Ray Tracing Implicit Surfaces

Overview

- Similar to **CSG**
  - Combine primitive objects to form complex object
- Primitives are "**density fields**"
- Combine by **summing** densities
- The surface is all points at which the density equals a user-defined **threshold**

Implicit Surface

- A surface not explicitly represented
- The surface consists of all points which satisfy a function

  $F(x,y,z) = 0$
- Usually, the implicit function is defined so that

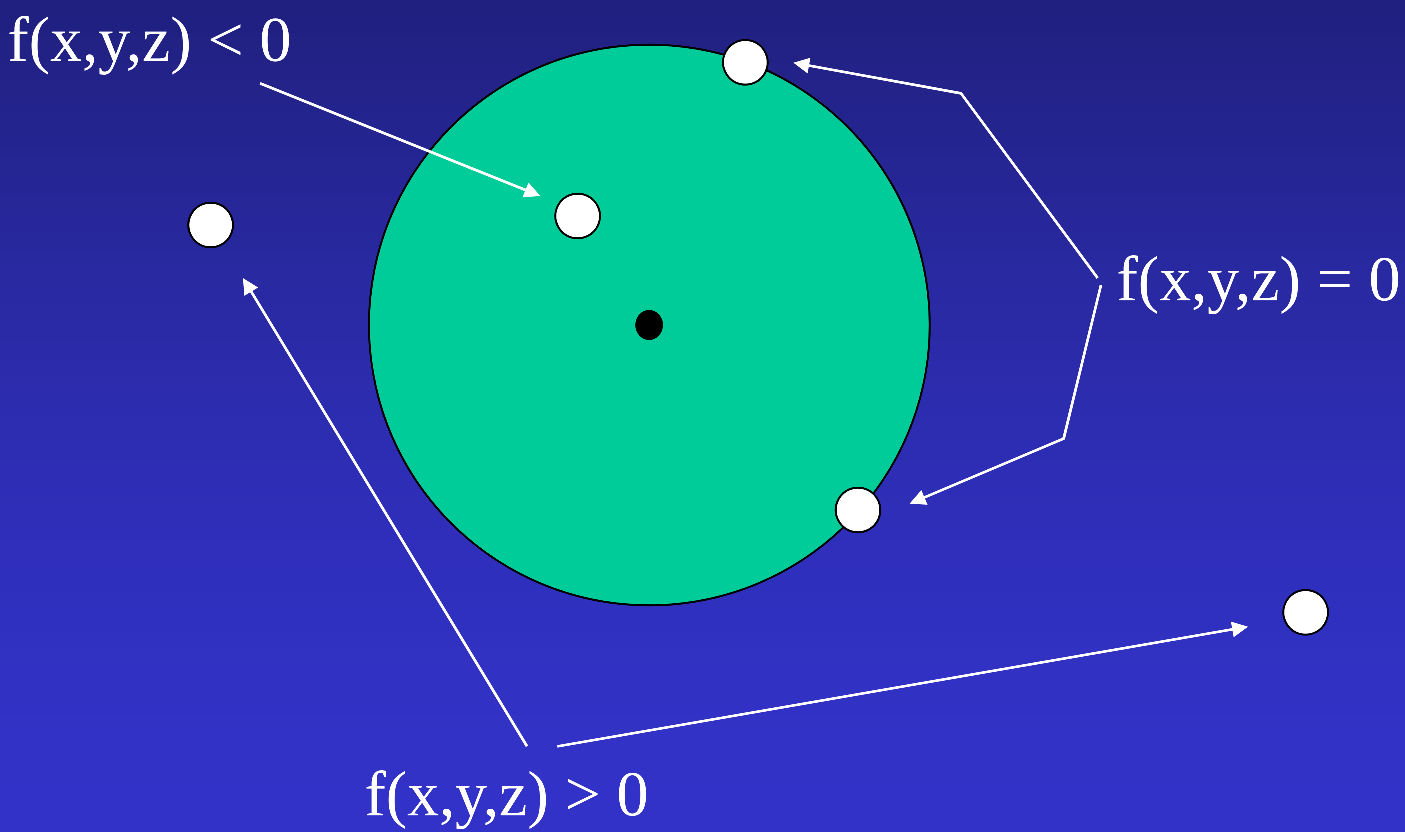
  $F(x,y,z) < 0$ => inside the object

  $F(x,y,z) > 0$ => outside the surface

  Sometimes $F(x,y,z)$ is based on a distance-to-a-central-element
- <u>The surface points have to be searched for!</u>

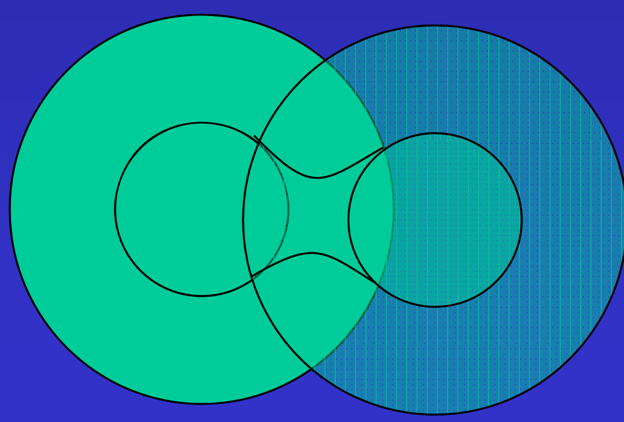
For example: single metaball

$$f(x,y,z) = x^2+y^2+z^2-r^2$$



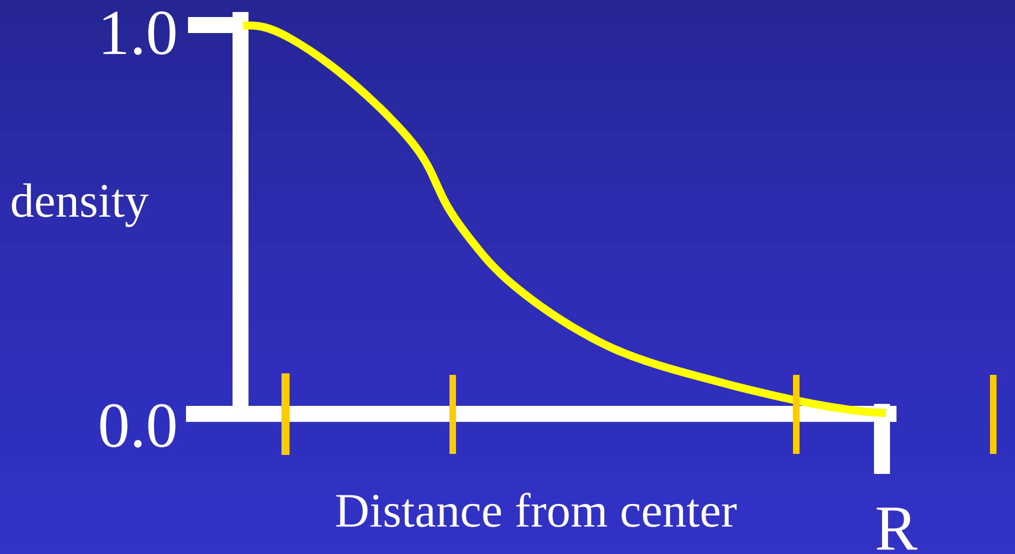
Multiple Implicits

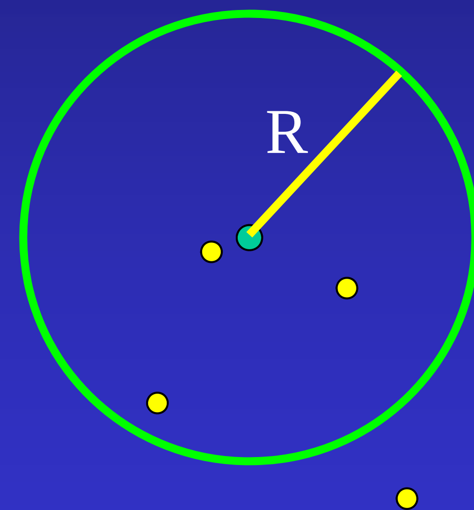
- Define each primitive as positive density field
- Sum densities
- Surface is defined at threshold
- Usually have finite radius of influence



Organic shapes

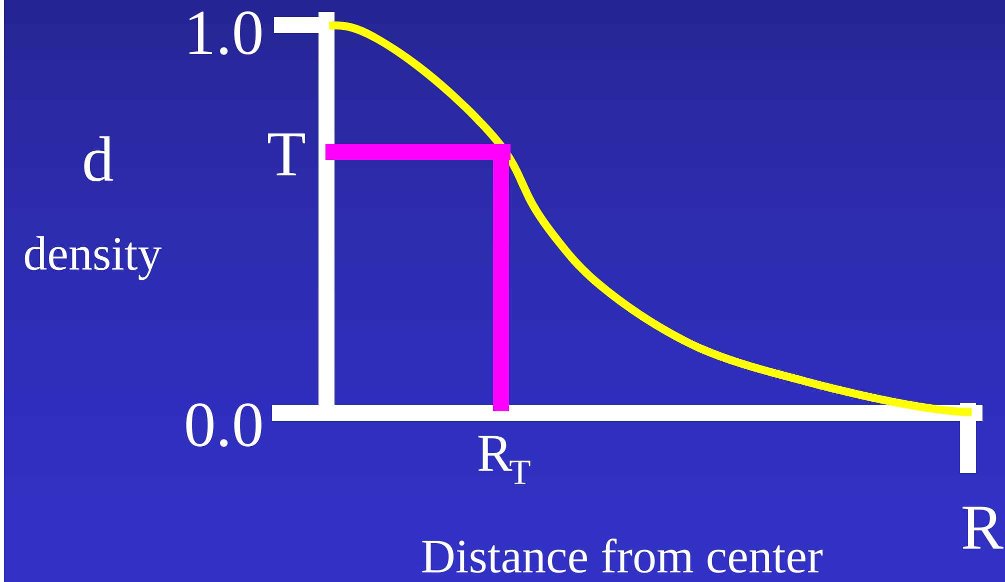
Density Function

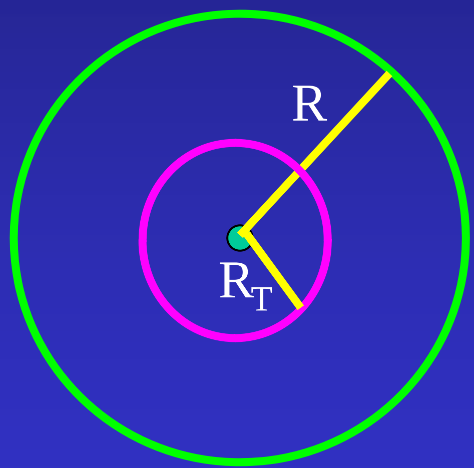




Threshold

- Define threshold that defines density of surface
- R_T is the radius of the isosurface (blob) in isolation

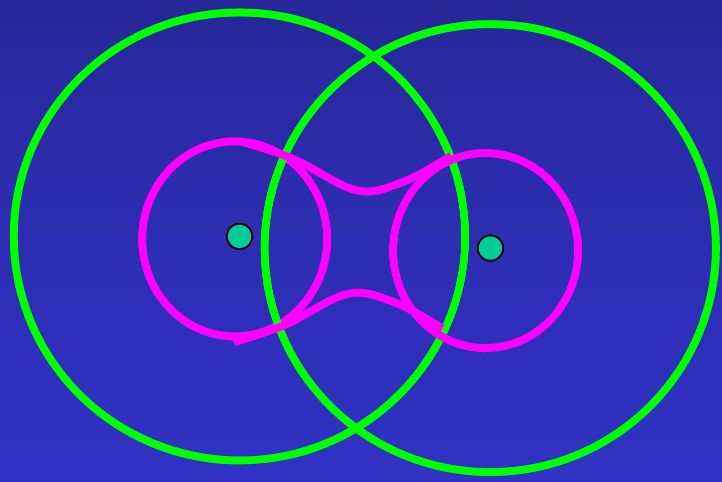




$$\{ p \mid d(p) - T = 0 \}$$

Blended Blobs

- Define surface as sum of densities



$$\left\{ p \middle| \sum d_i(p) - T = 0 \right\}$$

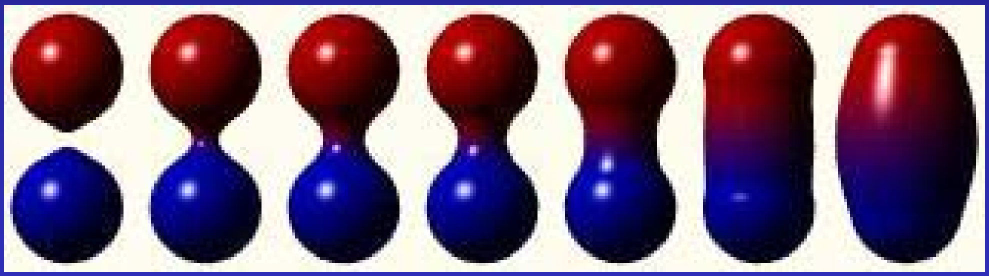
$$\left\{ p \middle| \sum w_i d_i(p) - T = 0 \right\}$$

Weighted Density Functions

- Define surface as <u>weighted sum of densities</u>

$$f(p) = \Sigma\, w_i\, d_i\, (p) - T = 0$$

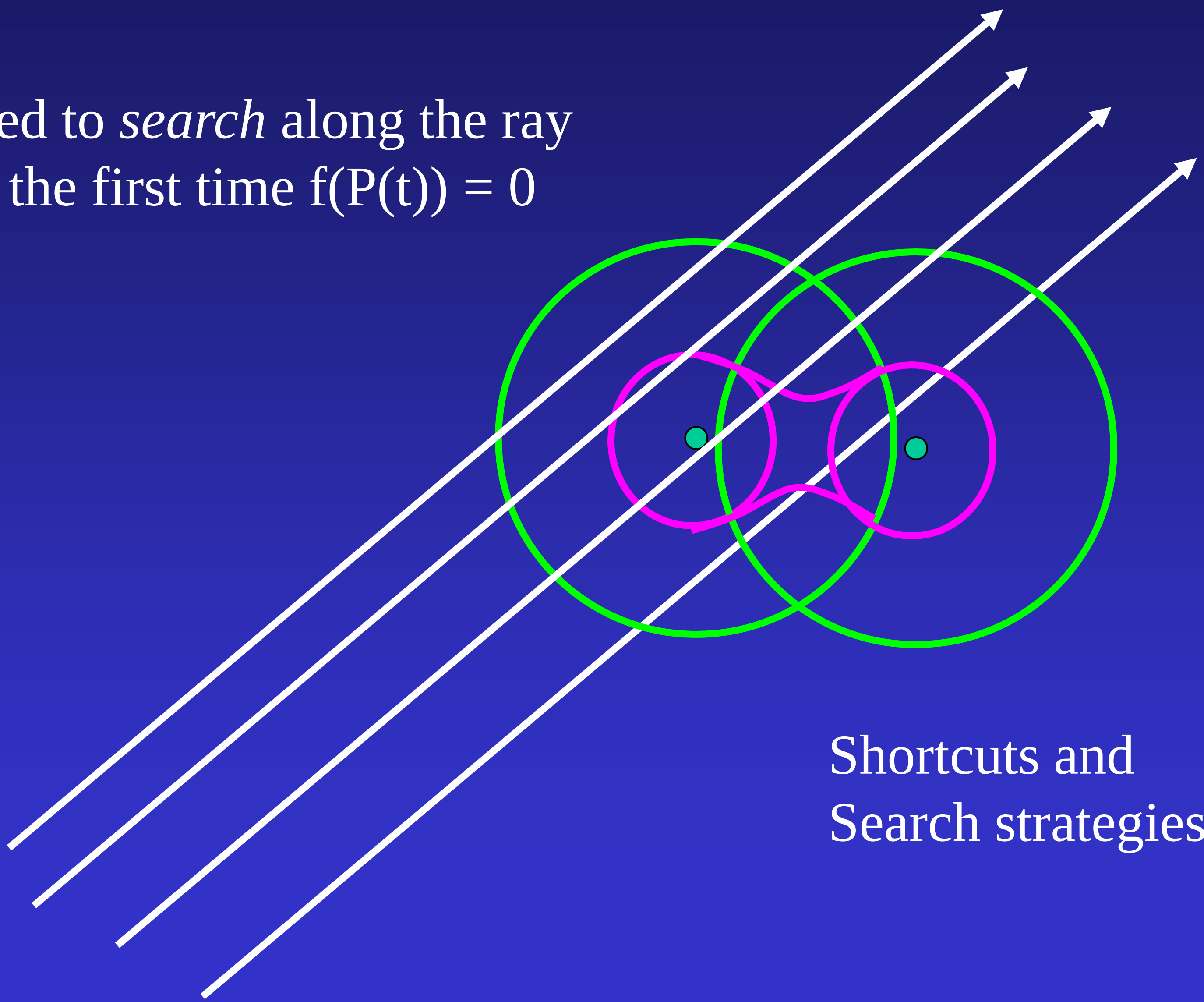
To keep the same radius, but increase blending, change weight, w_i, and the threshold, T, simultaneously.



Weights can be negative, too!

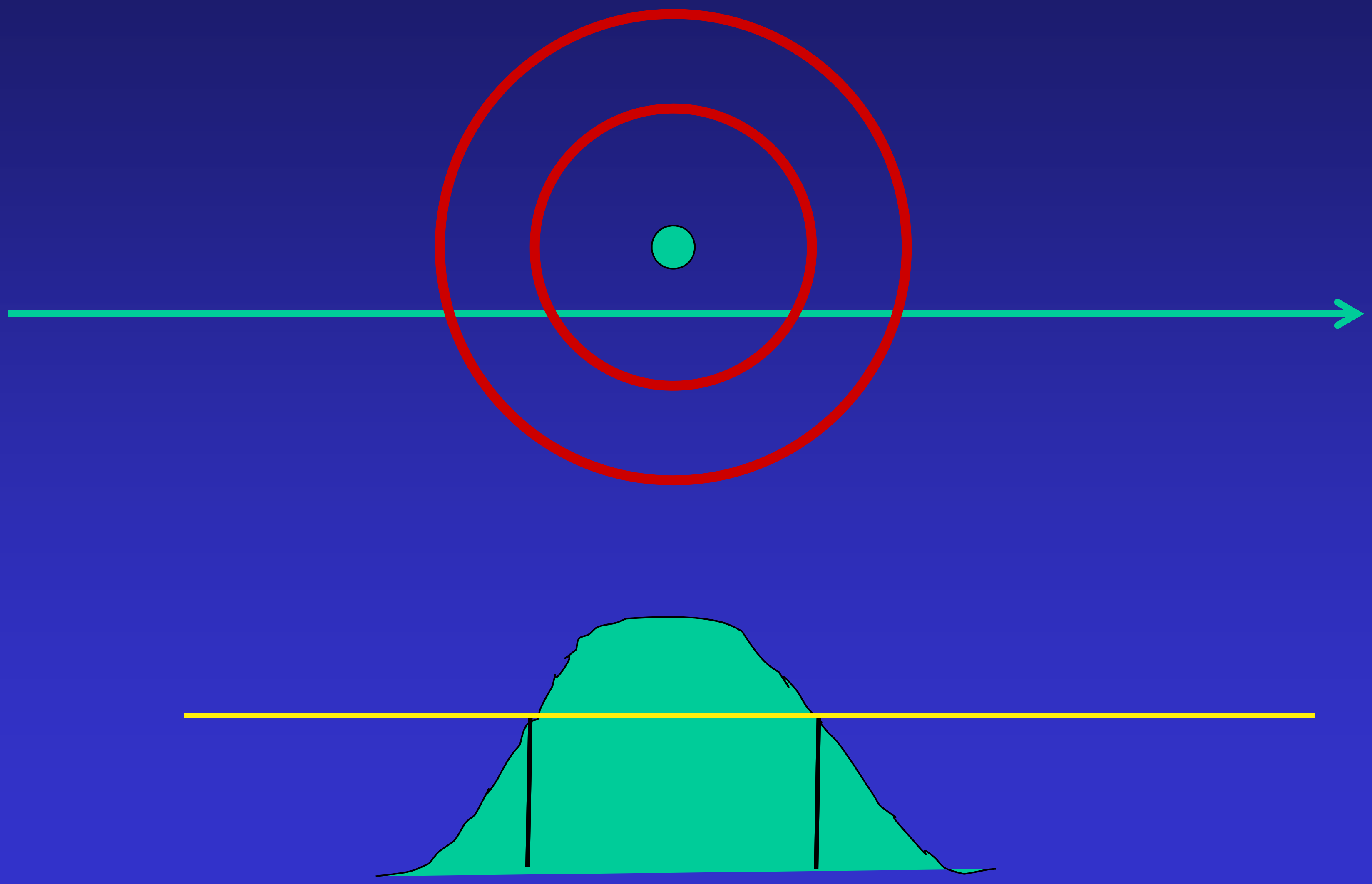
Ray Intersection

Need to *search* along the ray
for the first time $f(P(t)) = 0$



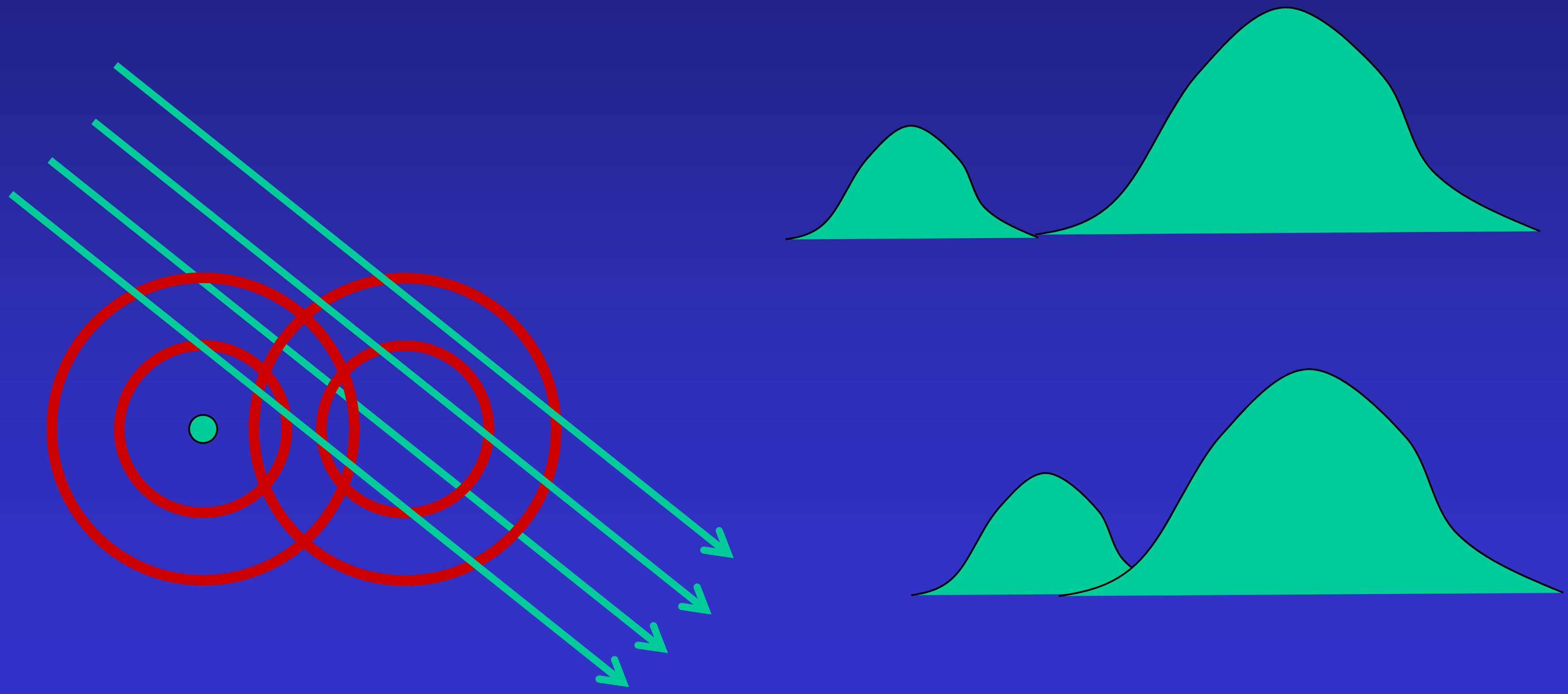
Shortcuts and
Search strategies?

Search for Intersection



Search for Intersection

Identify spans of interest: bounds on intersection

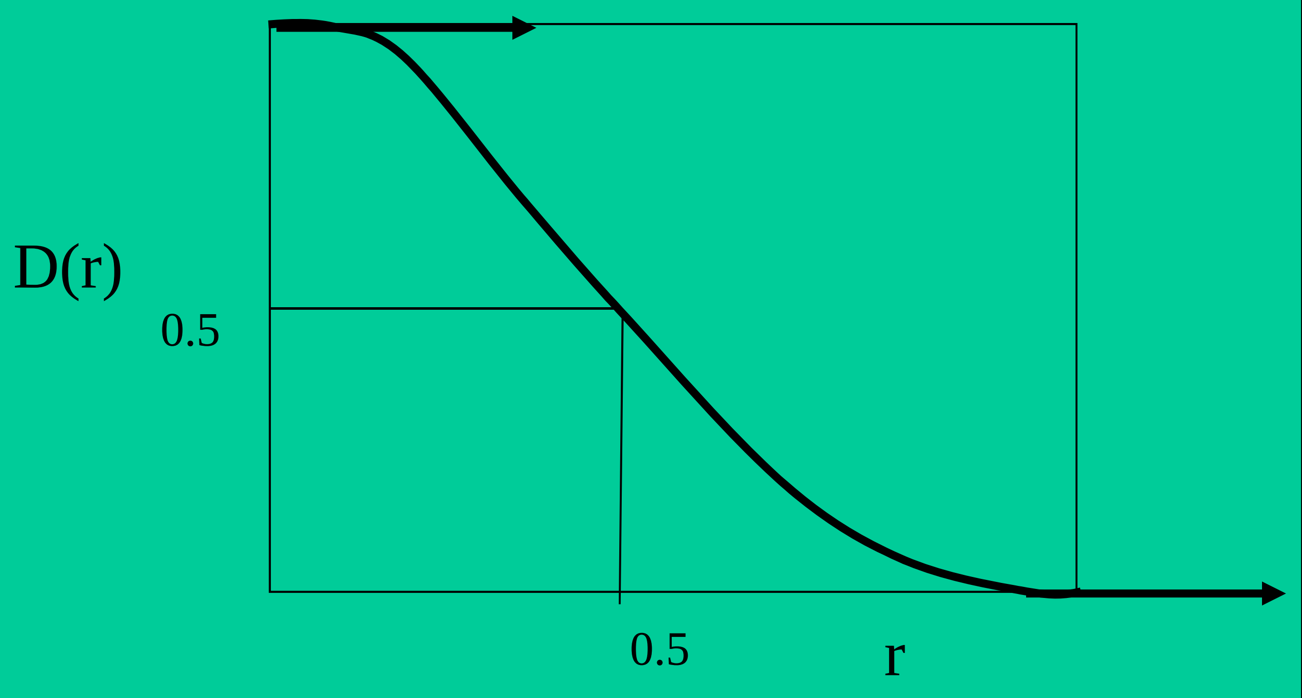


Density Functions

Define a density function that is:
- Easy to evaluate
- Blends smoothly
- Intuitive to use

Density functions proposed in the literature
- Exponential
- Piecewise cubic
- Cubic in distance squared

Density Functions



Distance-based Density Functions

$$d_i(p) = D(|P-C_i|/R) = D(r)$$

r is <u>normalized distance</u>

$D_1(r) = (1-r^2)^3 \qquad 0 <= r < 1$

$D_2(r) = 1 - (4/9)r^6 + (17/9)r^4 - (22/9)r^2 \qquad 0 <= r < 1$

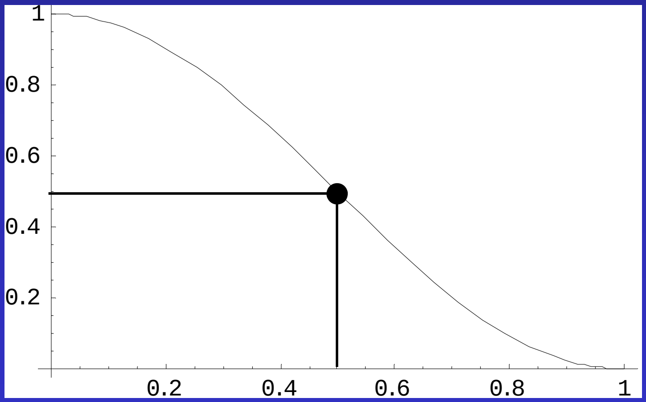
$D_3(r) = \exp(-ar^2)$

$D_4(r) = 1-3r^2 \qquad 0 <= r < 1/3$

$\qquad (3/2)(1-r)^2 \qquad 1/3 <= r < 1$

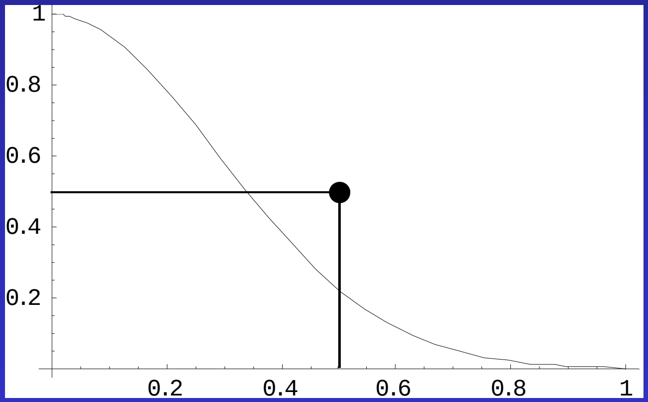
Distance-based Density Functions

$$D_2(r) = 1 - (4/9)r^6 + (17/9)r^4 - (22/9)r^2 \qquad 0 <= r < 1$$



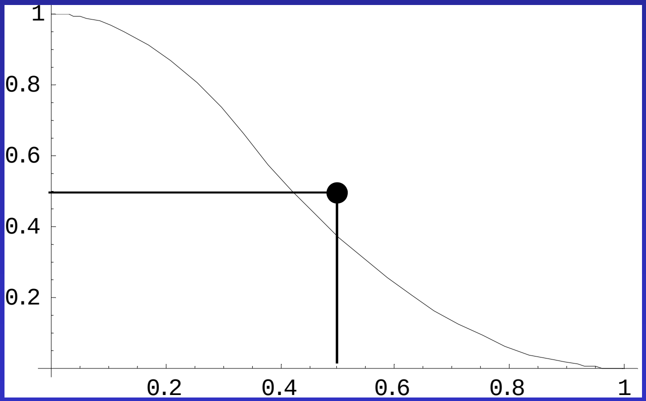
Distance-based Density Functions

$$D_3(r) = \exp(-ar^2)$$



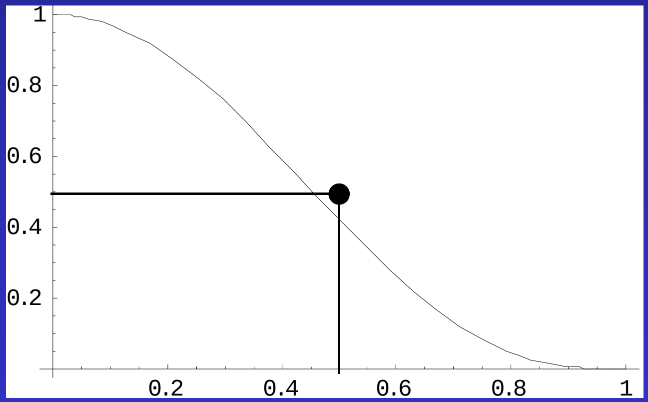
Distance-based Density Functions

$$D_4(r) = \begin{cases} 1-3r^2 & 0 <= r < 1/3 \\ (3/2)(1-r)^2 & 1/3 <= r < 1 \end{cases}$$



Distance-based Density Functions

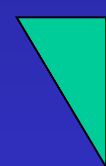
$D_1 (r) = (1-r^2)^3 \qquad 0 <= r < 1$

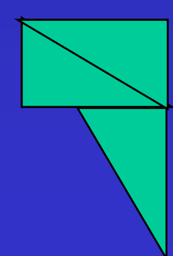


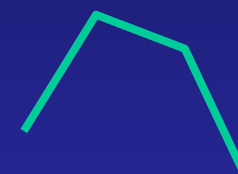
Distance-based Primitives

Point 

Line 

Polygon 

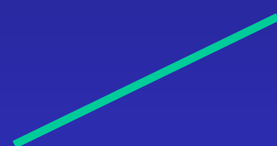
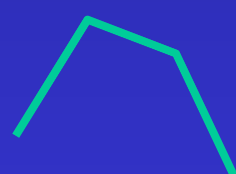
Polygonal mesh 

Polyline 

Polyhedron 

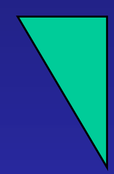
Anything you can efficiently compute the distance from

Distance-based Primitives

Point		Distance from point
Line		Distance from line or endpoints - partition by perpendiculars
Polyline		Distance from one of lines or points - partition by perpendiculars

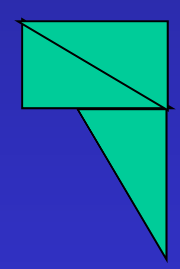
Distance-based Primitives

Polygon



Partition space by planes perpendicular to plane through an edge

Polygonal mesh



Same, for each face - two planes per edge

Distance-based Primitives

Polyhedra

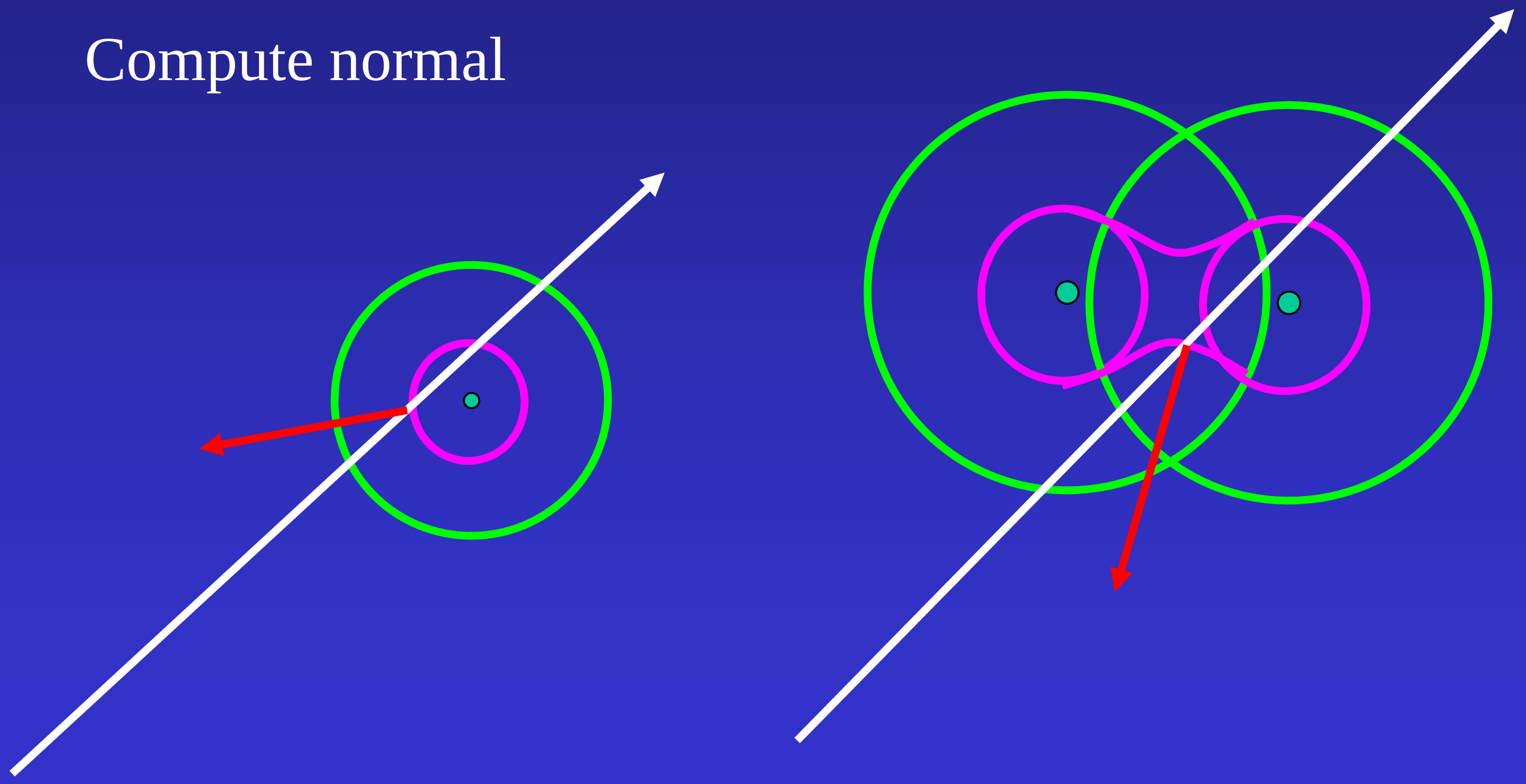
Convex?

Concave?

Display Considerations

Find point of intersection along ray: F(P(t)) = 0

Compute normal



Computing the Normal

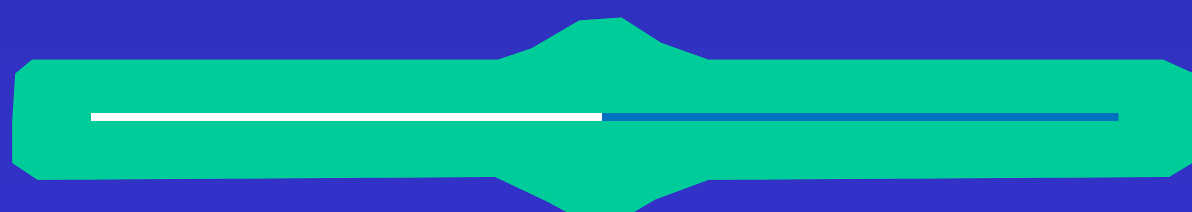
Form analytic expression of implicit function
And take partial derivatives
$N = (\delta F/\delta x, \delta F/\delta y, \delta F/\delta z)$

Take discrete approximation by sampling function
Compute gradient
$N = (F(x+dx,y,z)-F(x,y,z),\ F(x,y+dy,z)-F(x,y,z),\ F(x,y,z+dz)-F(x,y,z))$

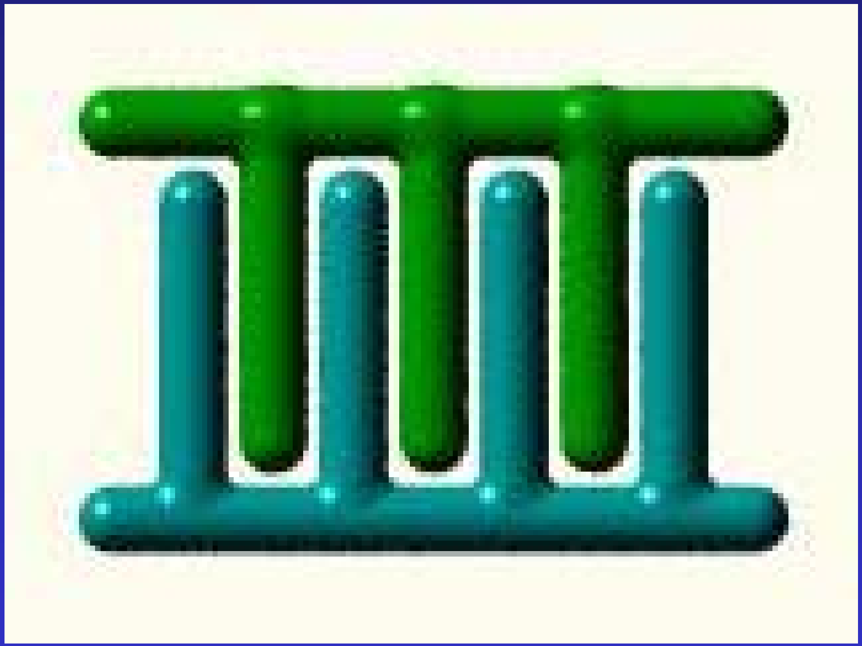
Bulge problem

One long primitive

Two side-by-side primitives



CSG-approach to control blending



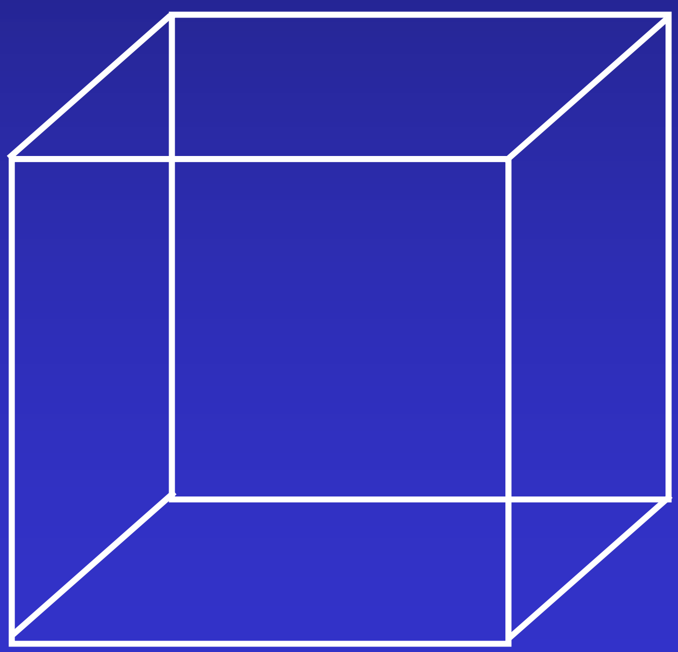
Use nodes to combine primitives by either summing or taking max of functions

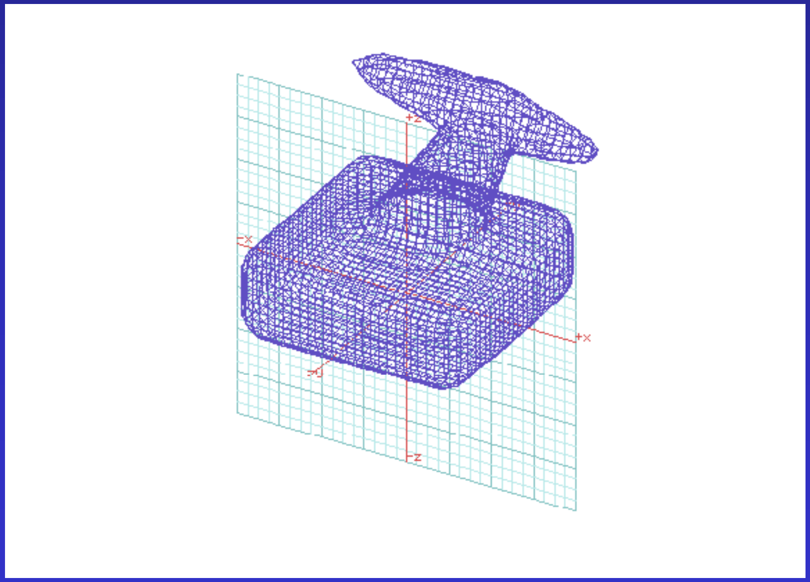
Complexity

- Bounding volumes
- Spatial subdivision - cellular bucket sort
- Hierarchical spatial subdivision – quadtree
- Binary spatial partitioning

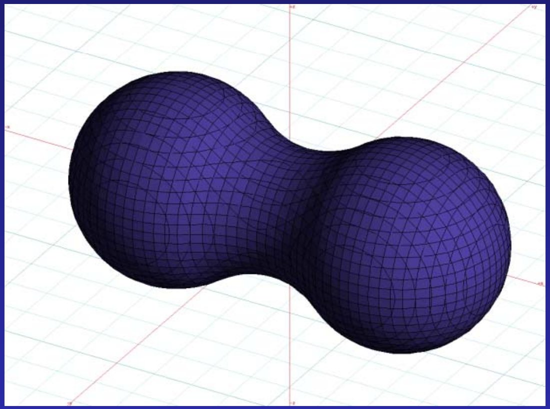
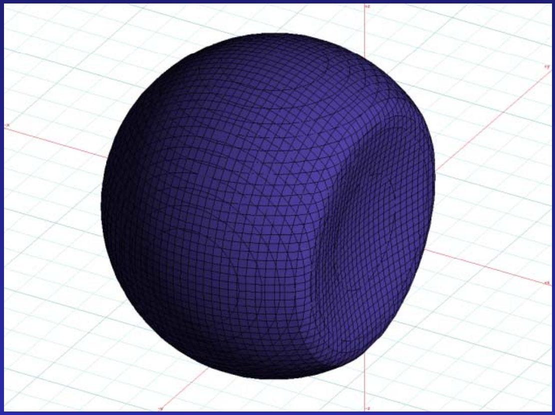
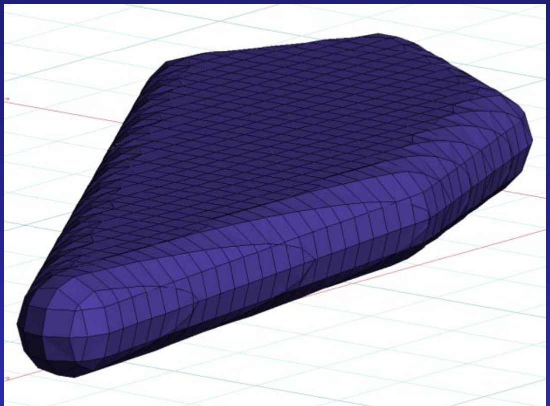
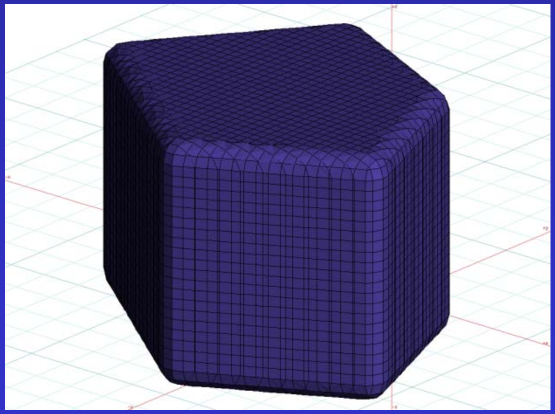
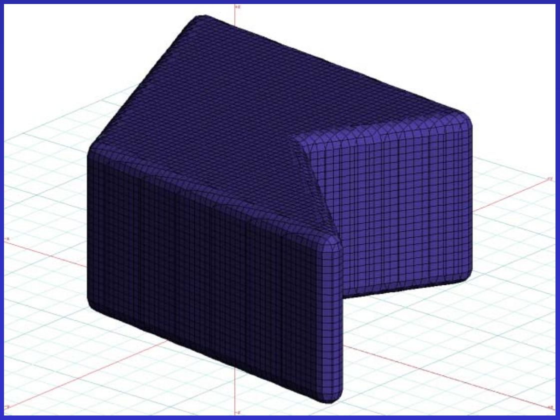
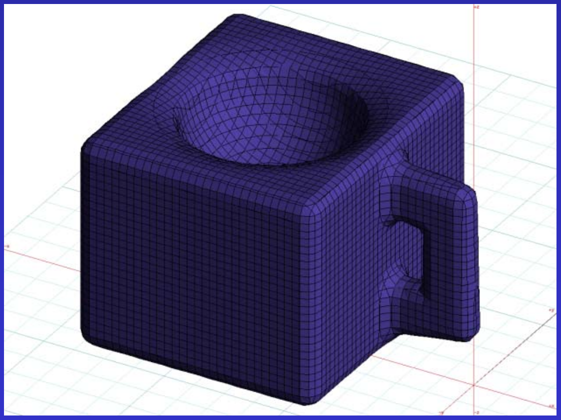
Display alternative

Marching cubes algorithm - construct surface fragments from isosurface intersections with grid cells

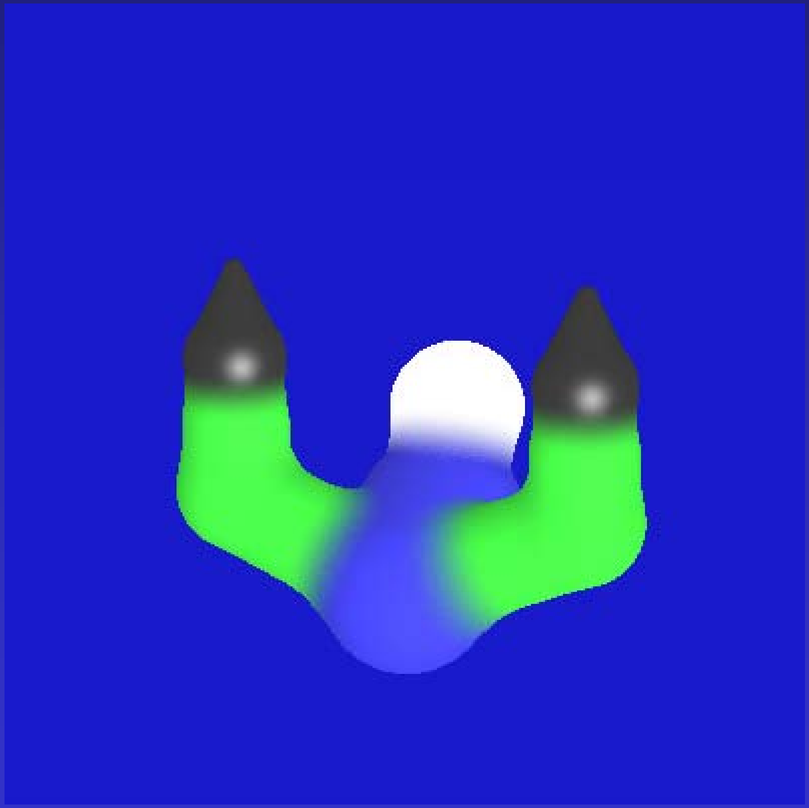




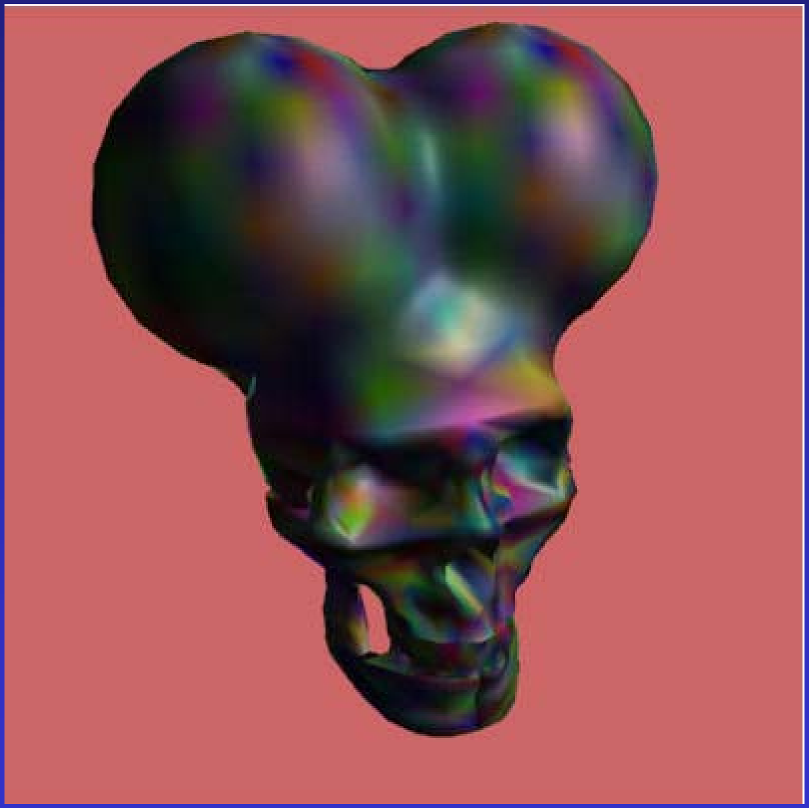
Distance-based Primitives

Examples



Examples



Examples