Object Representation

Implicit forms
\( F(x, y, z) = 0 \)

Testing

Explicit forms
Analytic form \( x = F(y, z) \)

Generating
Parametric form \( (x, y, z) = P(t) \)

Ray-Object Intersection

Implicit forms
\( F(x, y, z) = 0 \)

Ray: \( P(t) = (x, y, z) = source + t*direction = s + t*c \)

Solve for \( t \): \( F(P(t)) = 0 \)

Ray-Sphere Intersection

Implicit form for sphere at origin of radius 1
\[ F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0 \]

Ray: \( P(t) = (x, y, z) = s + tc = (s_x + tc_x, s_y + tc_y, s_z + tc_z) \)

Solve: …
\[
F(P(t)) = (s_x + tc_x)^2 + (s_y + tc_y)^2 + (s_z + tc_z)^2 - 1 = 0
\]
\[
= s_x^2 + s_y^2 + s_z^2 + 2(s_xc_x + s_yc_y + s_zc_z) + t^2(c_x^2 + c_y^2 + c_z^2) - 1 = 0
\]

Use quadratic equation…
Ray-Sphere Intersection

\[ A t^2 + B t + C = 0 \]

- \( A = |c|^2 \)
- \( B = 2s \cdot c \)
- \( C = |s|^2 - 1 \)

\[ t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

- \( B^2 - 4AC < 0 \) => no intersection
- \( = 0 \) => just grazes
- \( > 0 \) => two hits

Axis-Aligned Cuboid
(rectangular solid, rectangular parallelepiped)

Ray equation

\[ P(t) = s + tc \]

Planar equations

Solve for intersections with planes

- \( t_{11} = \frac{(x_1 - s_x)}{c_x} \)
- \( t_{12} = \frac{(x_2 - s_x)}{c_x} \)
- \( t_{13} = \frac{(y_1 - s_y)}{c_y} \)

For each intersection, note whether entering or leaving square side of plane

Ray is inside after last entering and before first leaving

Ray-Plane for arbitrary plane

\[ ax + by + cz + d = 0 \]

\[ n \cdot P = -d \]

\( V_1 \)
\( V_2 \)
\( V_3 \)
\( V_4 \)
Normal Vector

Given ordered sequence of points defining a polygon how do you find a normal vector for the plane?

Note: 2 normal vectors to a plane, colinear and one is the negation of the other

Ordered: e.g., clockwise when viewed from the front of the face

Right hand v. left hand space

\[ \mathbf{n} = (\mathbf{V}_1 - \mathbf{V}_2) \times (\mathbf{V}_3 - \mathbf{V}_2) \]

Ray-Plane

\[ A x + b y + c z + d = 0 \]
\[ \mathbf{n} \cdot \mathbf{P} = -d \]
\[ \mathbf{P} = s + t \mathbf{dir} \]
\[ \mathbf{n} \cdot (s + t \mathbf{dir}) = -d \]
\[ n_1 s + t (n \cdot \mathbf{dir}) = -d \]
\[ t = \frac{-d + n_1 s}{n \cdot \mathbf{dir}} \]
Ray-Polyhedron

Polyhedron - volume bounded by flat faces
Each face is defined by a ring of edges
Each edge is shared by 2 and only 2 faces

The polyhedron can be convex or concave
Faces can be convex or concave

Convex Polyhedron

volume bounded by finite number of infinite planes
Computing intersections is similar to cube but using ray-plane intersection and arbitrary number of planes

Intersection going from outside to inside
Intersection going from inside to outside

Record maximum entering intersection - enterMax
Record minimum leaving intersection - leaveMin
If (enterMax < leaveMin) polyhedron is intersected

Concave Polyhedron

Find closest face (if any) intersected by ray
Need ray-face (ray-polygon) intersection test
Ray-Polyhedron

1. Intersect ray with plane
2. Determine if intersection point is inside of 2D polygon
   A) Convex polygon
   B) Concave polygon

Ray-Convex Polygon

Test to see if point is on ‘inside’ side of each edge

\[ n \cdot (V \times E) > 0 \]

Ray-concave polygon

Project plane and point of intersection to 2D plane
2D point-inside-a-polygon test

2D point inside a polygon test

Project to plane of 2 smallest coordinates of normal vector
Form semi-infinite ray and count ray-edge intersections
2D point inside a polygon test

\[ \text{if}'((y < y_2) \& \& (y \geq y_1)) \text{ and } ((y < y_1) \& \& (y \geq y_2)) \]

Transformed objects

e.g., Ellipse is transformed sphere

Intersect ray with transformed object

Use inverse of object transformation to transform ray

Intersect transformed ray with untransformed object

Transformed objects

World space ray

\[ r(t) = x + tc \]

\[ r = [x_0, y_0, z_0, 1] \]

\[ c = [c_x, c_y, c_z, 0] \]

Object space ray

\[ R(t) = M^{-1}s + M^{-1}c \]

Intersect ray with object in object space

Transform intersection point and normal back to world space

\[ P_{\text{world}} = M_{\text{object}}^{-1}P_{\text{object}} \]

\[ N_{\text{world}} = (M^{-1})^T N_{\text{object}} \]

Ray-Cylinder

At closest points

\[ (P(t) - Q(t)) \cdot \text{dir}_1 = 0 \]

\[ (P(t) - Q(t)) \cdot \text{dir}_2 = 0 \]

\[ Q = s_2 + t_2 \cdot \text{dir}_2 \]

\[ P = s_1 + t_1 \cdot \text{dir}_1 \]
Ray-Cylinder

\[
\begin{align*}
(P(t_1) - Q(t_2)) \cdot \text{dir}_1 &= 0 \\
(P(t_1) - Q(t_2)) \cdot \text{dir}_2 &= 0 \\
(s_1 + t_1 \text{dir}_1 - (s_2 + t_2 \text{dir}_2)) \cdot \text{dir}_1 &= 0 \\
(s_1 + t_1 \text{dir}_1 - (s_2 + t_2 \text{dir}_2)) \cdot \text{dir}_2 &= 0
\end{align*}
\]

Ray-Ellipsoid

Geometric construction: all points \( p \) such that 
|\( p - a \) + |\( p - b \) = \( r \)

\[|P(t) - a| + |P(t) - b| = r\]

Ray-Quadric

\[
P(t) = (x, y, z) = s + tc = (s_x + tc_x, s_y + tc_y, s_z + tc_z)
\]

\[
Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gxx + Hyy + Izz + J = 0
\]

http://en.wikipedia.org/wiki/Quadric

Ray-Whatever

Algebraic solution v. Numeric solution

\[
P(P(t)) = 0
\]