Brief Review:
Vectors
Vectors

• Basics
• Normalizing a vector => unit vector
• Dot product
• Cross product
• Reflection vector
• Parametric form of a line
Basics

Vectors
• Have a direction and a length
• Do not have a position in space

Normal vector
• Is ‘normal’, or perpendicular, to a surface
• Are usually unit-length, also called ‘normalized’
Normalizing a Vector

- Compute the magnitude and divide through
- Produces a UNIT VECTOR
- Aka NORMALIZED VECTOR

To normalize \((x,y,z)\):

\[
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} \div \text{len} = \begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} \div \sqrt{x^2 + y^2 + z^2}
\]

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Dot Product

Scalar Product

\[ A \cdot B = A_x B_x + A_y B_y + A_z B_z \]

\[ A \cdot B = B \cdot A \]

\[ A \cdot B = |A| |B| \cos(\alpha) \]

- If A and B are unit vectors, \( A \cdot B = \cos(\alpha) \)
- If A is unit vector, \( A \cdot B = |B| \cos(\alpha) \) is the length of B projected onto A
Cross Product

Vector Product

\[ A \times B = (A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x) \]

\[ A \times B = -B \times A \]

- \[ A \times B \] is orthogonal to plane defined by \( A \) and \( B \)
- With length \[ |A \times B| = |A||B| \sin(\alpha) \]
- If \( A \) and \( B \) are unit vectors, \[ |A \times B| = \sin(\alpha) \]
- If \( B \) is unit vector, \[ |A \times B| \] is perpendicular distance from \( A \) to \( B \)

\[ |A \times B| = |A| \sin(\alpha) \]
Reflection Vector

In 3D, Reflect V about N to make R
Assume N is normalized

\[ R = V + 2S \]
\[ S = P - V \]
\[ k = |P| = N \cdot V \]
\[ P = kN = (V \cdot N)N \]
\[ R = V + 2(P - V) = V + 2((V \cdot N)N - V) = 2(V \cdot N)N - V \]
Parametric Equation of Line

- $P_0$ is point on line
- $V$ is direction of line
- Generalizes to any dimension (2D, 3D, etc)
- As $0 < u < 1.0$, $P(u)$ goes from $P_0$ to $P_0 + V$

$P(u) = P_0 + uV$