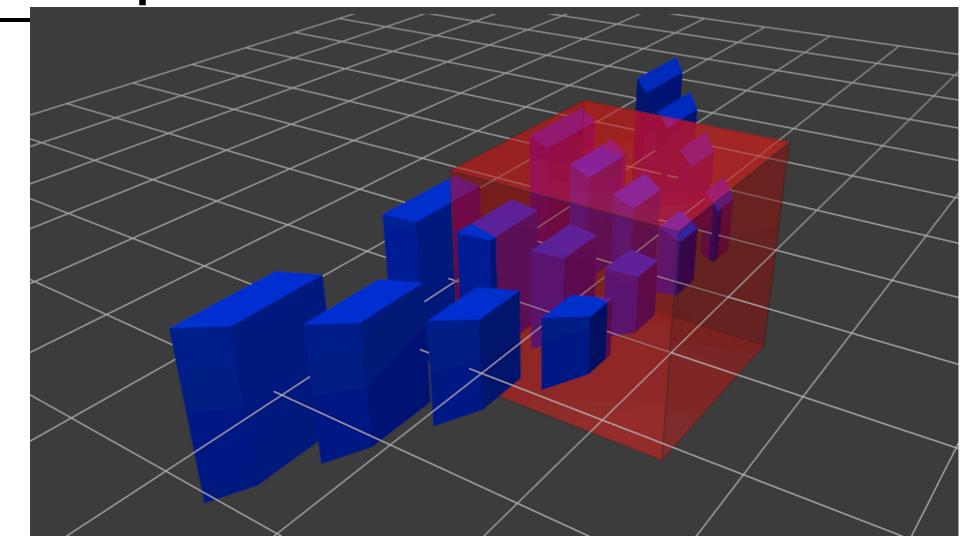
# CSE 5542 - Real Time Rendering Week 6

# OpenGL Perspective Matrix

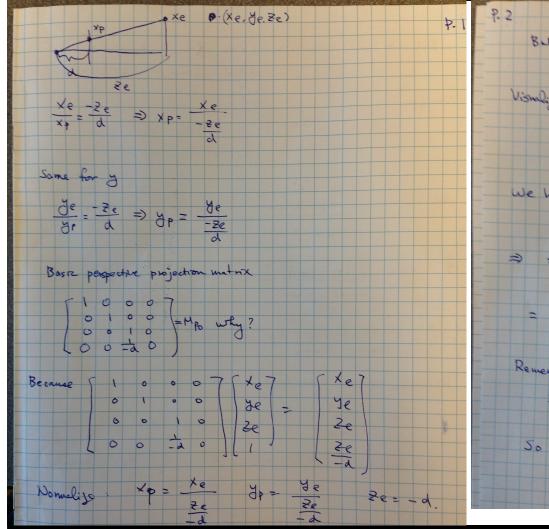
Courtesy: Prof. H-W. Shen

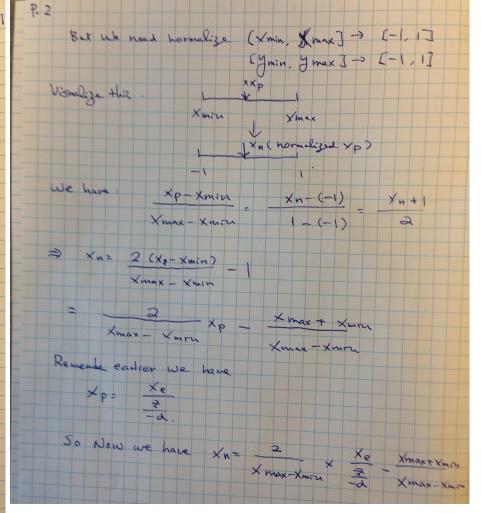


## Perspective Transform



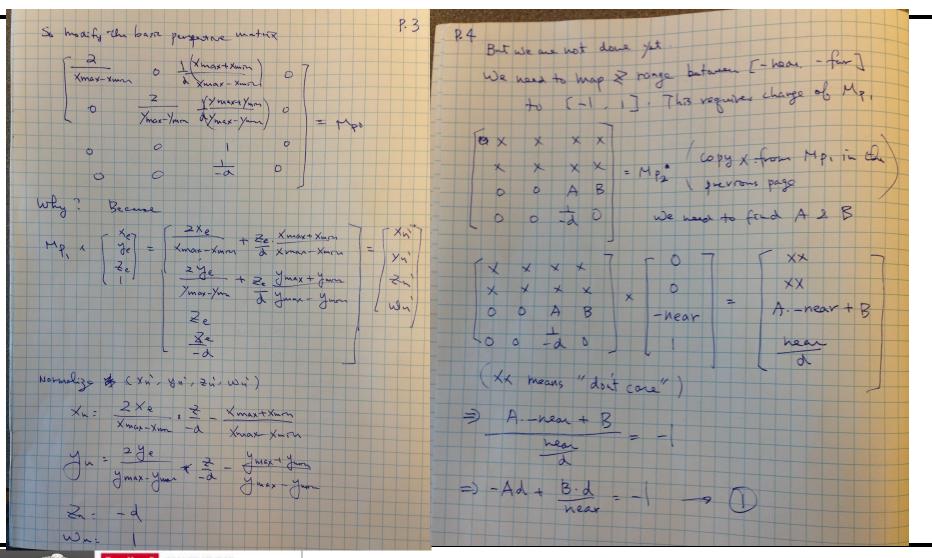




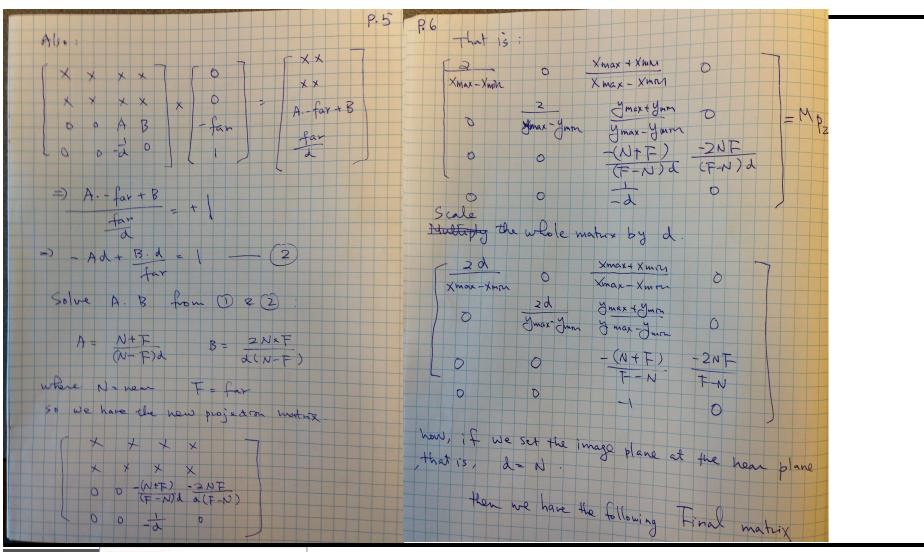






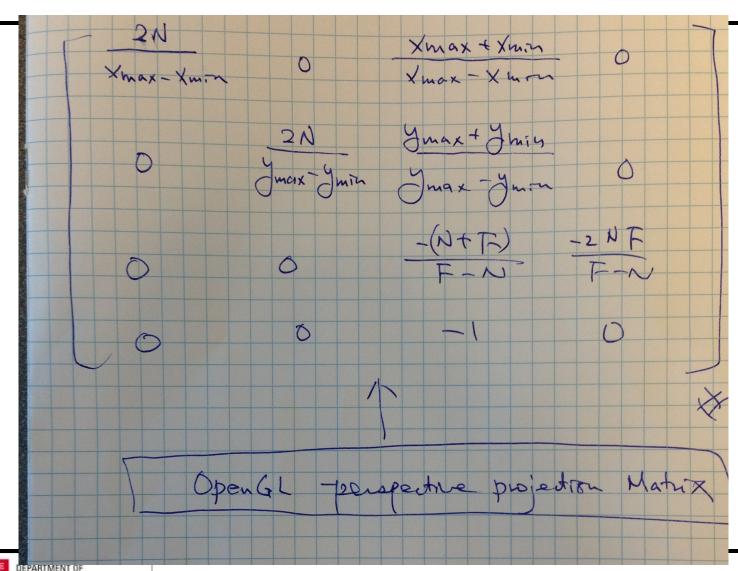














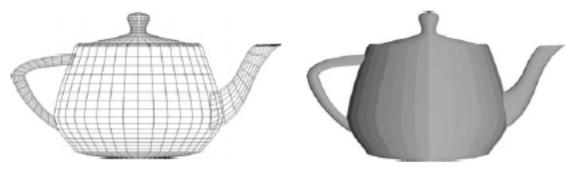


FIGURE 10.41 Rendered teapots.







# Modeling

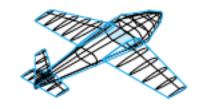


FIGURE 10.5 Model airplane



FIGURE 10.6 Cross-section curve.

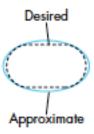
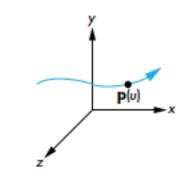


FIGURE 10.7 Approximatior of cross-section curve.



### Parametric Curve



$$x = x(u),$$

$$y = y(u),$$

$$z = z(u).$$

$$\frac{d\mathbf{p}(u)}{du} = \begin{bmatrix} \frac{dx(u)}{du} \\ \frac{dy(u)}{du} \\ \frac{dz(u)}{du} \end{bmatrix}$$

FIGURE 10.1 Parametric

### Parametric Curve

Consider a curve of the form2

$$\mathbf{p}(u) = \begin{bmatrix} x(u) \\ y(u) \\ z(u) \end{bmatrix}$$

A polynomial parametric curve of degree  $^{3}$  n is of the form

$$\mathbf{p}(u) = \sum_{k=0}^{n} u^k \mathbf{c}_k,$$

where each  $c_k$  has independent x, y, and z components; that is,

$$\mathbf{c}_{k} = \begin{bmatrix} c_{xk} \\ c_{yk} \\ c_{zk} \end{bmatrix}.$$

The n+1 column matrices  $\{c_k\}$  are the coefficients of p; they give us 3(n+1) degrees of freedom in how we choose the coefficients of a particular p. There is no coupling, however, among the x, y, and z components, so we can work with three independent equations, each of the form

$$p(u) = \sum_{k=0}^{n} u^k c_k,$$

where p is any one of x, y, or z. There are n+1 degrees of freedom in p(u). We can define our curves for any range interval of u:

$$u_{\min} \le u \le u_{\max}$$
;

however, with no loss of generality (see Exercise 10.3), we can assume that  $0 \le u \le 1$ . As the value of u varies over its range, we define a **curve segment**, as shown in Figure 10.3.

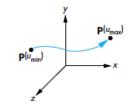


FIGURE 10.3 Curve segment.

#### Cubic Parametric Curves

$$\mathbf{p}(u) = \mathbf{c}_0 + \mathbf{c}_1 u + \mathbf{c}_2 u^2 + \mathbf{c}_3 u^3 = \sum_{k=0}^{3} \mathbf{c}_k u^k = \mathbf{u}^T \mathbf{c},$$

where

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix}, \qquad \mathbf{u} = \begin{bmatrix} 1 \\ u \\ u^2 \\ u^3 \end{bmatrix}, \qquad \mathbf{c}_k = \begin{bmatrix} c_{kx} \\ c_{ky} \\ c_{kz} \end{bmatrix}.$$

### **Control Points**

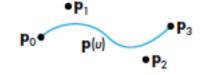


FIGURE 10.9 Curve segment and control points.

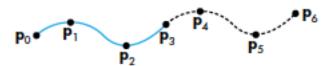


FIGURE 10.10 Joining of interpolating segments.



### Bezier

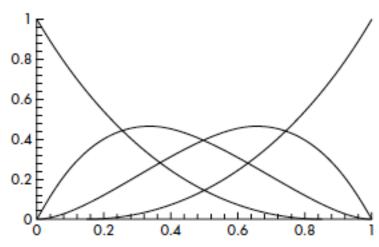


FIGURE 10.18 Blending polynomials for the Bézier cubic.

$$\mathbf{p}(u) = \sum_{t=0}^{3} b_t(u) \mathbf{p}_t,$$

$$\mathbf{p}(u) = \mathbf{b}(u)^T \mathbf{p},$$

$$\mathbf{b}(u) = \mathbf{M}_{B}^{T} \mathbf{u} = \begin{bmatrix} (1-u)^{3} \\ 3u(1-u)^{2} \\ 3u^{2}(1-u) \\ u^{3} \end{bmatrix}.$$

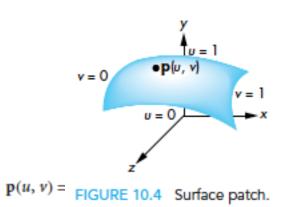
$$\mathbf{M}_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}.$$

### Parametric Surface

$$\mathbf{p}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix}$$

$$\begin{aligned}
x &= x(u, v), \\
y &= y(u, v), \\
z &= z(u, v),
\end{aligned}
\qquad \frac{\partial \mathbf{p}}{\partial u} = \begin{bmatrix} \frac{\partial x(u, v)}{\partial u} \\ \frac{\partial y(u, v)}{\partial u} \\ \frac{\partial z(u, v)}{\partial u} \end{bmatrix} \qquad \frac{\partial \mathbf{p}}{\partial v} = \begin{bmatrix} \frac{\partial x(u, v)}{\partial v} \\ \frac{\partial y(u, v)}{\partial v} \\ \frac{\partial z(u, v)}{\partial v} \end{bmatrix}$$

 $\nu/\partial \nu$ .



### Parametric Surface

$$\mathbf{p}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix} = \sum_{t=0}^{n} \sum_{j=0}^{m} \mathbf{c}_{tj} u^{t} v^{j}.$$

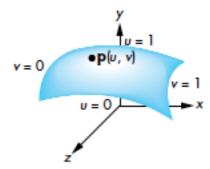


FIGURE 10.4 Surface patch.

### Bezier Surface Patches

$$\mathbf{p}(u, v) = \sum_{t=0}^{3} \sum_{j=0}^{3} b_{t}(u)b_{j}(v)\mathbf{p}_{tj} = \mathbf{u}^{T}\mathbf{M}_{B}\mathbf{P}\mathbf{M}_{B}^{T}\mathbf{v}.$$

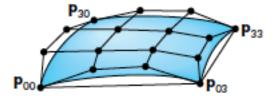


FIGURE 10.20 Bézier patch.

### Subdivision

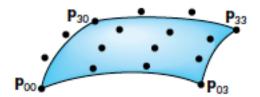


FIGURE 10.37 Cubic Bézier surface.

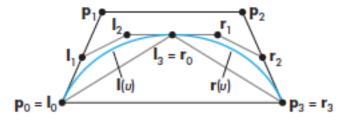
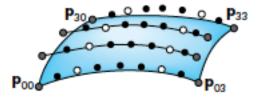
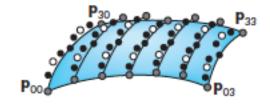


FIGURE 10.34 Convex hulls and control points.



- · New points created by subdivision
- o Old points discarded after subdivision
- Old points retained after subdivision

FIGURE 10.38 First subdivision of surface.



- · New points created by subdivision
- Old points discarded after subdivision
- Old points retained after subdivision

FIGURE 10.39 Points after second subdivision.

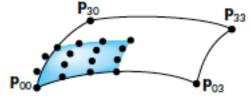


FIGURE 10.40 Subdivided quadrant.



### Code

```
void draw_patch(point4 p[4][4])
                                                                void divide_patch(point4 p[4][4], int n)
    points[n] = p[0][0];
                                                                    point4 q[4][4], r[4][4], s[4][4], t[4][4];
    n++;
                                                                    point4 a[4][4], b[4][4];
    points[n] = p[3][0];
                                                                    int k:
                                                                    if(n==0) draw_patch(p); /* draw patch if recursion done */
    n++:
    points[n] = p[3][3];
                                                                /* subdivide curves in u direction, transpose results, divide
    n++:
                                                                in u direction again (equivalent to subdivision in v) */
    points[n] = p[0][3];
    n++:
                                                                    else
                                                                        for(k=0; k<4; k++) divide_curve(p[k], a[k], b[k]);
void divide_curve(point4 c[4], point4 r[4], point4 1[4])
                                                                        transpose4(a);
                                                                        transpose4(b);
/* division of convex hull of Bezier curve */
                                                                        for(k=0; k<4; k++)
   int i:
                                                                           divide_curve(a[k], q[k], r[k]);
  point4 t:
                                                                           divide_curve(b[k], s[k], t[k]);
   for(i=0:i<3:i++)
      1[0][i]=c[0][i]:
      r[3][i]=c[3][i]:
                                                                /* recursive division of 4 resulting patches */
      1[1][i]=(c[1][i]+c[0][i])/2;
      r[2][i]=(c[2][i]+c[3][i])/2;
                                                                        divide_patch(q, n-1);
      t[i]=(1[1][i]+r[2][i])/2;
                                                                        divide_patch(r, n-1);
      1[2][i]=(t[i]+1[1][i])/2:
      r[1][i]=(t[i]+r[2][i])/2;
                                                                        divide_patch(s, n-1);
      1[3][i]=r[0][i]=(1[2][i]+r[1][i])/2;
                                                                        divide_patch(t, n-1);
   for(i=0; i<4; i++) 1[i][3] = r[i][3] = 1.0;
```

### Code for GL

Courtesy:

http://www.opengl-tutorial.org/beginners-tutorials/tutorial-3-matrices/



### **GLM**

OpenGL Mathematics (GLM) is a header only C++ mathematics library for graphics software based on the OpenGL Shading Language (GLSL).

Provides classes and functions designed and implemented following as strictly as possible the GLSL conventions and functionalities.

When a programmer knows GLSL, he knows GLM as well, making it really easy to use.



#### **C++**

```
glm::mat4 myMatrix;
glm::vec4 myVector;

// fill myMatrix and myVector somehow
glm::vec4 transformedVector = myMatrix * myVector;

// Again, in this order ! this is important.
```

### **GLSL**

```
mat4 myMatrix;
vec4 myVector;

// fill myMatrix and myVector somehow
vec4 transformedVector = myMatrix * myVector;

// Yeah, it's pretty much the same than GLM
```

# Identity

glm::mat4 myldentityMatrix = glm::mat4(1.0f);

#### **Translate**

#### GLM -

```
#include <glm/transform.hpp> // after <glm/glm.hpp> glm::mat4 myMatrix = glm::translate(10.0f, 0.0f, 0.0f); glm::vec4 myVector(10.0f, 10.0f, 10.0f, 0.0f); glm::vec4 transformedVector = myMatrix * myVector;
```

GLSL - vec4 transformedVector = myMatrix \* myVector;



### Scaling

// Use #include <glm/gtc/matrix\_transform.hpp> and #include <glm/gtx/transform.hpp>

glm::mat4 myScalingMatrix = glm::scale(2.0f, 2.0f, 2.0f);

#### Rotation

```
// Use #include <glm/gtc/matrix_transform.hpp> and #include
<glm/gtx/transform.hpp>
glm::vec3 myRotationAxis( ??, ??, ??);
glm::rotate( angle_in_degrees, myRotationAxis );
```

### Accumulating Transforms

TransformedVector =

TranslationMatrix \* RotationMatrix \* ScaleMatrix \* OriginalVector;

### In Code

#### **GLM**

glm::mat4 myModelMatrix = myTranslationMatrix \* myRotationMatrix \* myScaleMatrix;

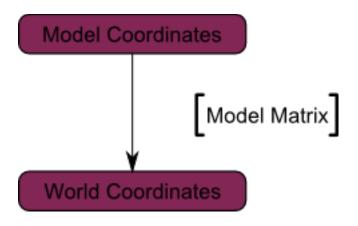
glm::vec4 myTransformedVector = myModelMatrix \* myOriginalVector;

#### **GLSL**

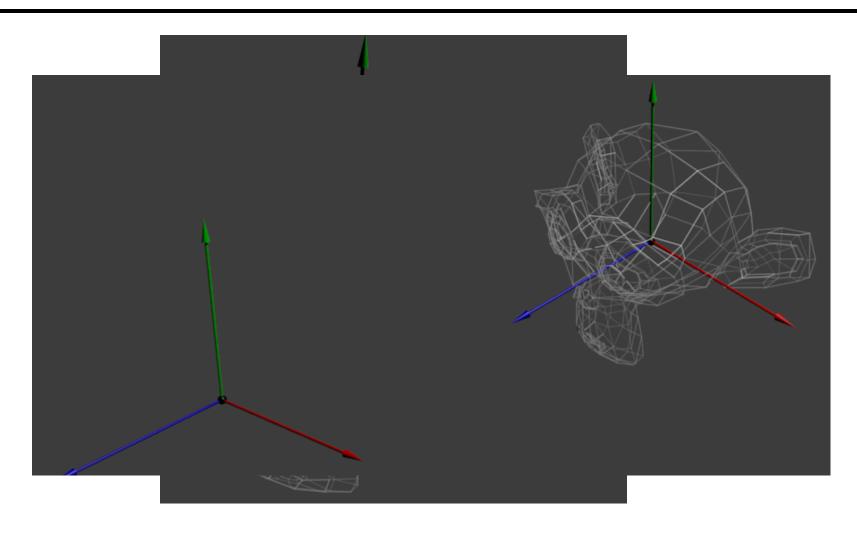
```
mat4 transform = mat2 * mat1;
vec4 out vec = transform * in vec;
```



### In Diagrams



### In Pictures

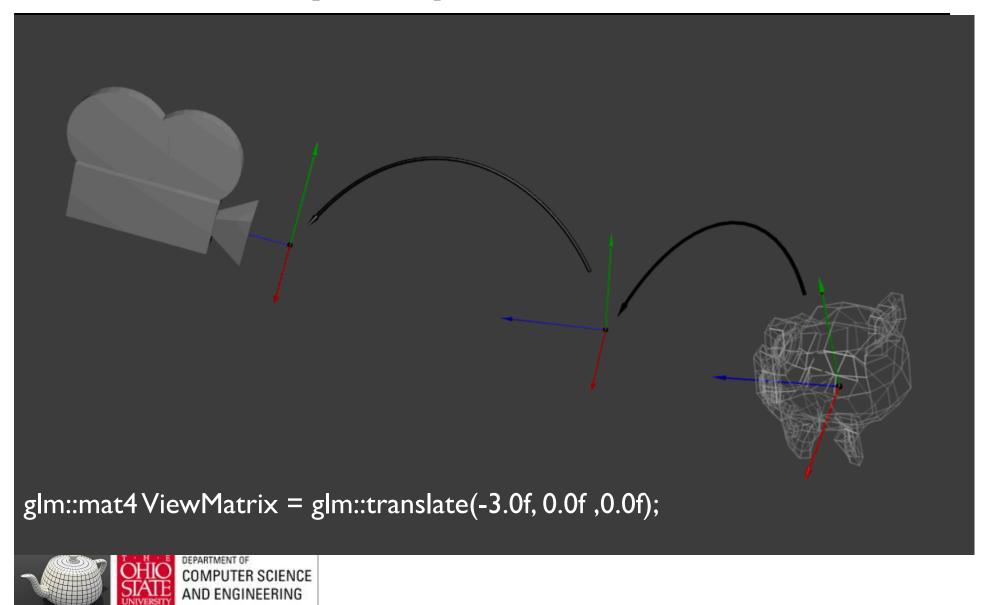








## Camera/Eye Space



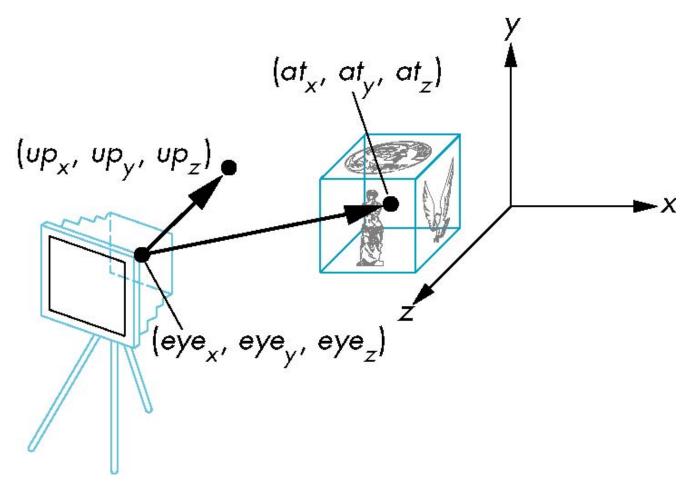
## Camera/Eye Space

```
glm::mat4 CameraMatrix = glm::LookAt (
  cameraPosition, // the position of your camera, in world space
  cameraTarget, // where you want to look at, in world space
            // probably glm::vec3(0,1,0),
  upVector
                // but (0,-1,0) would make you looking upside-down
);
                                                         Model Coordinates
                                                                      Model Matrix
    Transform objects from world to eye space
                                                         World Coordinates
                                                                      View Matrix
                                                        Camera Coordinates
```

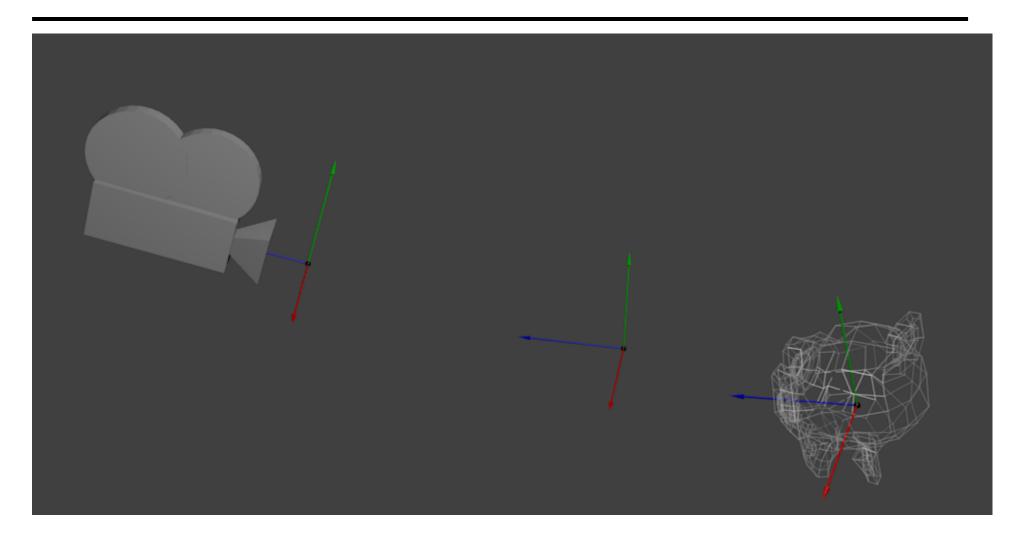


### gluLookAt

#### LookAt(eye, at, up)



### Camera Coordinate Frame





# Camera Space

#### Right hand coordinate system

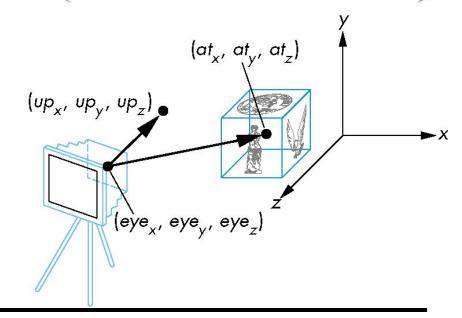
$$\vec{n} = at - eye$$

$$\vec{n} = \frac{\vec{n}}{\|\vec{n}\|}$$

$$\vec{u} = up \times \vec{n}$$

$$v = \vec{n} \times \vec{u}$$

$$\mathbf{V} = \begin{pmatrix} u_x & u_y & u_z & -eye \cdot \mathbf{u} \\ v_x & v_y & v_z & -eye \cdot \mathbf{v} \\ n_x & n_y & n_z & -eye \cdot \mathbf{n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



## Old Style

```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(0,0,1,0,0,0,0,1,0);
    ...
}
```

#### New World

- Create a view matrix

```
view = glm::lookAt(glm::vec3(0.0, 2.0, 2.0), glm::vec3(0.0, 0.0, 0.0), glm::vec3(0.0, 1.0, 0.0);
```

- Combine with modeling matrices

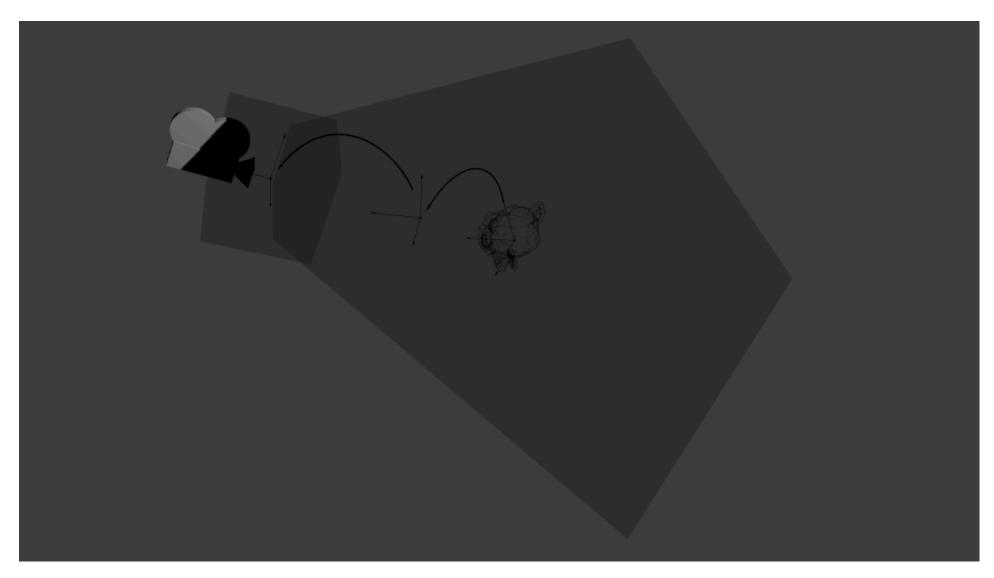
```
glm::mat4 model = glm::mat4(1.0f);
model = glm::rotate(model, angle, glm::vec3(0.0f, 0.0f, 1.0f));
model = glm::scale(model, scale_size, scale_size, scale_size);
glm::mat4 modelview = view * model;
```

# Working with Old World

```
glMatrixMode(GL_MODELVIEW);
  glLoadMatrixf(&modelview[0][0]);
// begin to draw your geometry
...
```

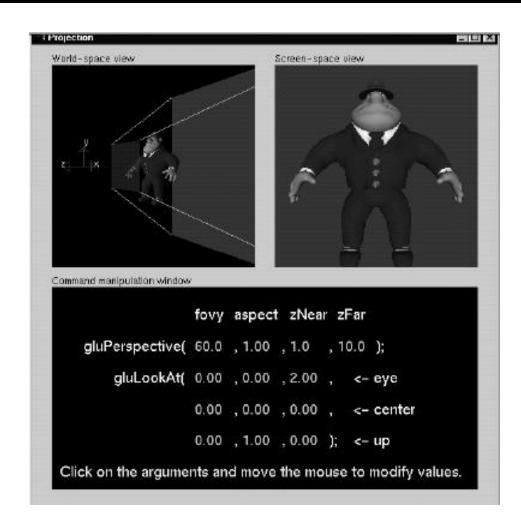
# Projection Matrices





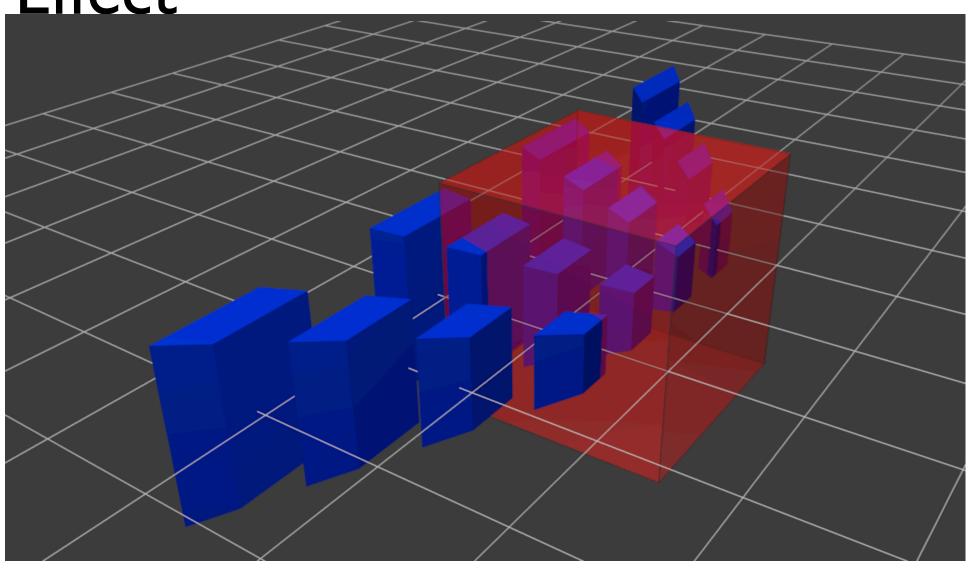


#### Demo



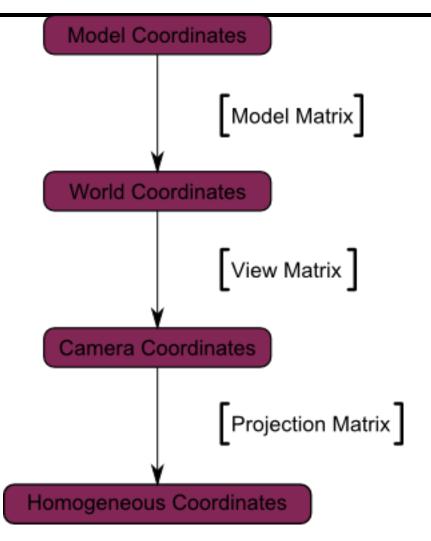
#### In Code

**Effect** 





### In Diagrams





#### More Code

```
C++:compute the matrix
glm::mat4 MVPmatrix = projection * view * model;
// Remember:inverted!

// GLSL:apply it
transformed_vertex = MVP * in_vertex;
```

### Combined



#### Generate Matrix

```
// Projection matrix: 45°
//Field of View, 4:3 ratio, display range: 0.1 unit <-> 100 units
glm::mat4 Projection = glm::perspective(45.0f, 4.0f / 3.0f, 0.1f, 100.0f);
// Camera matrix
glm::mat4 View = glm::lookAt(
  glm::vec3(4,3,3), // Camera is at (4,3,3), in World Space
  glm::vec3(0,0,0), // and looks at the origin
  glm::vec3(0,1,0) // Head is up (set to 0,-1,0 to look upside-down)
);
// Model matrix : an identity matrix (model will be at the origin)
glm::mat4 Model = glm::mat4(1.0f); // Changes for each model !
// Our ModelViewProjection : multiplication of our 3 matrices
glm::mat4 MVP = Projection *View * Model;
// Remember, matrix multiplication is the other way around
```



#### GLSL Takes Over

```
// Get a handle for our "MVP" uniform.

// Only at initialisation time.

GLuint MatrixID = glGetUniformLocation(programID, "MVP");

// Send our transformation to the currently bound shader,

// in the "MVP" uniform

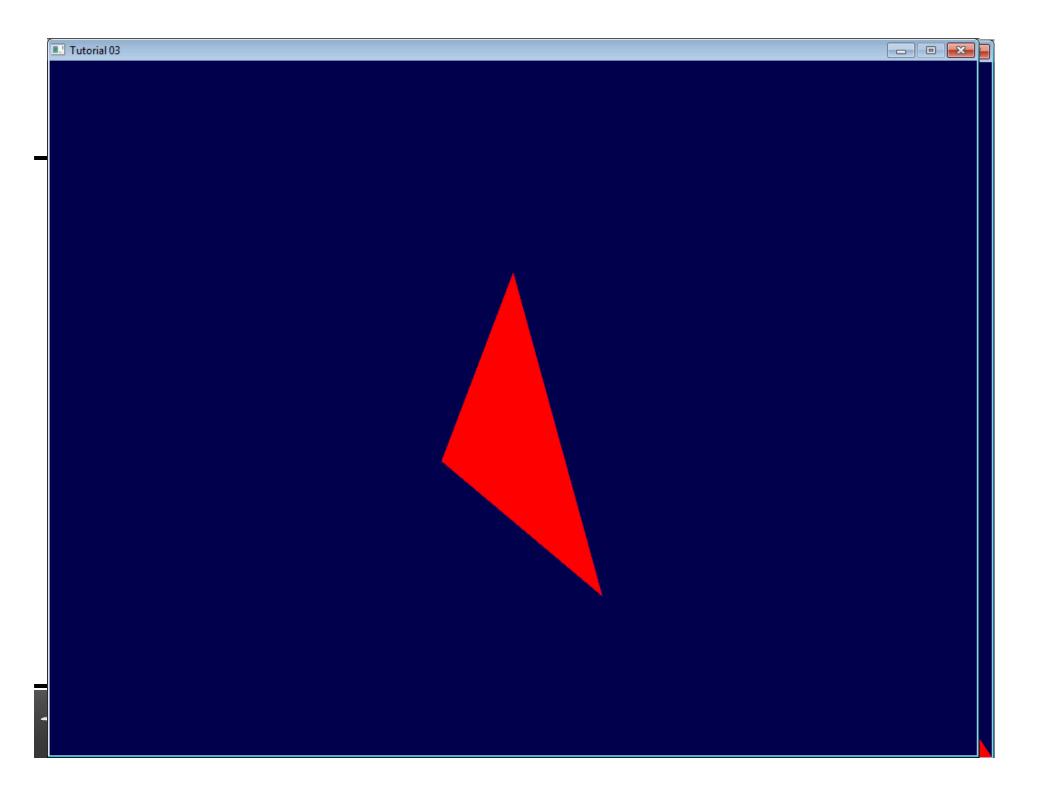
// For each model you render, since the MVP will be different

// (at least the M part)

glUniformMatrix4fv(MatrixID, I, GL_FALSE, &MVP[0][0]);
```

#### Use It

```
in vec3 vertexPosition modelspace;
uniform mat4 MVP;
void main(){
// Output position of the vertex, in clip space : MVP * position
  vec4 v = vec4(vertexPosition_modelspace, I);
// Transform an homogeneous 4D vector, remember ?
  gl Position = MVP * v;
```

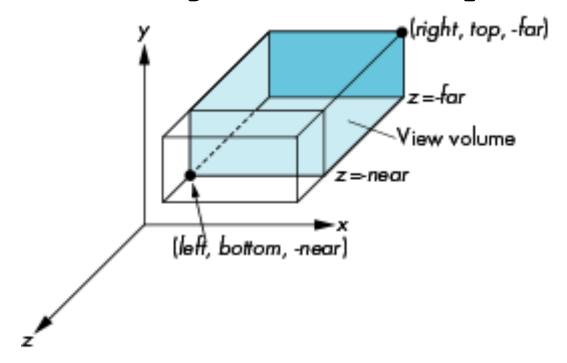


# Old Style



#### OpenGL Orthogonal Viewing

Ortho(left, right, bottom, top, near, far)

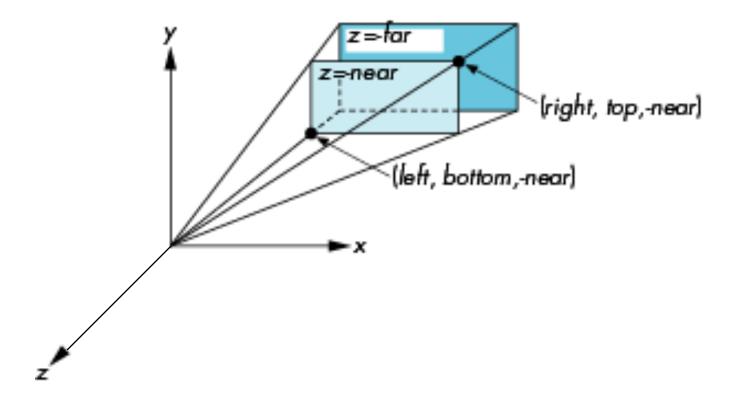


near and far measured from camera



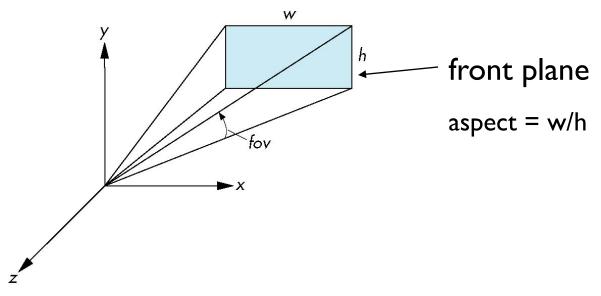
#### OpenGL Perspective

Frustum(left,right,bottom,top,near,far)



#### Using Field of View

- With Frustum it is often difficult to get the desired view
- Perpective(fovy, aspect, near, far) often provides a better interface



## Old Style

```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT);
    glMatrixMode(GL_PROJETION);
    glLoadIdentity();
    gluPerspective(fove, aspect, near, far);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(0,0,1,0,0,0,0,1,0);
    my_display(); // your display routine
}
```

#### Can Still GLM

- Set up the projection matrix

```
glm::mat4 projection = glm::mat4(1.0f);
projection = glm::perspective(60.0f, 1.0f, 1 f, 100.0f);
```

- Load the matrix to GL\_PROJECTION

```
glMatrixMode(GL_PROJECTION); glLoadMatrixf(&projection[0][0]);
```

### Next

