Texture Mapping:

Texture Mapping

Visual complexity on demand

Vary display properties over object

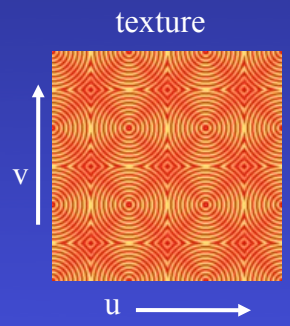
Visible pixel maps to location on object

Location on object
    used to lookup display attributes
Or
    as function parameters to generate attributes

2D Texture Mapping

Usually a 2D rectangular image or function

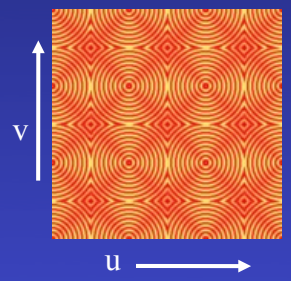
Parameterize using (u,v) texture coordinates



2D Texture Mapping

Need to parameterize surface similar to texture

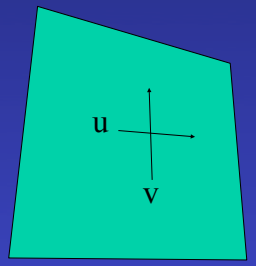
texture



sphere

latitude - longitude

quadrilateral



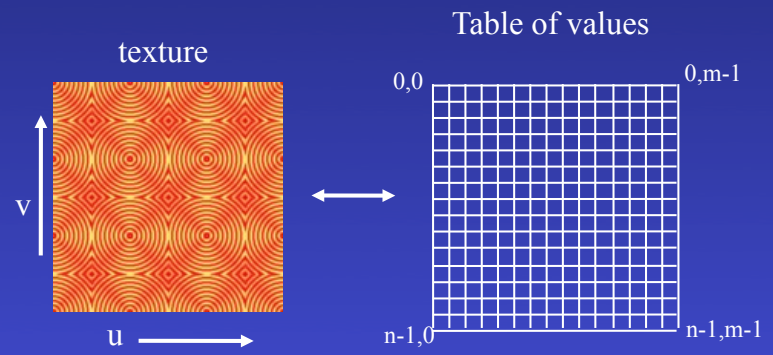
bilinear interpolation

Also - cylindrical is a common mapping

Simple examples - sphere and quadrilateral

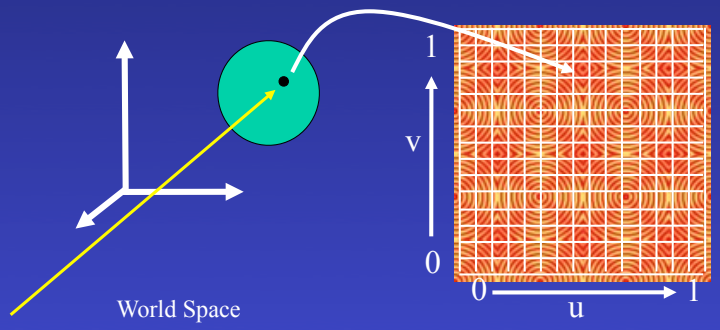
More interesting - triangle mesh - not dealt with here

Texture as table of values



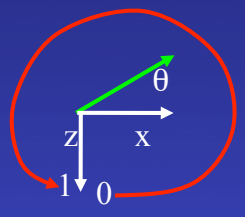
Values at grid intersections

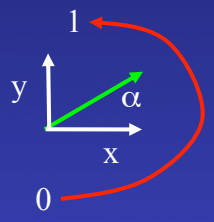
For sphere
Texture Map Coordinates



Map (x,yz) to u,v space to table values

For sphere
map sphere surface to (u,v)



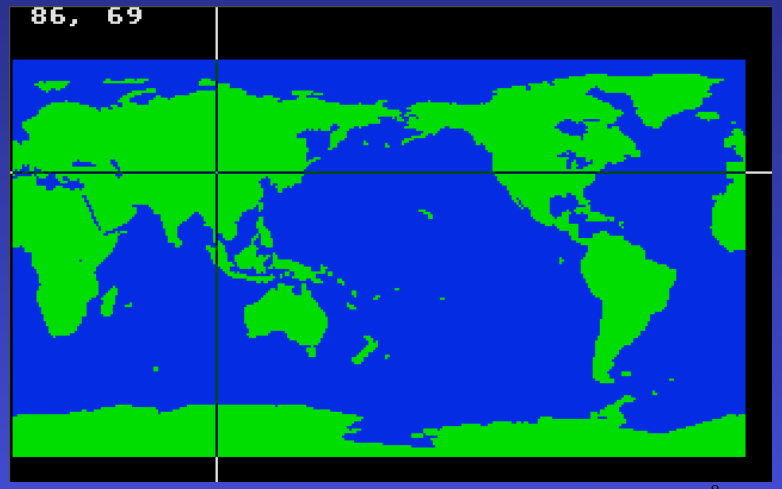


$$s = \frac{\tan^{-1}(z/x)}{\pi/2}$$
$$if\,(x > 0)\{\, u = (1+s)/4 \,\}$$
$$else\{u = 1/2 + (1-s)/4\}$$

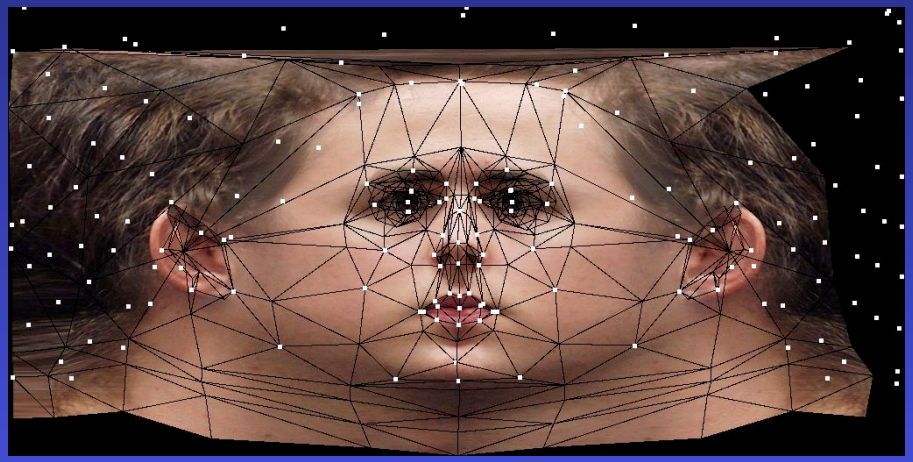
$$t = \frac{\tan^{-1}(\frac{y}{x})}{\pi/2}$$
$$v = \frac{t+1}{2}$$

BUT -
Has a seam
& distorts

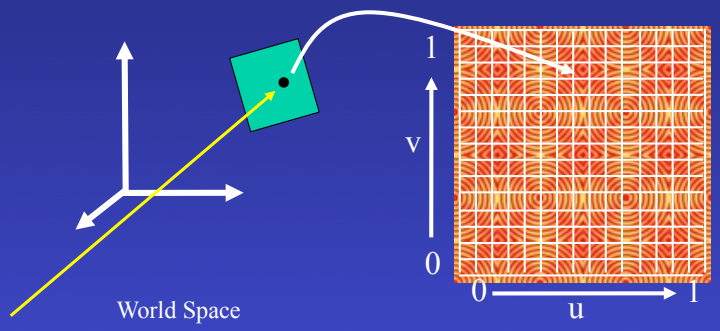
Spherical - e.g., Cartography



Cylindrical Mapping

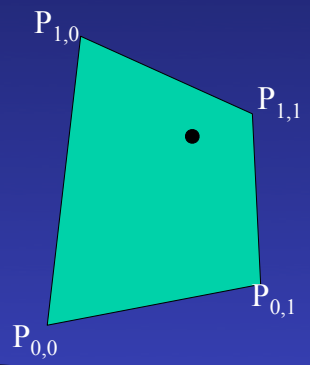


For quadrilateral Texture Map Coordinates



Map (x,yz) to u,v space to table values

World space point to u,v space



$$P_{u,0} = P_{0,0} + u(P_{1,0} - P_{0,0})$$

$$P_{u,1} = P_{0,1} + u(P_{1,1} - P_{0,1})$$

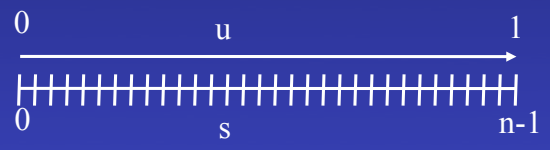
$$P_{u,v} = P_{u,0} + v(P_{u,1} - P_{u,0})$$

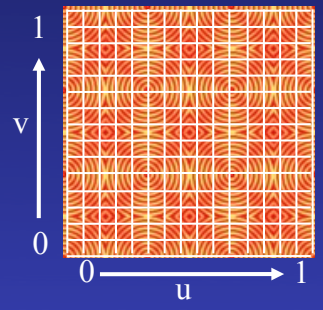
$$P_{u,v} = P_{0,0} + u(P_{1,0} - P_{0,0}) + v(P_{0,1} + u(P_{1,1} - P_{0,1}) - P_{0,0} + u(P_{1,0} - P_{0,0}))$$

$$P_{u,v} = P_{0,0} + u(P_{1,0} - P_{0,0}) + v(P_{0,1} - P_{0,0}) + uv(P_{1,1} - P_{0,1} + P_{1,0} - P_{0,0})$$

$$u = \frac{P_{u,v} - P_{0,0} - v(P_{0,1} - P_{0,0})}{(P_{1,0} - P_{0,0}) + v(P_{1,1} - P_{0,1} + P_{1,0} - P_{0,0})}$$

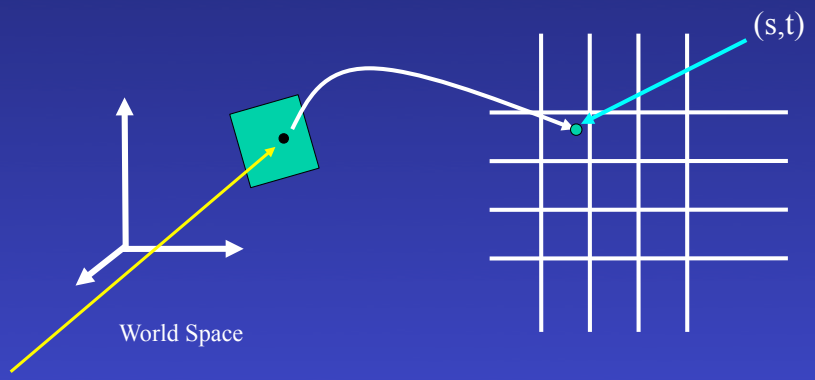
u,v space to table indice space





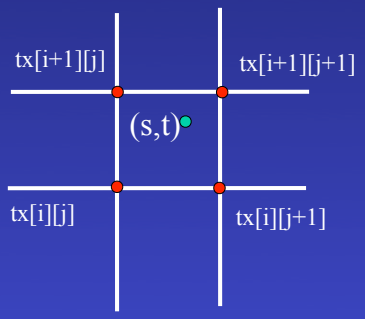
$$s = u(n-1)$$
$$t = m-1-v(m-1)$$

A closer look



Values only at the intesections
What value to use at non-intersection point?

Closer still

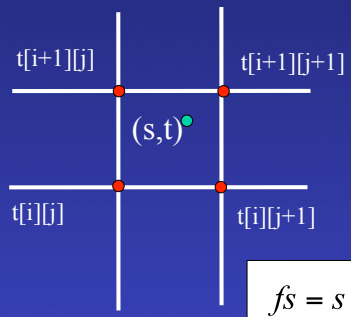


Use closest value?

$$i = \lfloor s + 0.5 \rfloor$$
$$j = \lfloor t + 0.5 \rfloor$$

$$txst = tx[i][j]$$

Closer still

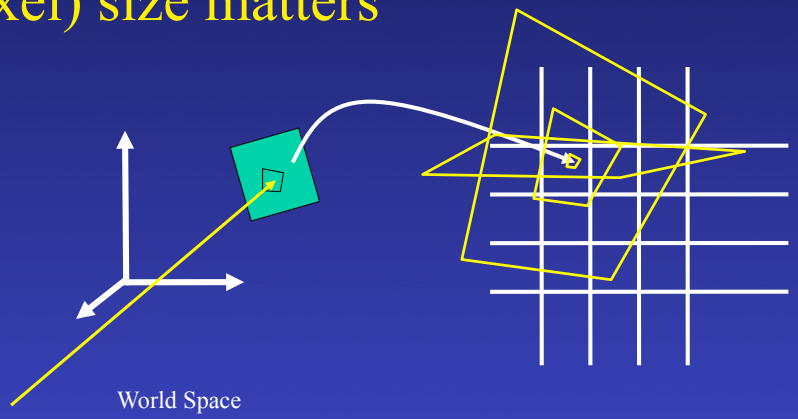


Interpolate 4 closest?

$$i = \lfloor s \rfloor$$
$$j = \lfloor t \rfloor$$

$$fs = s - \lfloor s \rfloor$$
$$ft = t - \lfloor t \rfloor$$
$$ts1 = tx[i][j] + fs(tx[i+1][j] - tx[i][j])$$
$$ts2 = tx[i][j+1] + fs(tx[i+1][j+1] - tx[i][j+1])$$
$$txst = ts1 + ft(ts2 - ts1)$$

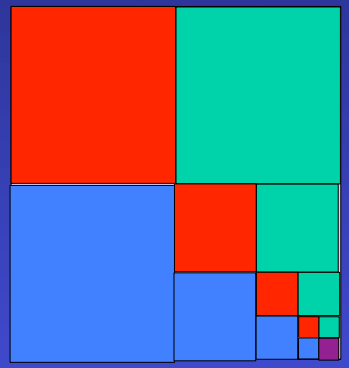
(Pixel) size matters



Can’t just use pixel center and expect good results in all cases - need to consider how entire pixel maps into texture space

One solution: Mip-mapping

Pre-filter texture, reducing resolution
(increase size of grid relative to pixel size)



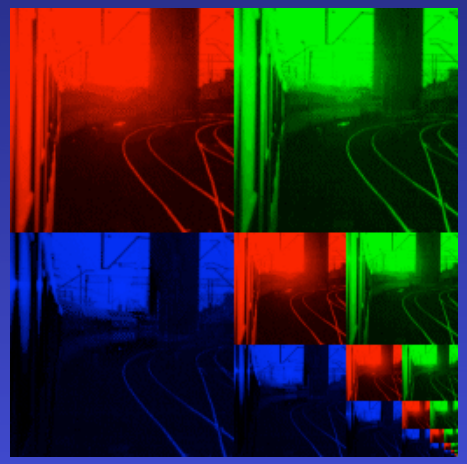
Successive table of values (r,g,b) at reduced resolution

Down to single pixel

Index into highest resolution one in which bilinear interpolation makes sense

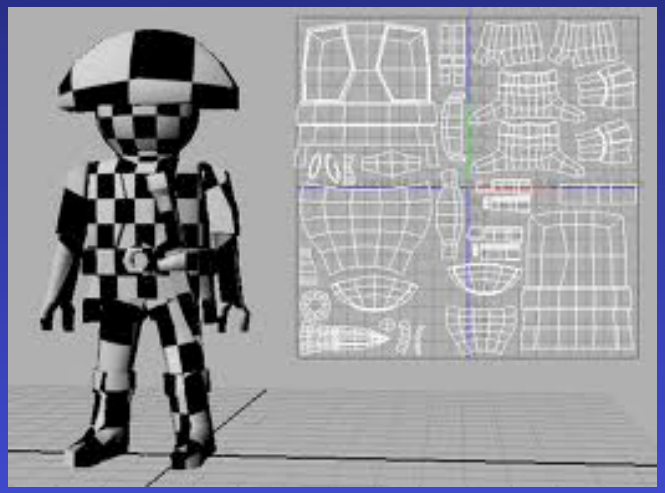
What size ‘makes sense’? Pixel size is less that grid

One solution: Mip-mapping



What size ‘makes sense’? Pixel size is less that grid

UV Mapping



What size ‘makes sense’? Pixel size is less that grid