Volumetric Data Modeling and Reduction in a Compressive Sensing Framework

Xie Xu∗ Mahsa Mirzargar† Alireza Entezari‡

Department of Computer and Information Science and Engineering
University of Florida

ABSTRACT
We propose an alternative volumetric data modeling and reduction approach via compressive sensing theory. We provide evidence that with a small set of randomly chosen Fourier samples of a dataset, it is possible to recover the dataset accurately. Our experiments demonstrate that the number of samples necessary for an accurate reconstruction is linearly proportional to the number of features, as opposed to the original resolution of the dataset. These experiment results motivate further research in the direction of custom-designed sparse representations for large-scale volumetric data.

Keywords: Volumetric data reduction, sampling and reconstruction, sparse representation.

1 MOTIVATION
Compressive sensing (CS) [1, 3, 4] has emerged as a powerful paradigm for efficient sampling and accurate reconstruction of signals. It allows us to accurately reconstruct a signal from only a few samples. CS has had a significant impact in scientific and biomedical imaging such as seismic imaging [5] and MRI [7]. While there are significant efforts on 2D imaging, sparse representations for 3D volumetric data have not been well studied in the context of compressive sensing. Note that the application of CS on MRI is not truly 3D CS, since the sensing is not performed in the 3D frequency domain, but is only a sequence of 2D measurements [7, 8].

The central theme of CS revolves around the assumption that most natural signals are either sparse or that they can be sparsely represented in a certain basis or domain. For example, most images and volumetric data are either sparse themselves, or, when transformed to the wavelet (or ridgelets, curvelets, shearlet, ...) domain, lead to sparse representations. For a dataset of size $N$ with a relatively small number, $k$, of features or non-zeros, the necessary number of samples for accurate reconstruction is only $O(k \log N / k)$. Motivated by this, we are considering an alternative data modeling and reduction approach for volumetric data via compressive sensing, that is, keeping only a few “compressively sensed” samples instead of the original high resolution data, and later reconstructing the original data from those saved samples.

The remarkable advantage of the CS framework is that at the pre-processing (i.e., data sensing/sampling) stage, no prior information is required about the feature domain – just the assumption that the data is sparse in some domain. The information about features (i.e., sparse representations) only enters the picture at the reconstruction phase. This means that once the data is sensed using the CS framework, we can continue to refine the definition of features through dictionary learning or other sources of domain-knowledge for reconstruction purpose after the data reduction stage. Once such features of interest are refined, they can be reconstructed from the “old” compressively sensed data without having to re-run the original high resolution simulations.

2 APPROACH
The number of samples required by the compressive sensing theory depends on the sparsity of the signal (or the sparsity of the signal in a transform domain). In our experiments, we examined the “four-to-one practical rule” [2] as a starting point for deciding the number of samples, that is, we experiment with the number of samples, $n$, about four times the number of non-zero elements (features), $k$, of the unknown signal: $n \approx 4k$. During the sensing step, we apply discrete cosine transform (DCT) on the ground truth dataset of size $N$ and randomly pick the DCT coefficients, thus obtain a measurement of length $n$. This so-called partial Fourier sampling meets the Restricted Isometry Property [1] that is required by the CS theory for accurate reconstruction. For clarity of presentation, we define the sampling rate as $\rho := n/N$ and signal sparsity rate to be $\beta := k/N$. When applying the “four-to-one” rule exactly, we have $\rho = 4\beta$.

Unlike traditional Shannon-Nyquist sampling theorem, reconstruction algorithms for CS problems are no longer linear. Instead, the algorithms involve convex optimization or other iterative (greedy) methods. $l_1$-minimization solver $\text{11}_\text{LS}$ [6] and iterative solver Regularized Orthogonal Matching Pursuit (ROMP) [9] are chosen for recovery in our experiments. We note that iterative solvers may take longer time and return less accurate recovery results than optimization solvers when the unknown signal is not very sparse or the problem size is large because they only bring limited components into the solution per iteration. The accuracy of recovery is measured in terms of SNR (Signal to Noise Ratio) over the entire volumetric data. Our experiments are carried out, non-parallelized, in MATLAB on a workstation. We examine both sparse datasets and datasets sparse in a transform domain. For later case, we exploit the Haar wavelet transform domain for its efficiency and simplicity.

The aneurysm dataset has been sampled at a resolution of $256 \times 256 \times 256$. It is a sparse dataset in the space domain, of which only $\beta = 1\%$ of data elements are non-zero. We randomly choose the dataset’s DCT coefficients and experiment with the sampling rate $\rho = 3\%$ and $\rho = 4\%$ (i.e., we keep $n = \rho \times 256 \times 256 \times 256$ samples). Figure 1 compares the volume rendering images, as well as SNR’s, of the ground truth data and the recovered data. Because the aneurysm dataset is extremely sparse ($\beta = 1\%$), the greedy method outperforms the convex optimization with a reasonable accuracy in a shorter time. Since the reported SNR’s are fairly high, the difference among images in the figure are not visible with the current transfer function used for rendering. We also observed that the larger the sampling rate ($\rho$) is, the faster the recovery method converges. So, when the efficiency of reconstruction is of concern, we can increase the measurement rate $\rho$.

The turbulent combustion simulation datasets have five fields, and each of them includes 122 time steps. Our ground truth datasets have dimensions of $256 \times 512 \times 64$. These datasets are sparse in (Haar) wavelet transform domain. In these cases, the sparsity, $\beta$,…

∗e-mail: xie@cise.ufl.edu
†e-mail: smahsa@cise.ufl.edu
‡e-mail: entezari@cise.ufl.edu
is referred to the sparsity of the wavelet coefficients. Since $\beta$ is relatively large here, iterative algorithms such as ROMP are not a suitable choice. Therefore, 1_\text{ls} is used for our experiments on these datasets. Our experiments on “YOH” (mass fraction of OH) and “Mixture Fraction” fields of the datasets on different time steps are summarized in Table 1. In these experiments, we have sampling rate $\rho = 4\beta$. The combustion evolves along the time (i.e., having different sparsities). Figure 2 shows the rendering of both ground truth and recovered dataset of the YOH field at time step 41 listed in Table 1. We can observe the accurate recovery from the images.

Our experiments on both sparse signal and signal sparse in a transform domain provide compelling evidence for the application of compressive sensing theory on the volumetric datasets: by saving only a few random samples of the signal (thus decreasing the data storage and I/O requirements), we can recover the data with high accuracy. Note that these samples are obtained without any prior knowledge of the unknown signal.

3 Future Work

Although solvers such as 1_\text{ls} have been designed to deal with medium to large scale problems, those $\ell_1$ minimization or greedy methods still have high computation complexity and the reconstruction times in our experiments are still on the order of hours. While this is a disadvantage in small workstations, in large computing environments where there are ample computing power, the cost of reconstruction maybe mitigated by the inherent I/O delays. Recently some researchers have started investigating parallel algorithms for compressive sensing related problems. The proposed frameworks in this paper make a stronger case for investigation of sparse recovery algorithms tuned for parallel (e.g., GPU) implementation.

The saving from measurement depends on finding a good transform domain in which the signal is sufficiently sparse. Ideally, we would like that the number of samples is much smaller than the number of elements of the signal, that is, $n \ll N$, and then we can simply keep the samples instead of the full signal, thus saving the storage space and data I/O latencies. The ability of the CS framework, in terms of data reduction, is determined by how sparsely we can represent the features in the reconstruction stage. With domain knowledge or learning methods we can further improve the accuracy and efficiency of our experiments.

ACKNOWLEDGEMENTS

The aneurysm dataset is made available by Division of Neuroradiology, University of Erlangen, Germany. The multi-field turbulent combustion dataset is made available by Dr. Jacqueline Chen at Sandia Laboratories through US Department of Energy’s SciDAC Institute for Ultrascale Visualization. The research is supported by NSF Grants IIS-1048508 and CCF-1018149.

REFERENCES