]
Bidirectional Reflection Distribution Functions

Reciprocity: \( f_r(p, \omega_i, \omega_o) = f_r(p, \omega_o, \omega_i) \)

Energy Conservation:

\[
\int_{H^2(n)} f_r(p, \omega_o, \omega_l) \cos \theta d\omega_l \leq 1
\]
Bidirectional Scattering Distribution Functions

Bidirectional Reflection Distribution Function (BRDF)

\[ f_r(p, \omega_o, \omega_i) \]

Bidirectional Transmittance Distribution Function (BTDF)

\[ f_t(p, \omega_o, \omega_i) \]

Bidirectional Scattering Distribution Function (BSDF)

\[ f(p, \omega_o, \omega_i) \]
Bidirectional Scattering Distribution Functions

\[ dL_o(p, \omega_o) = f(p, \omega_o, \omega_i)L_i(p, \omega_i)|\cos \theta_i|d\omega_i \]

\[ L_o(p, \omega_o) = \int_{S^2(n)} f(p, \omega_o, \omega_i)L_i(p, \omega_i)|\cos \theta_i|d\omega_i \]