

# Verification of Smooth and Close Collision-Free Cruise Control



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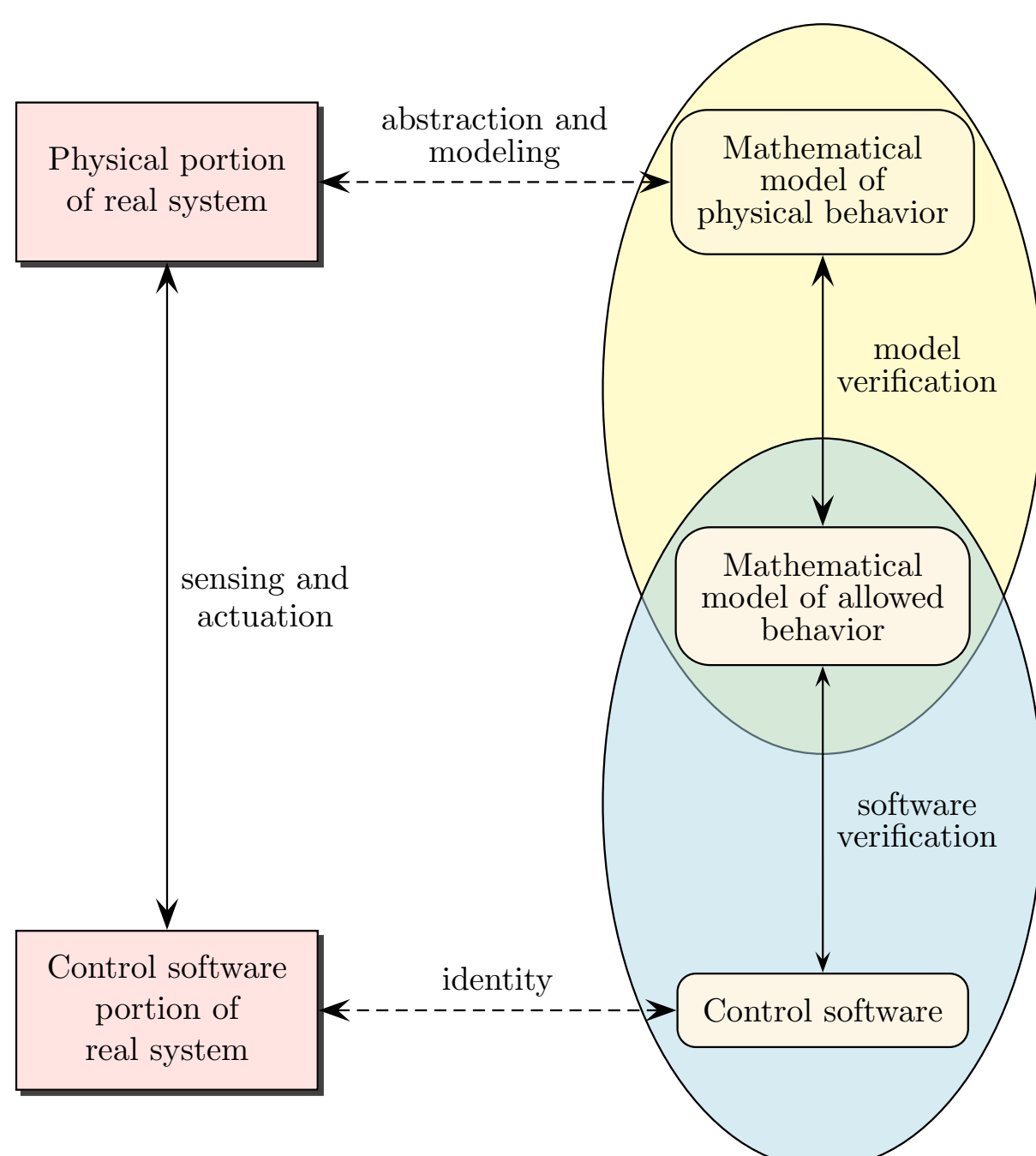
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## Abstract

Modern adaptive cruise control technologies are designed to improve the comfort or safety of the driver; however, no safety guarantees are asserted by these designs. Furthermore, existing theoretical work in the safety verification of adaptive cruise control algorithms require both discrete braking modes and overly conservative separation distances to make such safety guarantees. Thus, existing work in safety verification both risks reducing driver comfort while also eliminating any of the performance gains typically associated with automated highways. Our work extends verification of automated highway systems to mitigate both of these problems. Motivated by optimal control and verification of software systems, we have developed safety conditions for adaptive cruise control algorithms that do not require discontinuous braking and also allow for substantially lower following distances than existing work in the verification of autonomous highway systems. Moreover, we demonstrate a novel approach for verifying software in hybrid systems by embedding the continuous dynamics into the software specifications. The result is a verified software paradigm consistent with the vision of Hoare's verifying compiler.

## Tools for Verifying Cyber-Physical Systems



CPS Concrete-Abstract Correspondence

```
havoc dt
assume 0.0 < dt and dt < rho
```

### physical loop

```
maintains
bl = #bl and bf = #bf and
afMax = #afMax and rho = #rho and
af = #af and dt = #dt and
0.0 <= t and t < rho + dt and
vl = VEL(#vl, -bl, t) and
xl = POS(#xl, #vl, -bl, t) and
vf = VEL(#vf, af, t) and
xf = POS(#xf, #vf, af, t) and
xl >= xf

while IsGreater (rho, t) do
variable zero, dv, dx: Real

dv := Replica (dt)
Multiply (dv, bl)
Subtract (vl, dv)
if IsGreater (zero, vl) then
Clear (vl)
end if
dx := Replica (dt)
Multiply (dx, vl)
Add (xl, dx)

dv := Replica (dt)
Multiply (dv, af)
Add (vf, dv)
if IsGreater (zero, vf) then
Clear (vf)
end if
dx := Replica (dt)
Multiply (dx, vf)
Add (xf, dx)

Add (t, dt)
end loop
```

Augment Annotated Code with Physical Loop

### Prove:

$$VEL(vl_4, -bl_0, t_{11}) - dt_9 \times bl_0 = VEL(vl_4, -bl_0, t_{11} + dt_9)$$

### Given:

$$0.0 < bl_0$$

$$0.0 < bf_0$$

$$0.0 < afMax_0$$

$$bf_0 \leq bl_0$$

$$0.0 < rho_0$$

$$MINGAP(vl_2, bl_0, vf_2, bf_0, afMax_0, rho_0) \leq xl_2 - xf_2$$

$$0.0 \leq vl_2$$

$$0.0 \leq vf_2$$

$$0.0 \leq vl_4$$

$$0.0 \leq vf_4$$

$$MINGAP(vl_4, bl_0, vf_4, bf_0, afMax_0, rho_0) \leq xl_4 - xf_4 - bf_0 \leq af_8$$

$$af_8 \leq afMax_0$$

$$MINGAP(VEL(vl_4, -bl_0, rho_0), bl_0, VEL(vf_4, af_8, rho_0), bf_0, afMax_0, rho_0) \leq POS(xl_4 - xf_4, vl_4, -bl_0, rho_0) - POS(0.0, vf_4, af_8, rho_0)$$

$$0.0 < dt_9$$

$$dt_9 < rho_0$$

$$t_{11} < rho_0$$

$$0.0 \leq t_{11}$$

$$t_{11} < rho_0 + dt_9$$

$$POS(xf_4, vf_4, af_8, t_{11}) \leq POS(xl_4, vl_4, -bl_0, t_{11})$$

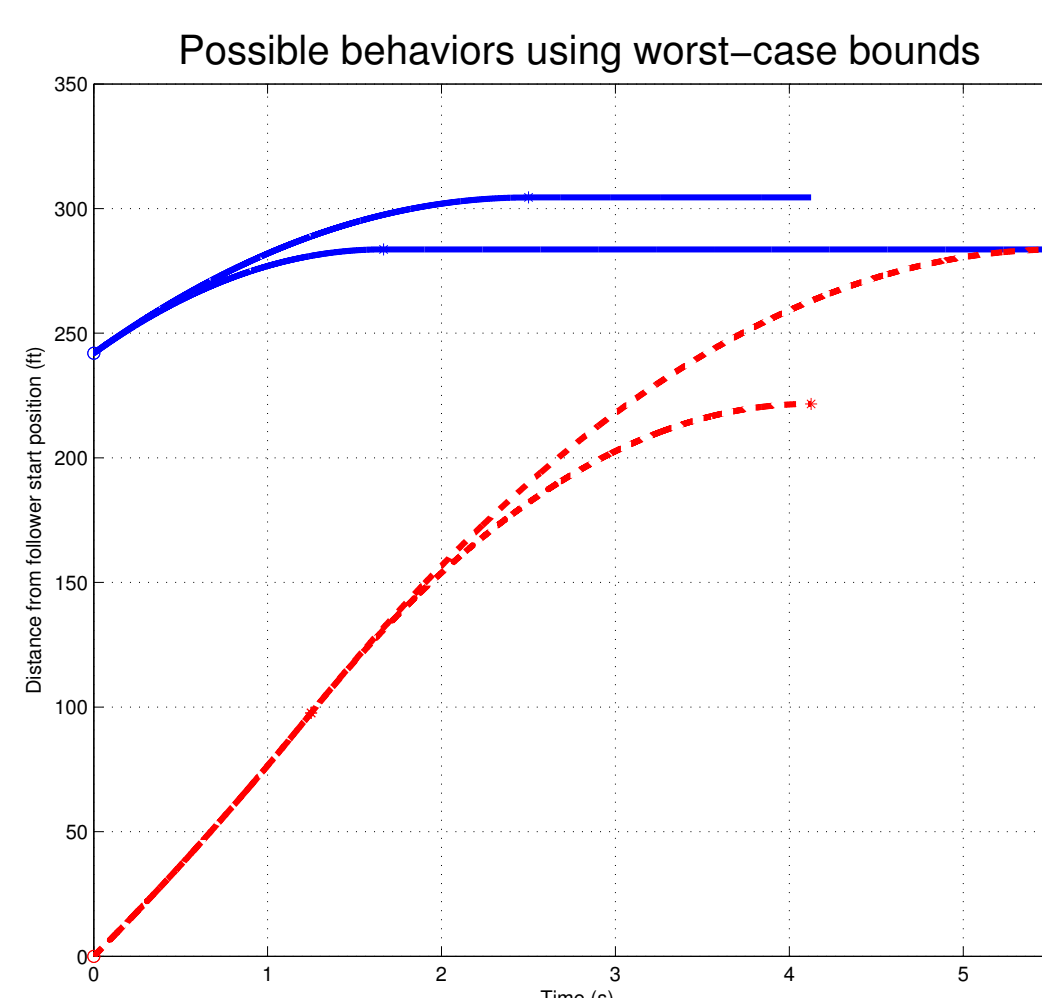
$$0.0 \leq VEL(vl_4, -bl_0, t_{11}) \leq dt_9 \times bl_0$$

$$VEL(vf_4, af_8, t_{11}) + dt_9 \times af_8 < 0.0$$

### Example Verification Condition (VC)

## Conventional Verification of ACC

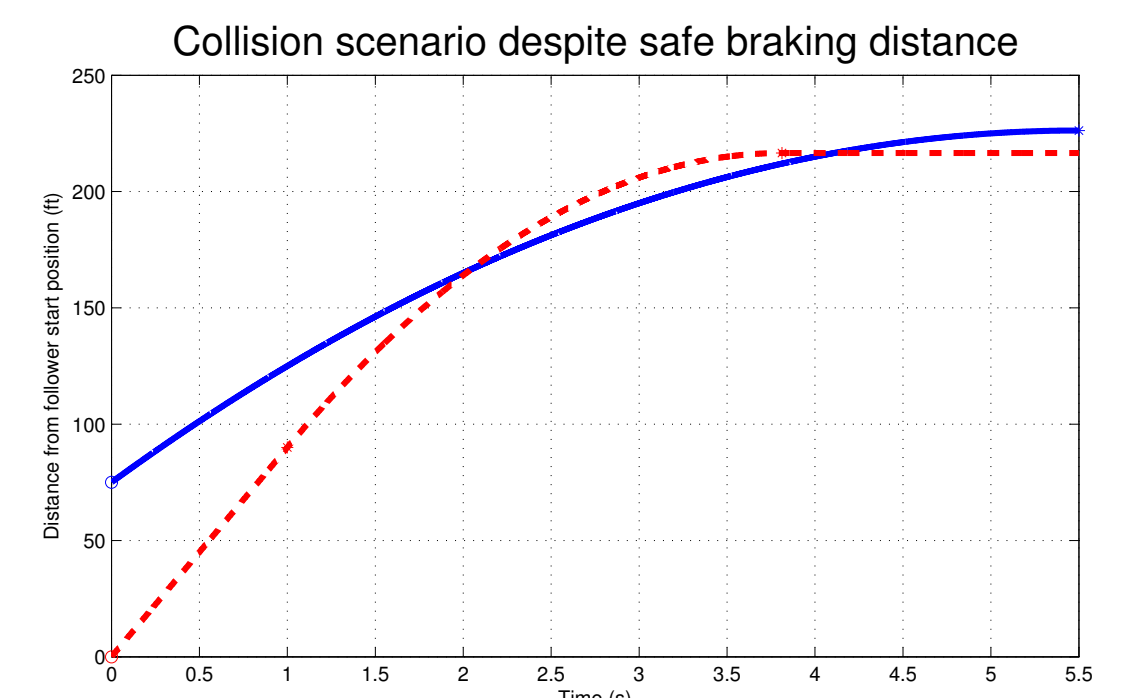
- Assume global upper and lower braking bounds
- Must apply minimum brake whenever worst-case collision scenario is possible
- Acceleration-safe distance grows as distance between bounds grow



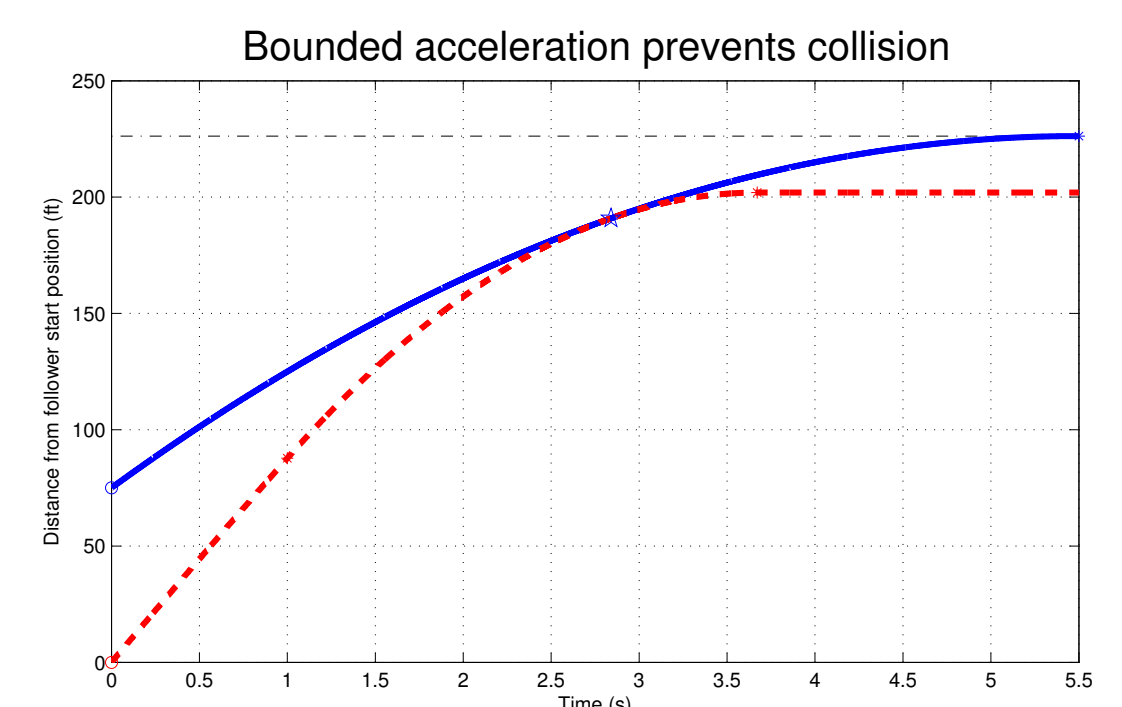
Specifications using worst-case stopping distances

## Heterogeneous Smooth-and-Close ACC

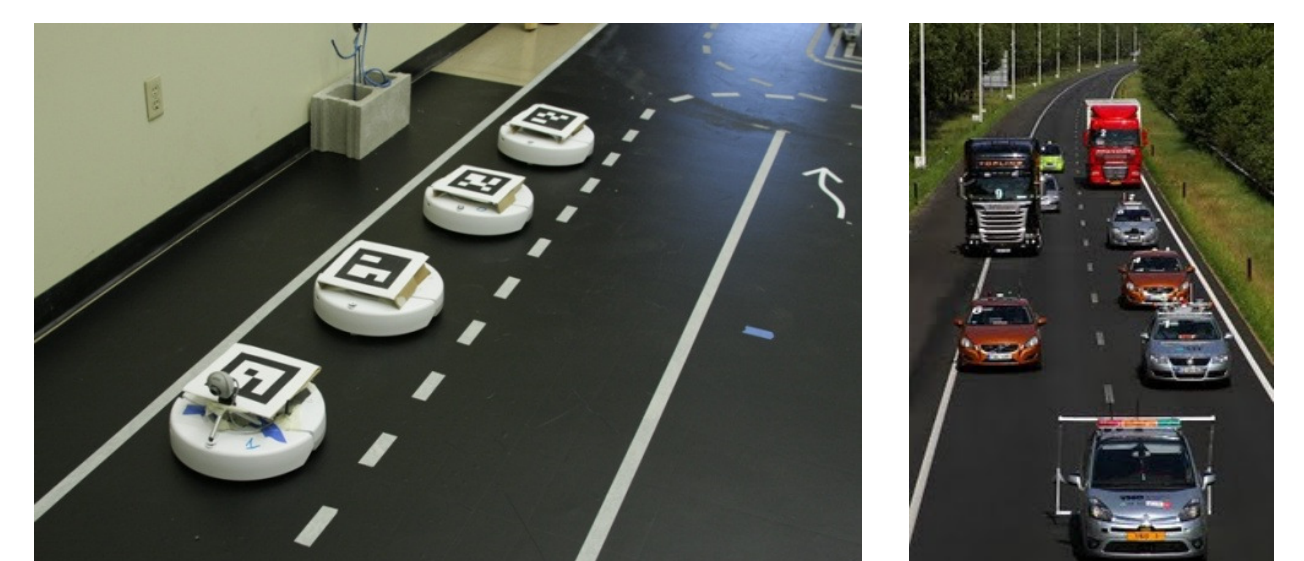
- Local braking is known
- Upper bound on leader is known (e.g., plate tag)
- Adjust upper bound on safe local acceleration
- Stopping-distance condition not sufficient



### Collision Using Stopping-Distance Logic



Marginally Safe Stop after Evasive Acceleration



Mixed-Traffic Adaptive Cruise Control (ACC)

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