Object Intersection
Object Representation

Implicit forms
\[ F(x,y,z) = 0 \]

Explicit forms
Analytic form \( x = F(y,z) \)
Parametric form \( (x,y,z) = P(t) \)
Ray-Object Intersection

Implicit forms
\[ F(x,y,z) = 0 \]

Ray: \( P(t) = (x,y,z) = \text{source} + t*\text{direction} = s + t*c \)

Solve for \( t \): \( F(P(t)) = 0 \)
Ray-Sphere Intersection

Implicit form for sphere at origin of radius 1

\[ F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0 \]

Ray:

\[ P(t) = (x, y, z) = s + tc = (s_x + tc_x, s_y + tc_y, s_z + tc_z) \]

Solve: …

\[ F(P(t)) = (s_x + tc_x)^2 + (s_y + tc_y)^2 + (s_z + tc_z)^2 - 1 = 0 \]

\[ = s_x^2 + s_y^2 + s_z^2 + 2t(s_xc_x + s_yc_y + s_zc_z) + t^2(c_x^2 + c_y^2 + c_z^2) - 1 = 0 \]

Use quadratic equation…

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Ray-Sphere Intersection

\[ At^2 + Bt + C = 0 \]

A = \(|c|^2\)
B = 2s \cdot c
C = |s|^2 - 1

\[ t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

\[ B^2 - 4AC < 0 \implies \text{no intersection} \]
\[ = 0 \implies \text{just grazes} \]
\[ > 0 \implies \text{two hits} \]
Axis-Aligned Cuboid
(rectangular solid, rectangular parallelepiped)

Ray equation

\[ P(t) = s + tc \]

Planar equations

Solve for intersections with planes

\[ t_{x1} = (x1 - s_x)/c_x \]

\[ t_{x2} = (x2 - s_x)/c_x \]

\[ t_{y1} = (y1 - s_y)/c_y \]

\[ t_{y2} = (y2 - s_y)/c_y \]

\[ t_{z1} = (z1 - s_z)/c_z \]

\[ t_{z2} = (z2 - s_z)/c_z \]

\[ \ldots \]
For each intersection, note whether entering or leaving square side of plane.

Ray is inside after last entering and before first leaving.
Ray-Plane for arbitrary plane

\[ ax + by + cz + d = 0 \]

\[ n \cdot P = -d \]
Normal Vector

Given ordered sequence of points defining a polygon how do you find a normal vector for the plane?

Note: 2 normal vectors to a plane, colinear and one is the negation of the other

Ordered: e.g., clockwise when viewed from the front of the face

Right hand v. left hand space
Normal Vector

\[ n = (V_1 - V_2) \times (V_3 - V_2) \]
Normal Vector

$m_x = \Sigma (y_i - y_{next(i)}) (z_i - z_{next(i)})$

$m_y = \Sigma (z_i - z_{next(i)}) (x_i - x_{next(i)})$

$m_z = \Sigma (x_i - x_{next(i)}) (y_i - y_{next(i)})$
Ray-Plane

Ax + by + cz + d = 0

n · P = -d

P = s + t*dir

n·(s+t*dir) = -d
n·s + t*(n·dir) = -d
\[ t = \frac{-(d + n·s)}{(n·dir)} \]
**Ray-Polyhedron**

**Polyhedron** - volume bounded by flat faces
Each face is defined by a ring of edges
Each edge is shared by 2 and only 2 faces

The polyhedron can be convex or concave

Faces can be convex or concave
Convex Polyhedron

volume bounded by finite number of infinite planes

Computing intersections is similar to cube but using ray-plane intersection and arbitrary number of planes

Intersection going from outside to inside
Intersection going from inside to outside

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Convex Polyhedron

Intersection going from outside to inside
Intersection going from inside to outside

Record maximum entering intersection - enterMax
Record minimum leaving intersection - leaveMin

If (enterMax < leaveMin) polyhedron is intersected
Concave Polyhedron

Find closest face (if any) intersected by ray

Need ray-face (ray-polygon) intersection test
Ray-Polyhedron

1. Intersect ray with plane

2. Determine if intersection point is inside of 2D polygon
   
   A) Convex polygon
   
   B) Concave polygon
Ray-Convex Polygon

Test to see if point is on ‘inside’ side of each edge

\[ n \cdot (V \times E) > 0 \]

Dot product of normal
Cross product of ordered edge
vector from edge source to point of intersection

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Ray-concave polygon

Project plane and point of intersection to 2D plane
2D point-inside-a-polygon test

Project to plane of 2 smallest coordinates of normal vector

Form semi-infinite ray and count ray-edge intersections

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2D point inside a polygon test

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2D point inside a polygon test

\[ \text{if} \left( ((y < y_2) \& \& (y \geq y_1)) \right) \right| \left( ((y < y_1) \& \& (y \geq y_2)) \right) \]
Transformed objects

e.g., Ellipse is transformed sphere

Intersect ray with transformed object

Use inverse of object transformation to transform ray
Intersect transformed ray with untransformed object
**Transformed objects**

World space ray

\[ r(t) = s + tc \]

\[ s = [s_x, s_y, s_z, 1] \]

\[ c = [c_x, c_y, c_z, 0] \]

Object space ray

\[ R(t)^T = M^{-1}s^T + M^{-1}c^T \]

Intersect ray with object in object space

Transform intersection point and normal back to world space

\[ P_{world}^T = MP_{object}^T \]

\[ N_{world}^T = (M^{-1})^T N_{object}^T \]
Ray-Cylinder

\[ P = s_1 + t_1 \times \text{dir}_1 \]

\[ Q = s_2 + t_2 \times \text{dir}_2 \]

At closest points

\[ (P(t_1) - Q(t_2)) \times \text{dir}_1 = 0 \]

\[ (P(t_1) - Q(t_2)) \times \text{dir}_2 = 0 \]
Ray-Cylinder

\[(P(t_1) - Q(t_2)) \cdot dir_1 = 0\]
\[(P(t_1) - Q(t_2)) \cdot dir_2 = 0\]

\[(s_1 + t_1 dir_1 - (s_2 + t_2 dir_2)) \cdot dir_1 = 0\]
\[(s_1 + t_1 dir_1 - (s_2 + t_2 dir_2)) \cdot dir_2 = 0\]
Ray-Ellipsoid

Geometric construction: all points \( p \) such that \( |p-a| + |p-b| = r \)

\[ |P(t)-a| + |P(t)- b| = r \]
Ray-Quadric

\[ P(t) = (x, y, z) = s + tc = (s_x + tc_x, s_y + tc_y, s_z + tc_z) \]

\[ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 \]

http://en.wikipedia.org/wiki/Quadric

Ray-Whatever

Algebraic solution v. Numeric solution

\[ F(P(t)) = 0 \]