Procedural Models

- How to easily generate complex data?
- Data from parameterized procedure and/or digitized
- Data amplification
- Ray trace directly v. convert to polygonal models and use z-buffer

Basic Procedural Approaches

- composition from primitive shapes
- extrusion
- surface of revolution
- lofting
- sweep operator

Composition

Procedurally generated
controlled randomness

Extrusion
Surface of revolution

Lofting

Sweep Operator

Combines extrusion, lofting, and surface of revolution

More sophisticated approaches

- CSG
- Subdivision surfaces
- Curves & surfaces
- Fractals
- Implicit functions
CSG: constructive solid geometry

- primitive shapes
- union, difference, intersection
- tree structure representation
- Ray trace directly
- Evaluate to polyhedron with Boolean operators
- OpenGL rendering using stencil buffer
- http://www.opengl.org/resources/code/samples/advanced/advanced97/notes/node11.html

Subdivision surfaces

- Initial coarsely defined geometry
- Refine geometry by
  - rounding corners,
  - subdividing faces and edges,
  - smoothing faces

What is resulting limit surface?
- What is continuity of limit surface?

Subdivision surfaces

- Round corners
- Catmull-Clark
- Doo-Sabin
- Loop
- Butterfly

Catmull-Clark Subdivision
Loop Subdivision

Curved surface patches
- Hermite curve & patches
- Bezier curve & patches
- Catmull-Rom spline
- B-spline

Hermite Curve
- Given: starting and ending points and tangents
- Determine cubic equation that satisfies constraints

\[ P(u) = au^3 + bu^2 + cu + d \]
\[ P'(u) = 3au^2 + 2bu + c \]

\[ P(0) = p_0 = d \]
\[ P(1) = p_1 = a + b + c + d \]
\[ P'(0) = t_0 = c \]
\[ P'(1) = t_1 = 3a + 2b + c \]

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Hermite Patch
- Given: starting and ending points and tangents
- Determine bicubic equation that satisfies constraints

\[ P(u, v) = u MBM^T v \]
Fractal geometry
- Objects that exhibit similarity under scale
  - e.g., Koch curve

  ![Koch curve]

  Mathematically curious - continuous, infinite length, nowhere differentiable, bounded

Serpinski Gaskets

Other fractal shapes
- Mandelbrot set
- Coastlines
- Mountains

statistically self-similar under scale

Create mountains
1. Subdivide triangle
2. Displace upward
3. Reduce amplitude of displacement
4. Recurse on subdivisions

![Mountains]
Implicit functions & isosurfaces

- $f(x,y,z) = 0$
- “density” function
- distance-based implicit functions
- Marching Cubes algorithm

Marching Cubes: convert to polygons