CSE 5542 - Real Time Rendering
Week 6,7,8

THE OHIO STATE UNIVERSITY
DEPARTMENT OF
COMPUTER SCIENCE
AND ENGINEERING

OpenGL Perspective Matrix

Courtesy: Prof. H-W. Shen

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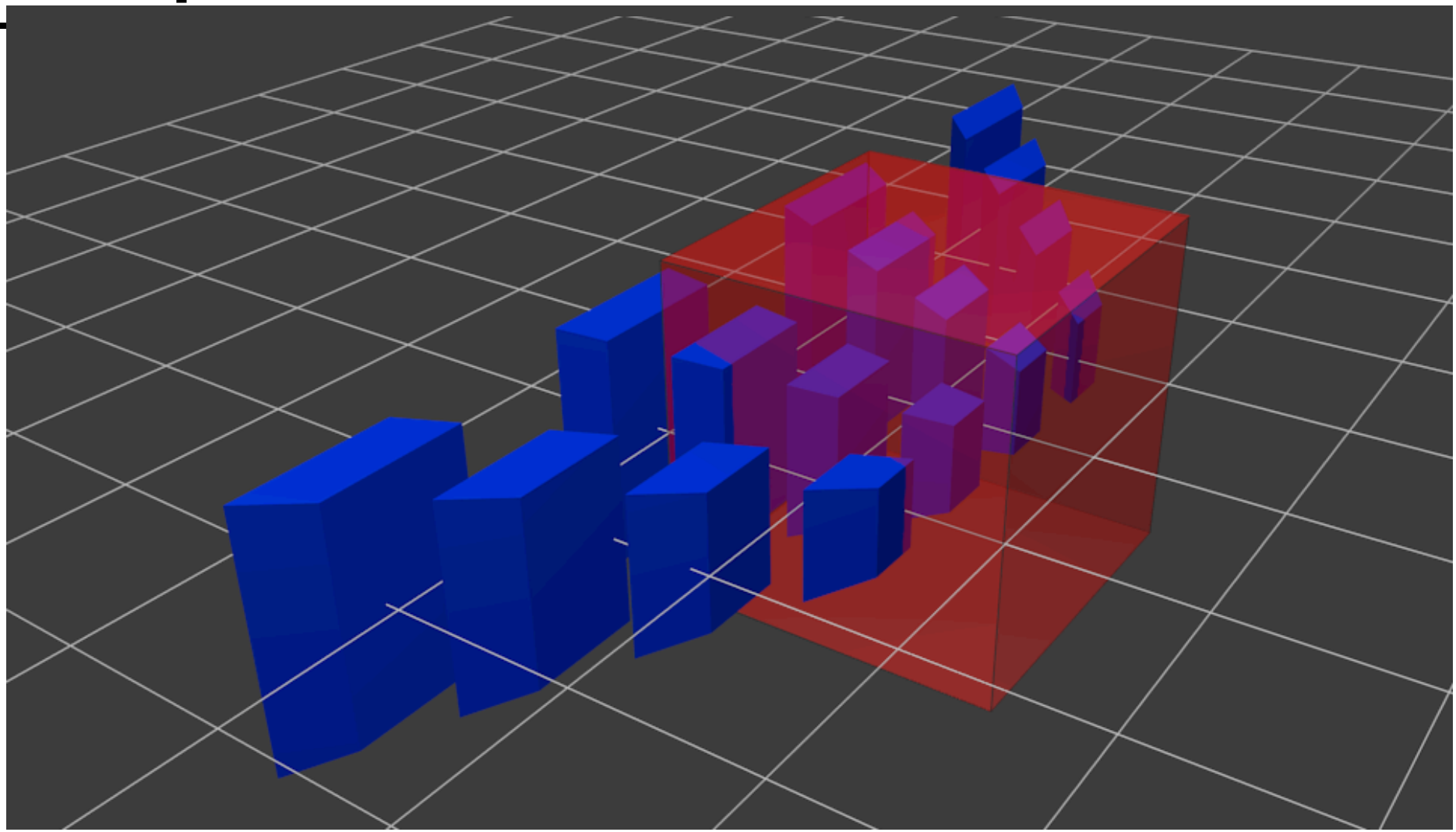
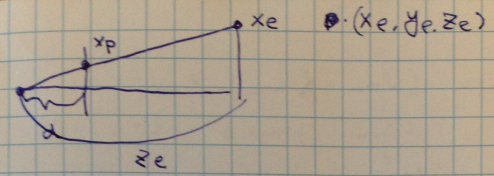
Perspective Transform





$$\frac{x_e}{x_p} = \frac{-z_e}{d} \Rightarrow x_p = \frac{x_e}{\frac{-z_e}{d}}$$

Same for y

$$\frac{y_e}{y_p} = \frac{-z_e}{d} \Rightarrow y_p = \frac{y_e}{\frac{-z_e}{d}}$$

Basic perspective projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{-d} & 0 \end{bmatrix} = M_{Po} \quad \text{why?}$$

Because

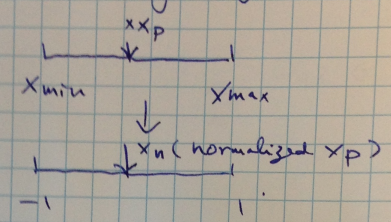
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{-d} & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \\ z_e \\ \frac{z_e}{-d} \end{bmatrix}$$

Normalize: $x_p = \dfrac{x_e}{\frac{z_e}{-d}}$ $\quad y_p = \dfrac{y_e}{\frac{z_e}{-d}}$ $\quad z_e = -d.$

But we need normalize $[x_{min}, x_{max}] \to [-1, 1]$

$[y_{min}, y_{max}] \to [-1, 1]$

Visualize this.



We have:

$$\frac{x_p - x_{min}}{x_{max} - x_{min}} = \frac{x_n - (-1)}{1 - (-1)} = \frac{x_n + 1}{2}$$

$$\Rightarrow x_n = \frac{2(x_p - x_{min})}{x_{max} - x_{min}} - 1$$

$$= \frac{2}{x_{max} - x_{min}} x_p - \frac{x_{max} + x_{min}}{x_{max} - x_{min}}$$

Remember earlier we have

$$x_p = \frac{x_e}{\frac{z}{-d}}.$$

So Now we have $x_n = \dfrac{2}{x_{max} - x_{min}} \times \dfrac{x_e}{\frac{z}{-d}} - \dfrac{x_{max} + x_{min}}{x_{max} - x_{min}}$

So modify the basic perspective matrix

$$\begin{bmatrix} \frac{2}{x_{max}-x_{min}} & 0 & \frac{1}{d}\left(\frac{x_{max}+x_{min}}{x_{max}-x_{min}}\right) & 0 \\ 0 & \frac{2}{y_{max}-y_{min}} & \frac{1}{d}\left(\frac{y_{max}+y_{min}}{y_{max}-y_{min}}\right) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{-d} & 0 \end{bmatrix} = M_{p_1}$$

Why? Because

$$M_{p_1} \times \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2x_e}{x_{max}-x_{min}} + \frac{z_e}{d} \cdot \frac{x_{max}+x_{min}}{x_{max}-x_{min}} \\ \frac{2y_e}{y_{max}-y_{min}} + \frac{z_e}{d} \frac{y_{max}+y_{min}}{y_{max}-y_{min}} \\ z_e \\ \frac{z_e}{-d} \end{bmatrix} = \begin{bmatrix} x_n' \\ y_n' \\ z_n' \\ w_n' \end{bmatrix}$$

Normalize (x_n', y_n', z_n', w_n')

$$x_n = \frac{2x_e}{x_{max}-x_{min}} \times \frac{z}{-d} - \frac{x_{max}+x_{min}}{x_{max}-x_{min}}$$

$$y_n = \frac{2y_e}{y_{max}-y_{min}} \times \frac{z}{-d} - \frac{y_{max}+y_{min}}{y_{max}-y_{min}}$$

$$z_n = -d$$

$$w_n = 1$$

But we are not done yet.

We need to map z range between $[-near, -far]$ to $[-1, 1]$. This requires change of M_{p_1}

$$\begin{bmatrix} x & x & x & x \\ x & x & x & x \\ 0 & 0 & A & B \\ 0 & 0 & \frac{1}{-d} & 0 \end{bmatrix} = M_{p_2}$$

(copy x from M_{p_1} in the previous page)

We need to find A & B

$$\begin{bmatrix} x & x & x & x \\ x & x & x & x \\ 0 & 0 & A & B \\ 0 & 0 & \frac{1}{-d} & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -near \\ 1 \end{bmatrix} = \begin{bmatrix} xx \\ xx \\ A \cdot -near + B \\ \frac{near}{d} \end{bmatrix}$$

(xx means "don't care")

$$\Rightarrow \frac{A \cdot -near + B}{\frac{near}{d}} = -1$$

$$\Rightarrow -Ad + \frac{B \cdot d}{near} = -1 \rightarrow (1)$$

Also:

$$\begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ 0 & 0 & A & B \\ 0 & 0 & -\frac{1}{d} & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -far \\ 1 \end{bmatrix} = \begin{bmatrix} \times \\ \times \\ A \cdot -far + B \\ \frac{far}{d} \end{bmatrix}$$

$$\Rightarrow \frac{A \cdot -far + B}{\frac{far}{d}} = +1$$

$$\Rightarrow -Ad + \frac{B \cdot d}{far} = 1 \quad \text{—— (2)}$$

Solve A. B from (1) & (2):

$$A = \frac{N+F}{(N-F)d} \qquad B = \frac{2N \times F}{d(N-F)}$$

where N = near F = far

so we have the new projection matrix.

$$\begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ 0 & 0 & \frac{-(N+F)}{(F-N)d} & \frac{-2NF}{d(F-N)} \\ 0 & 0 & -\frac{1}{d} & 0 \end{bmatrix}$$

That is:

$$\begin{bmatrix} \frac{2}{x_{max}-x_{min}} & 0 & \frac{x_{max}+x_{min}}{x_{max}-x_{min}} & 0 \\ 0 & \frac{2}{y_{max}-y_{min}} & \frac{y_{max}+y_{min}}{y_{max}-y_{min}} & 0 \\ 0 & 0 & \frac{-(N+F)}{(F-N)d} & \frac{-2NF}{(F-N)d} \\ 0 & 0 & \frac{1}{-d} & 0 \end{bmatrix} = M_{p_2}$$

Scale ~~Multiply~~ the whole matrix by d.

$$\begin{bmatrix} \frac{2d}{x_{max}-x_{min}} & 0 & \frac{x_{max}+x_{min}}{x_{max}-x_{min}} & 0 \\ 0 & \frac{2d}{y_{max}-y_{min}} & \frac{y_{max}+y_{min}}{y_{max}-y_{min}} & 0 \\ 0 & 0 & -\frac{(N+F)}{F-N} & \frac{-2NF}{F-N} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

now, if we set the image plane at the near plane, that is, $d = N$.

then we have the following Final matrix

$$\begin{bmatrix} \frac{2N}{x_{max}-x_{min}} & 0 & \frac{x_{max}+x_{min}}{x_{max}-x_{min}} & 0 \\ 0 & \frac{2N}{y_{max}-y_{min}} & \frac{y_{max}+y_{min}}{y_{max}-y_{min}} & 0 \\ 0 & 0 & \frac{-(N+F)}{F-N} & \frac{-2NF}{F-N} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

OpenGL perspective projection Matrix

Code for GL

Courtesy:
http://www.opengl-tutorial.org/beginners-tutorials/tutorial-3-matrices/

GLM

OpenGL Mathematics (GLM) is a header only C++ mathematics library for graphics software based on the OpenGL Shading Language (GLSL).

Provides classes and functions designed and implemented following as strictly as possible the GLSL conventions and functionalities.

When a programmer knows GLSL, he knows GLM as well, making it really easy to use.

C++

```
glm::mat4 myMatrix;
glm::vec4 myVector;

// fill myMatrix and myVector somehow

glm::vec4 transformedVector = myMatrix * myVector;

 // Again, in this order ! this is important.
```

GLSL

```
mat4 myMatrix;
vec4 myVector;

// fill myMatrix and myVector somehow
vec4 transformedVector = myMatrix * myVector;

// Yeah, it's pretty much the same than GLM
```

Identity

```
glm::mat4 myIdentityMatrix = glm::mat4(1.0f);
```

Translate

GLM -

```
#include <glm/transform.hpp> // after <glm/glm.hpp>
glm::mat4 myMatrix = glm::translate(10.0f, 0.0f, 0.0f);
glm::vec4 myVector(10.0f, 10.0f, 10.0f, 0.0f);
glm::vec4 transformedVector = myMatrix * myVector;
```

GLSL -

```
vec4 transformedVector = myMatrix * myVector;
```

Scaling

```
// Use #include <glm/gtc/matrix_transform.hpp> and #include
<glm/gtx/transform.hpp>

glm::mat4 myScalingMatrix = glm::scale(2.0f, 2.0f ,2.0f);
```

Rotation

```
// Use #include <glm/gtc/matrix_transform.hpp> and #include
<glm/gtx/transform.hpp>

glm::vec3 myRotationAxis( ??, ??, ??);

glm::rotate( angle_in_degrees, myRotationAxis );
```

Accumulating Transforms

TransformedVector =
TranslationMatrix * RotationMatrix * ScaleMatrix * OriginalVector;

In Code

GLM

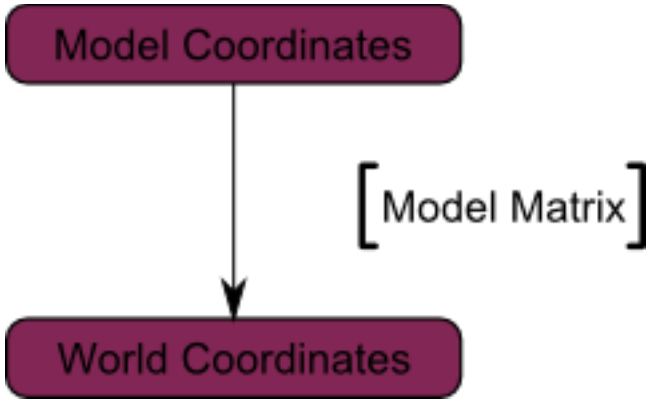
```
glm::mat4 myModelMatrix = myTranslationMatrix * myRotationMatrix *
myScaleMatrix;

glm::vec4 myTransformedVector = myModelMatrix * myOriginalVector;
```

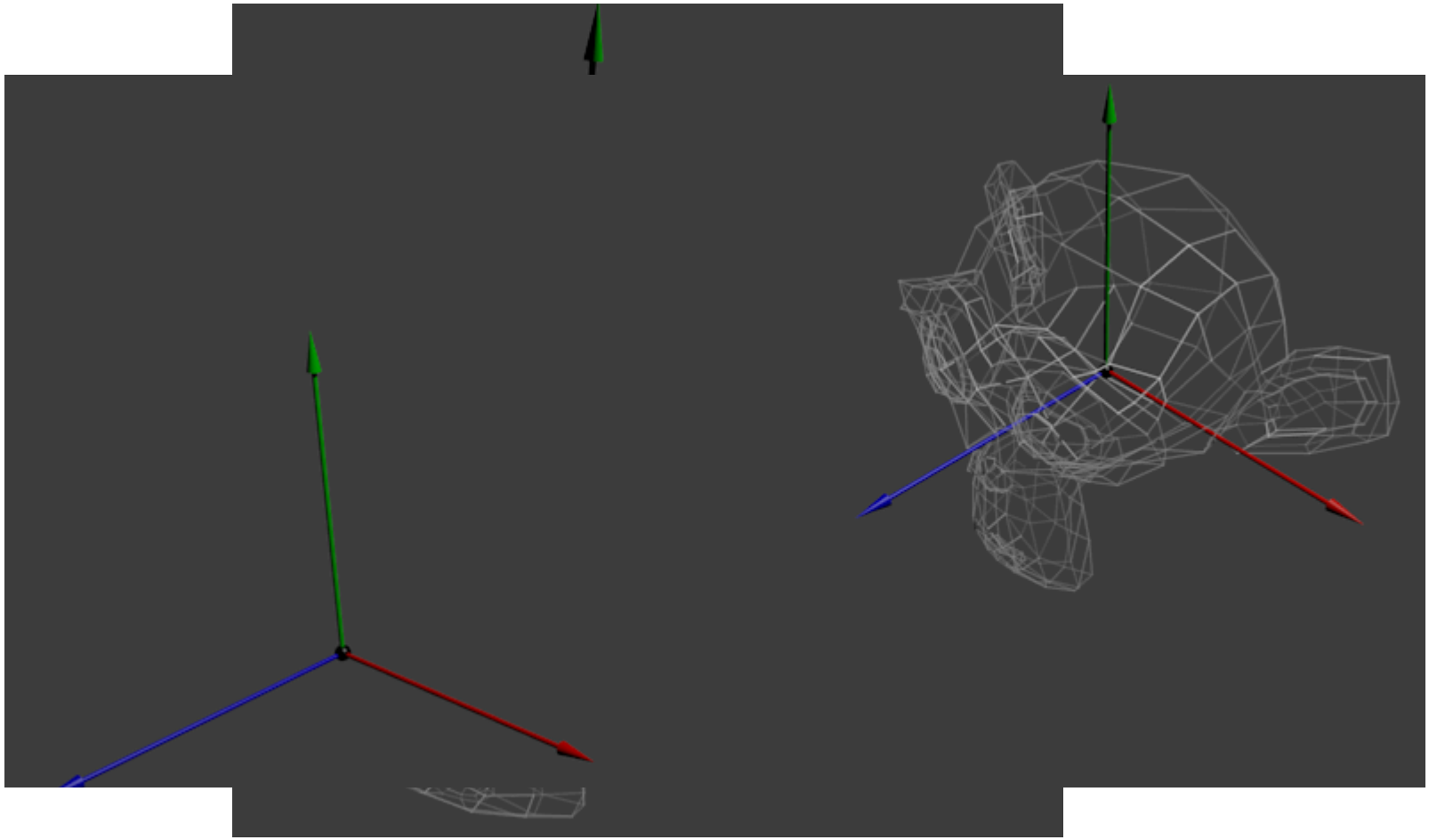
GLSL

```
mat4 transform = mat2 * mat1;
vec4 out_vec = transform * in_vec;
```

In Diagrams

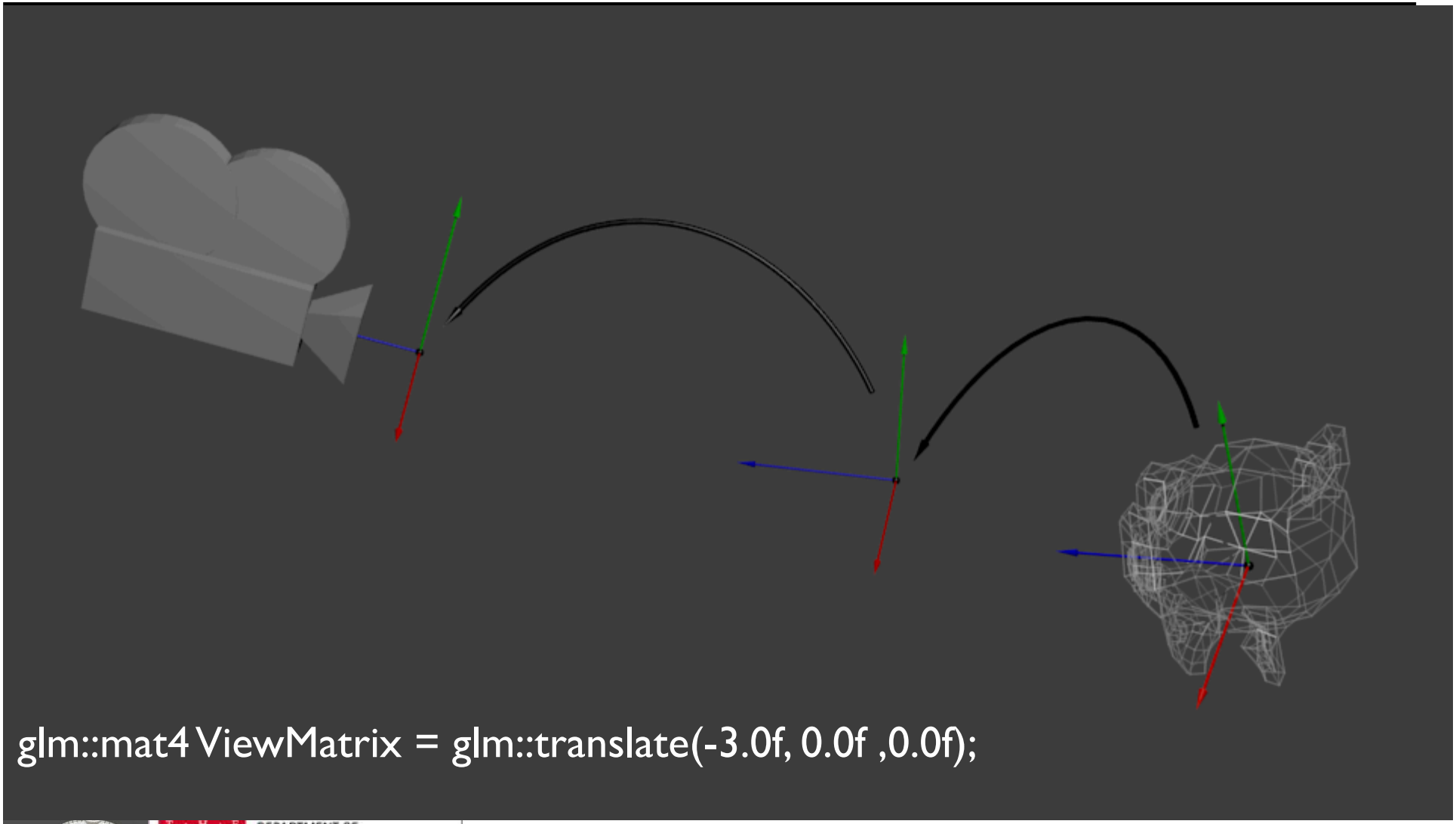


In Pictures




Camera

Camera/Eye Space

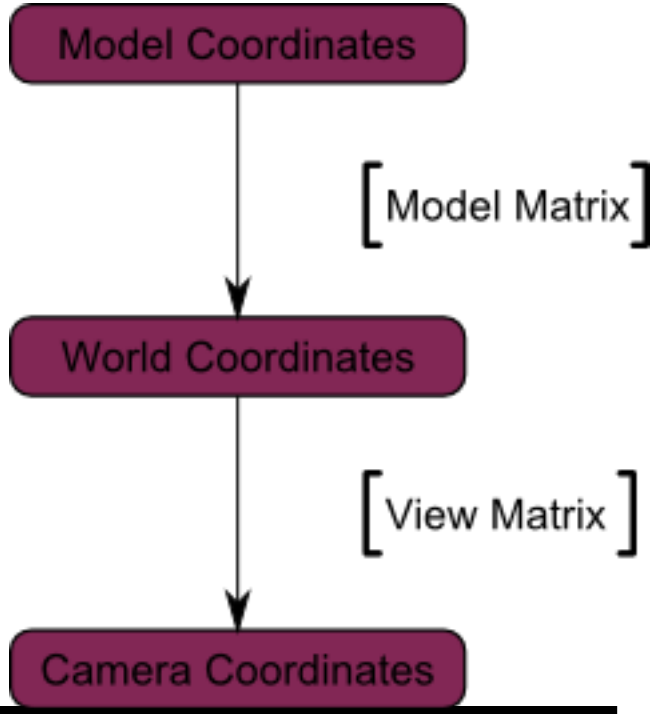


```
glm::mat4 ViewMatrix = glm::translate(-3.0f, 0.0f ,0.0f);
```

Camera/Eye Space

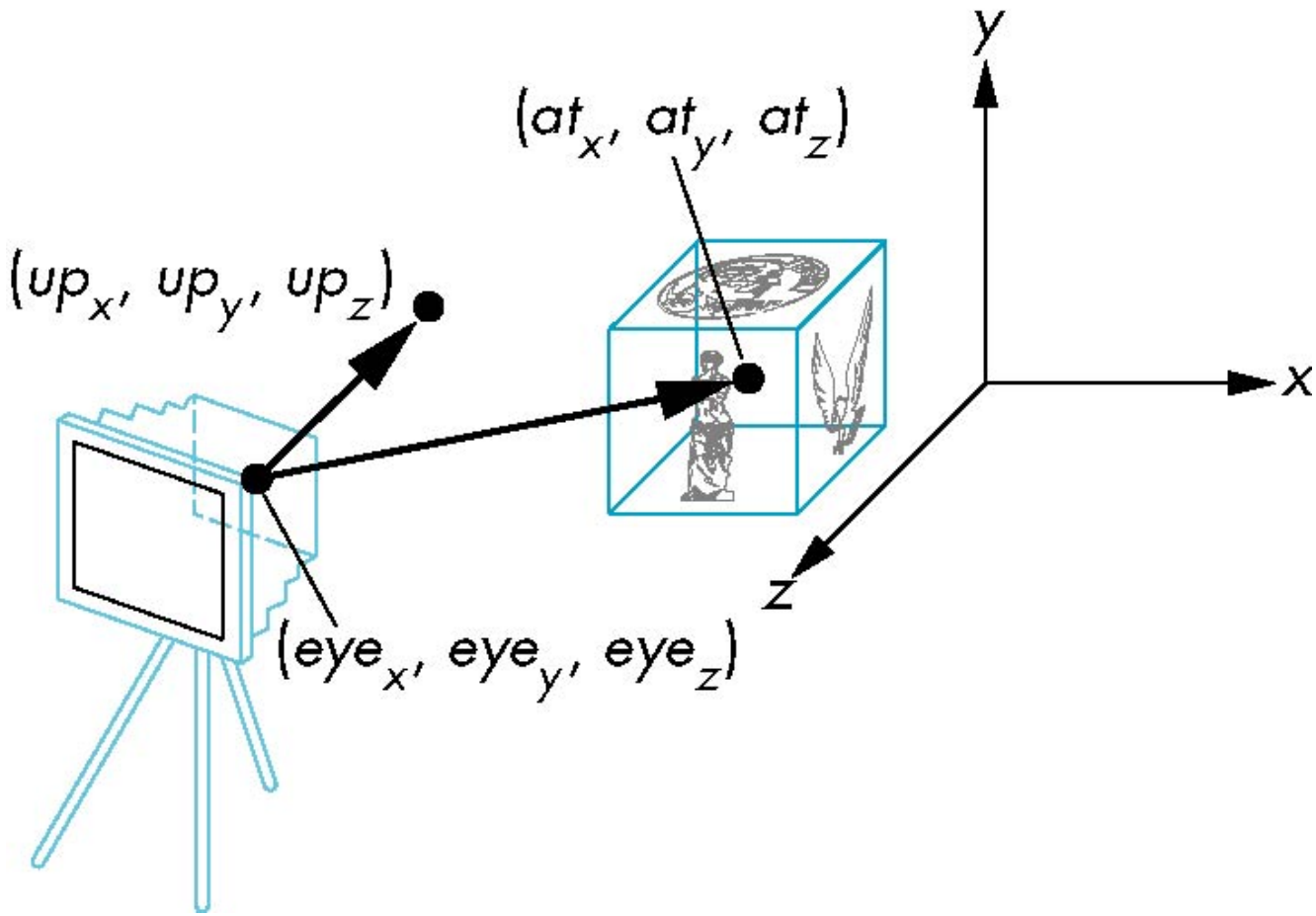
```
glm::mat4 CameraMatrix = glm::LookAt (
    cameraPosition, // the position of your camera, in world space
    cameraTarget,   // where you want to look at, in world space
    upVector        // probably glm::vec3(0,1,0),
                    // but (0,-1,0) would make you looking upside-down
);
```

<u>Transform objects from world to eye space</u>



gluLookAt

LookAt(eye, at, up)



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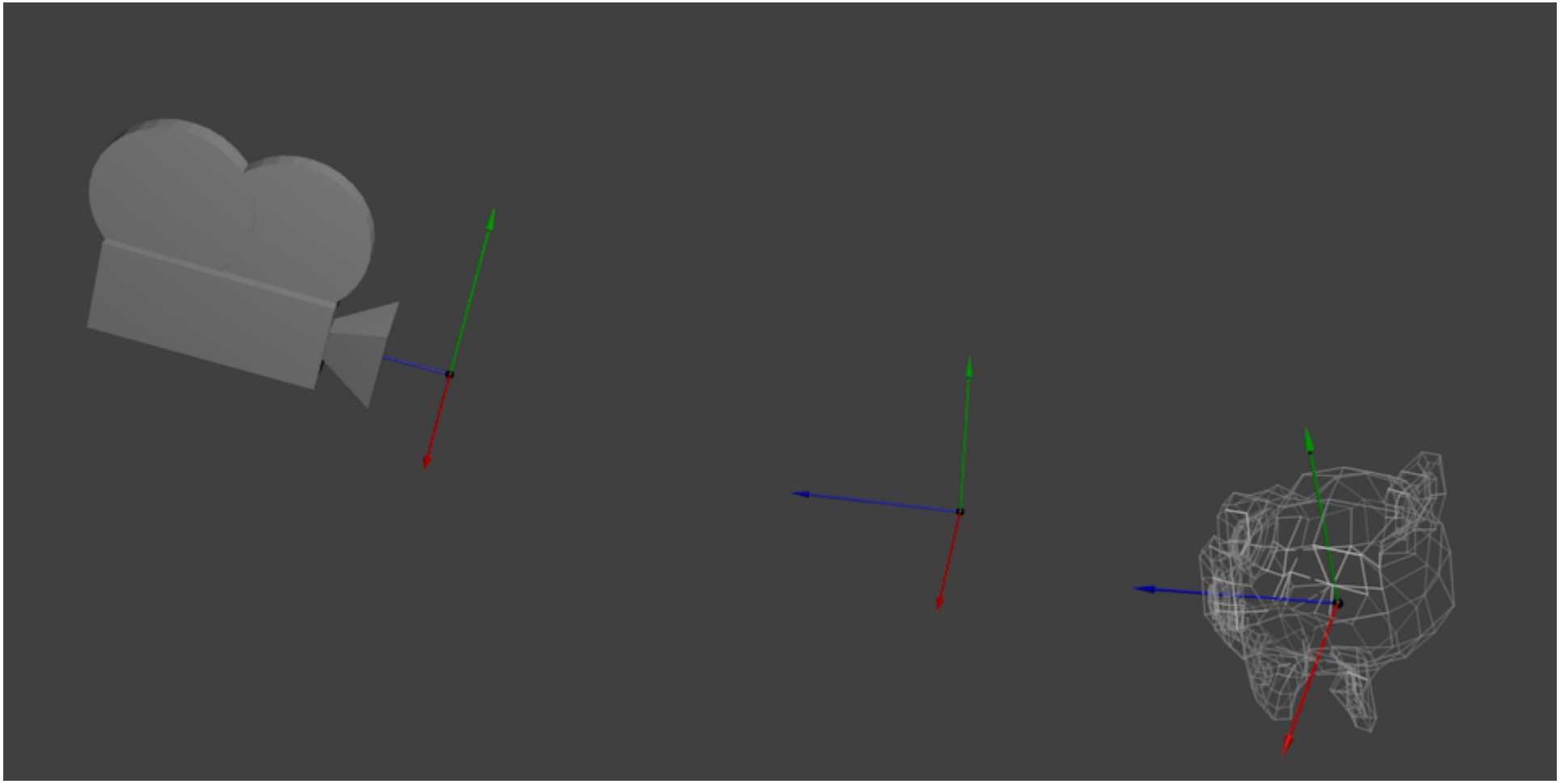
Camera Coordinate Frame



Camera Space

Right hand coordinate system

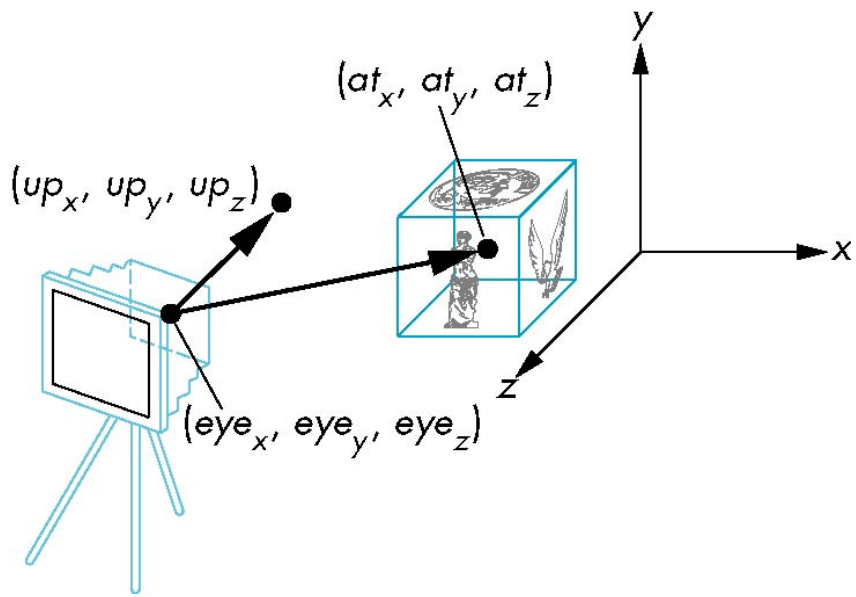
$$\vec{n} = at - eye$$

$$\vec{n} = \frac{\vec{n}}{\|\vec{n}\|}$$

$$\vec{u} = up \times \vec{n}$$

$$v = \vec{n} \times \vec{u}$$

$$\boldsymbol{V} = \begin{pmatrix} u_x & u_y & u_z & -eye \cdot \mathbf{u} \\ v_x & v_y & v_z & -eye \cdot \mathbf{v} \\ n_x & n_y & n_z & -eye \cdot \mathbf{n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Old Style

```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(0,0,1,0,0,0,0,1,0);
    …
}
```

New World

- Create a view matrix

```
view = glm::lookAt(glm::vec3(0.0, 2.0, 2.0), glm::vec3(0.0, 0.0, 0.0), glm::vec3(0.0, 1.0, 0.0));
```

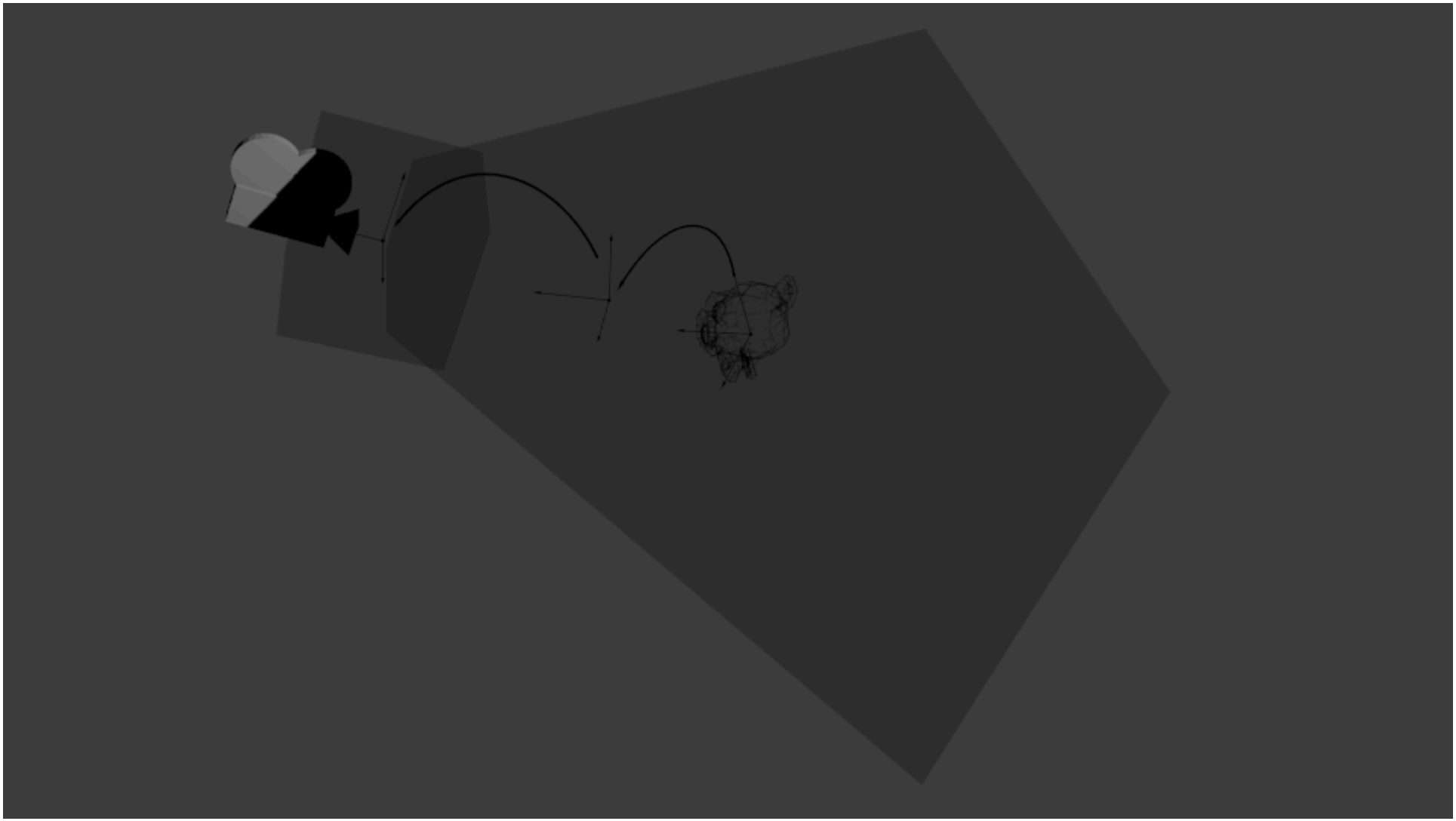
- Combine with modeling matrices

```
glm::mat4 model = glm::mat4(1.0f);
 model = glm::rotate(model, angle, glm::vec3(0.0f, 0.0f, 1.0f));
 model = glm::scale(model, scale_size, scale_size, scale_size);

 glm::mat4 modelview = view * model;
```

Working with Old World

```
glMatrixMode(GL_MODELVIEW);
 glLoadMatrixf(&modelview[0][0]);

// begin to draw your geometry
…
```

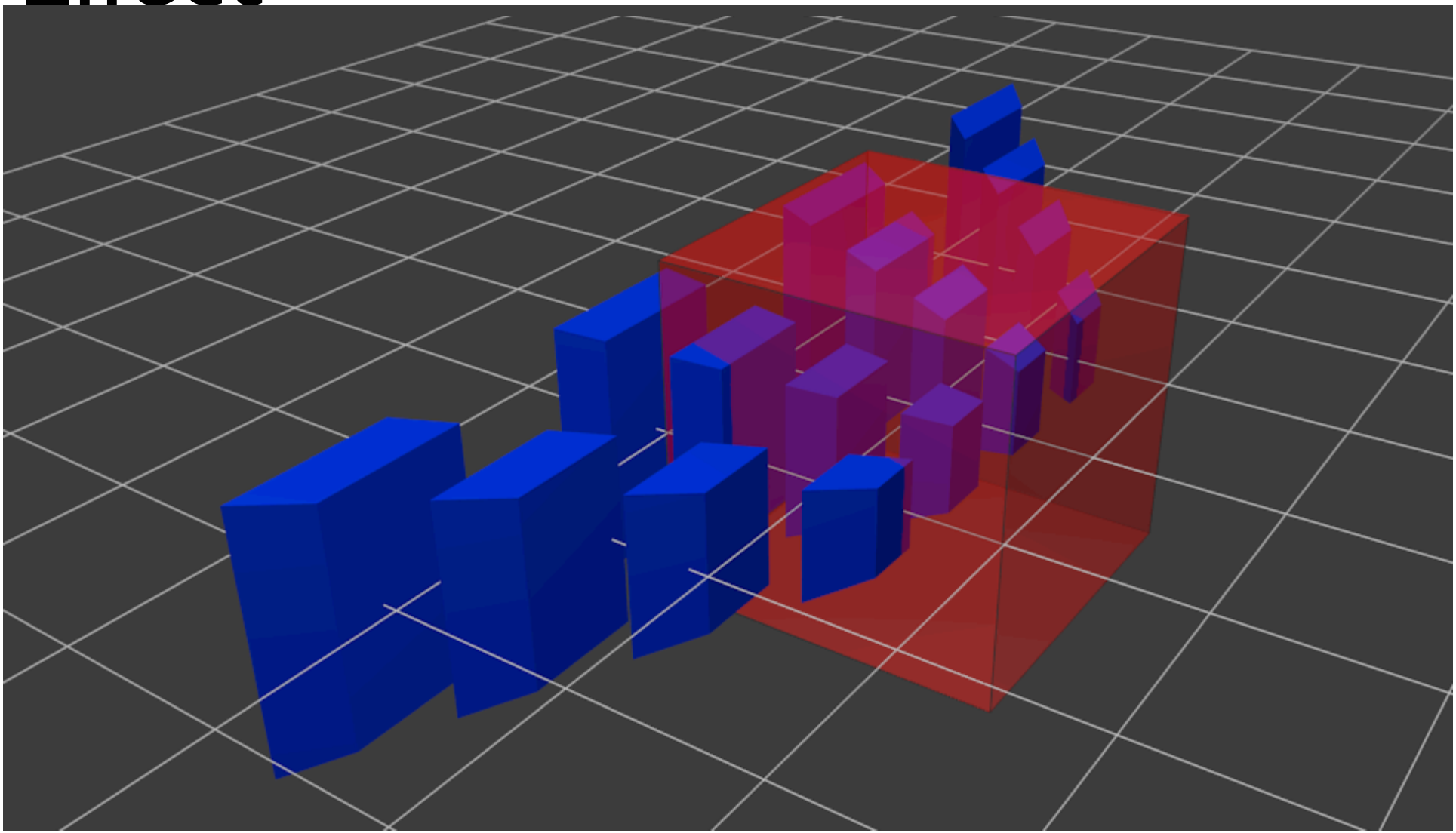
Projection Matrices



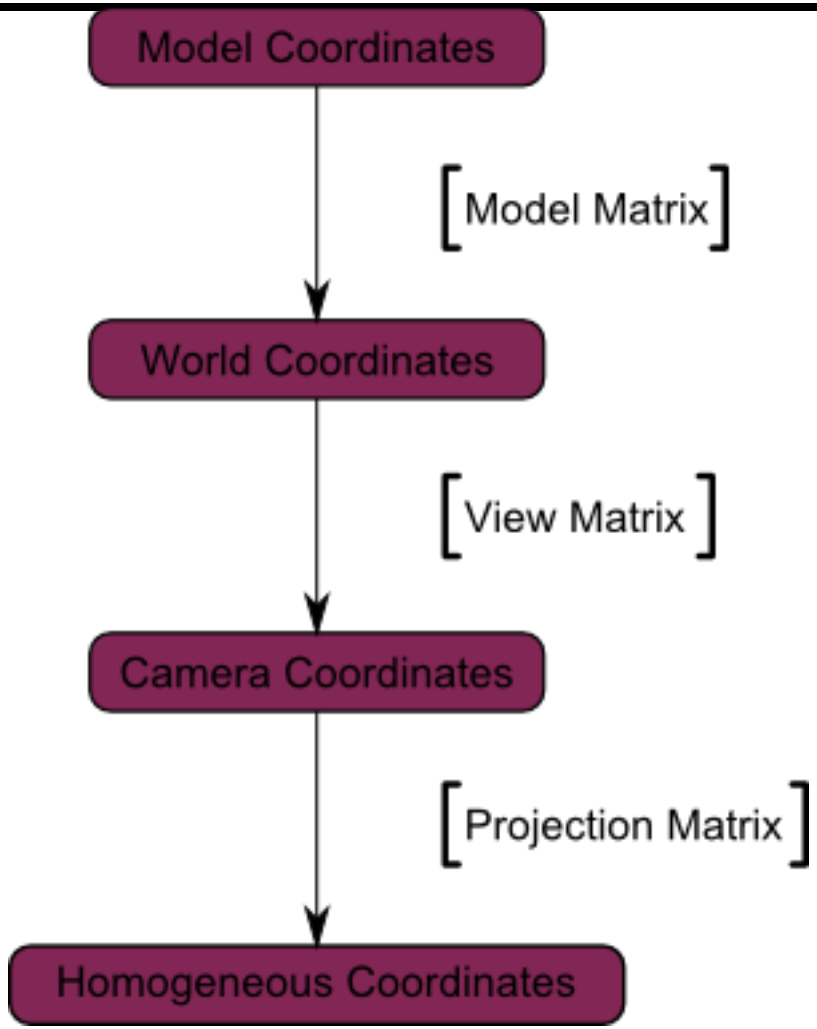
In Code

```
// Generates a really hard-to-read matrix, but a normal, standard 4x4 matrix nonetheless

glm::mat4 projectionMatrix = glm::perspective(

    FoV,        // The horizontal Field of View, in degrees : the amount of "zoom".
                // Think "camera lens". Usually between 90° (extra wide) and 30° (quite zoomed in)
    4.0f / 3.0f, // Aspect Ratio. Depends on the size of your window.
                //Notice that 4/3 == 800/600 == 1280/960, sounds familiar ?
    0.1f,       // Near clipping plane. Keep as big as possible, or you'll get precision issues.

    100.0f      // Far clipping plane. Keep as little as possible.
);
```

Effect



In Diagrams


Model Coordinates
[Model Matrix]
World Coordinates
[View Matrix]
Camera Coordinates
[Projection Matrix]
Homogeneous Coordinates

More Code

C++ : compute the matrix

```
glm::mat4 MVPmatrix = projection * view * model;
// Remember : inverted !
```

```
// GLSL : apply it
transformed_vertex = MVP * in_vertex;
```

Combined

Generate Matrix

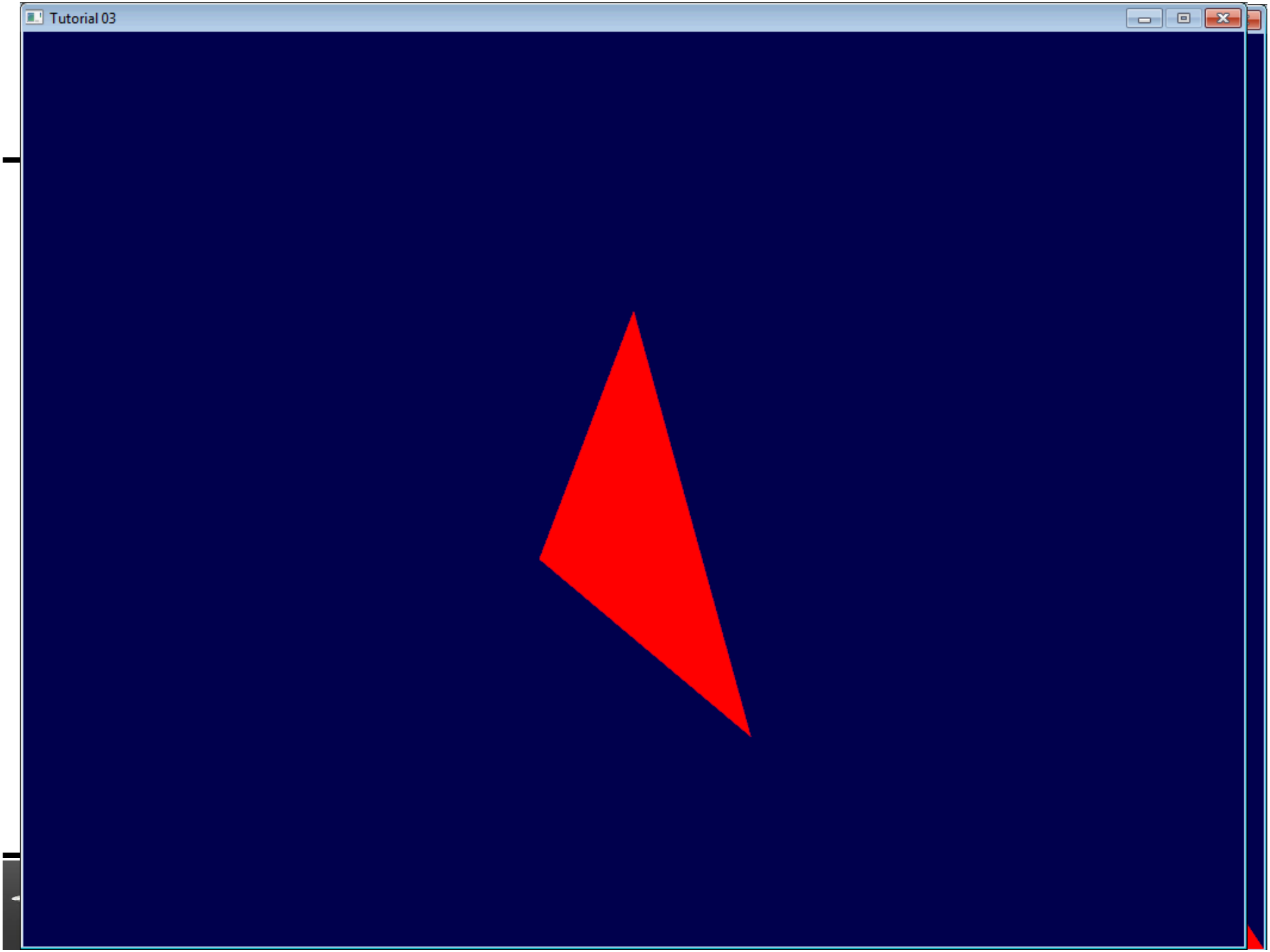
```
// Projection matrix : 45°
//Field of View, 4:3 ratio, display range : 0.1 unit <-> 100 units
glm::mat4 Projection = glm::perspective(45.0f, 4.0f / 3.0f, 0.1f, 100.0f);

// Camera matrix
glm::mat4 View       = glm::lookAt(
    glm::vec3(4,3,3), // Camera is at (4,3,3), in World Space
    glm::vec3(0,0,0), // and looks at the origin
    glm::vec3(0,1,0)  // Head is up (set to 0,-1,0 to look upside-down)
);
// Model matrix : an identity matrix (model will be at the origin)
glm::mat4 Model      = glm::mat4(1.0f);  // Changes for each model !
// Our ModelViewProjection : multiplication of our 3 matrices
glm::mat4 MVP        = Projection * View * Model;
// Remember, matrix multiplication is the other way around
```

GLSL Takes Over

```
// Get a handle for our "MVP" uniform.
// Only at initialisation time.
GLuint MatrixID = glGetUniformLocation(programID, "MVP");

// Send our transformation to the currently bound shader,
// in the "MVP" uniform
// For each model you render, since the MVP will be different
// (at least the M part)

glUniformMatrix4fv(MatrixID, 1, GL_FALSE, &MVP[0][0]);
```

Use It

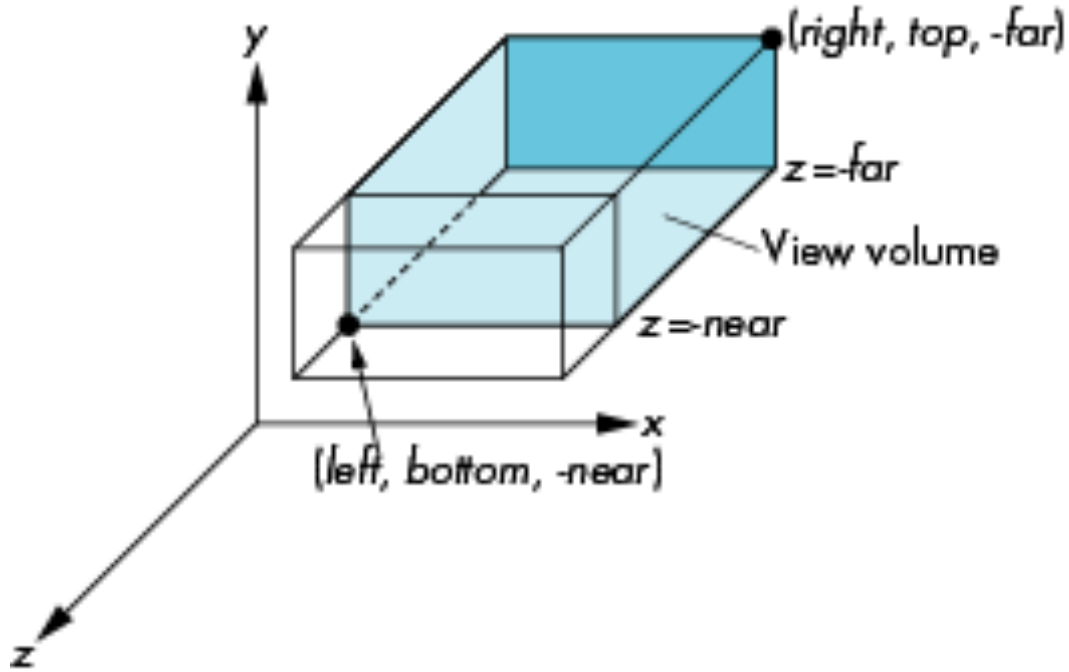
```
in vec3 vertexPosition_modelspace;
uniform mat4 MVP;

void main(){
// Output position of the vertex, in clip space : MVP * position

    vec4 v = vec4(vertexPosition_modelspace,1);

// Transform an homogeneous 4D vector, remember ?
    gl_Position = MVP * v;

}
```


Tutorial 03

Old Style

OpenGL Orthogonal Viewing

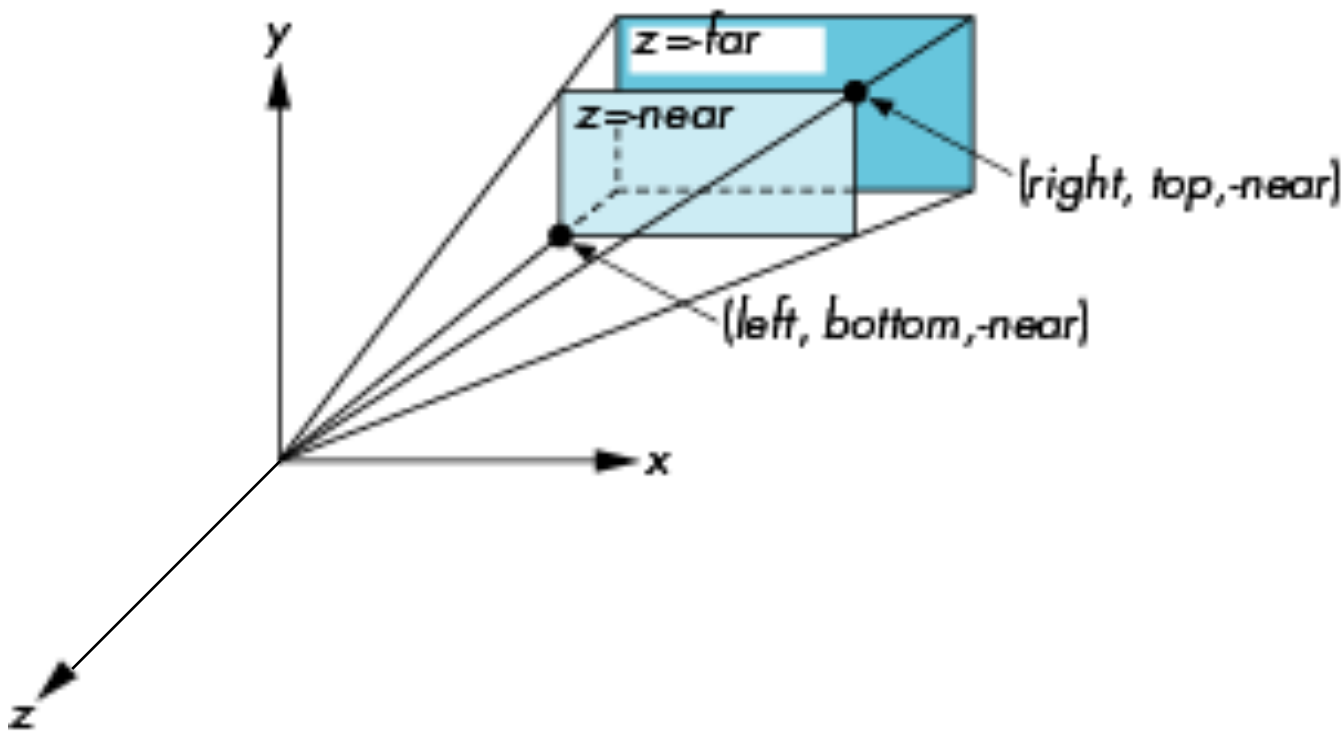
`Ortho(left,right,bottom,top,near,far)`



`near` and **`far`** measured <u>from</u> camera

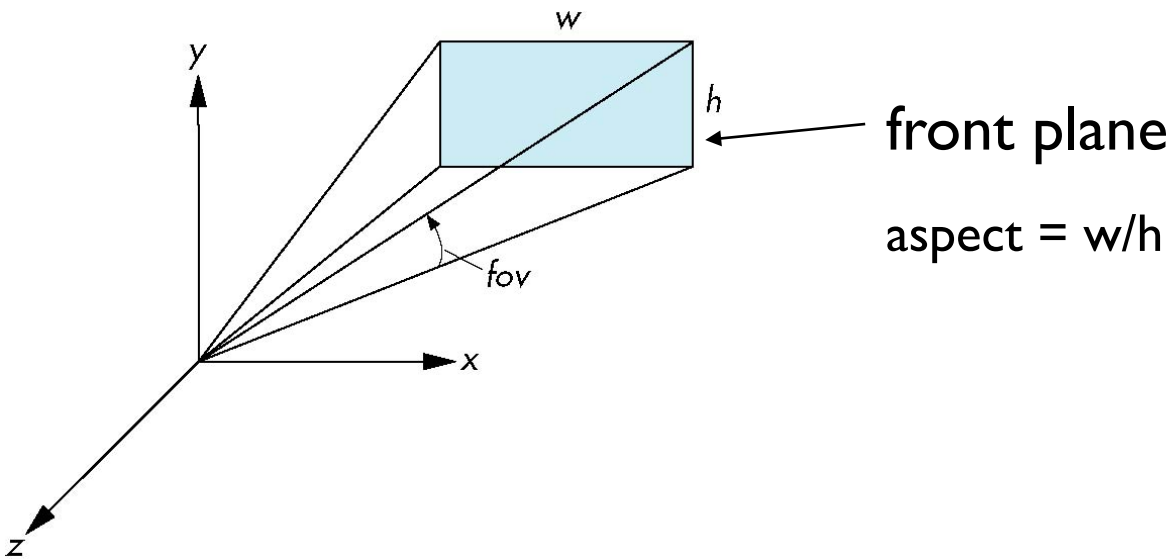
OpenGL Perspective

Frustum(left,right,bottom,top,near,far)



Using Field of View

- With Frustum it is often difficult to get the desired view
- Perpective(fovy, aspect, near, far) often provides a better interface



Old Style

```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT);
    glMatrixMode(GL_PROJETION);
    glLoadIdentity();
    gluPerspective(fove, aspect, near, far);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(0,0,1,0,0,0,0,1,0);
    my_display();    // your display routine
}
```

Can Still GLM

- Set up the projection matrix

```
glm::mat4 projection = glm::mat4(1.0f);
projection = glm::perspective(60.0f,1.0f,.1f,100.0f);
```

- Load the matrix to GL_PROJECTION

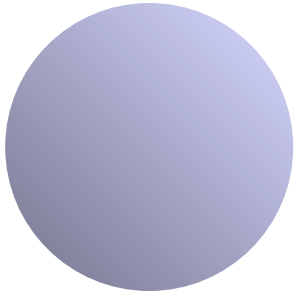
```
glMatrixMode(GL_PROJECTION);
glLoadMatrixf(&projection[0][0]);
```

Next

Why we need shading

- Just attach color *glColor*

- But



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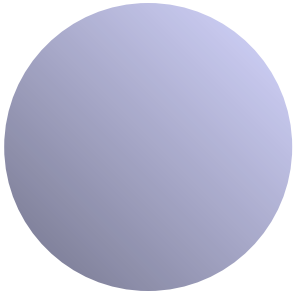
Shading

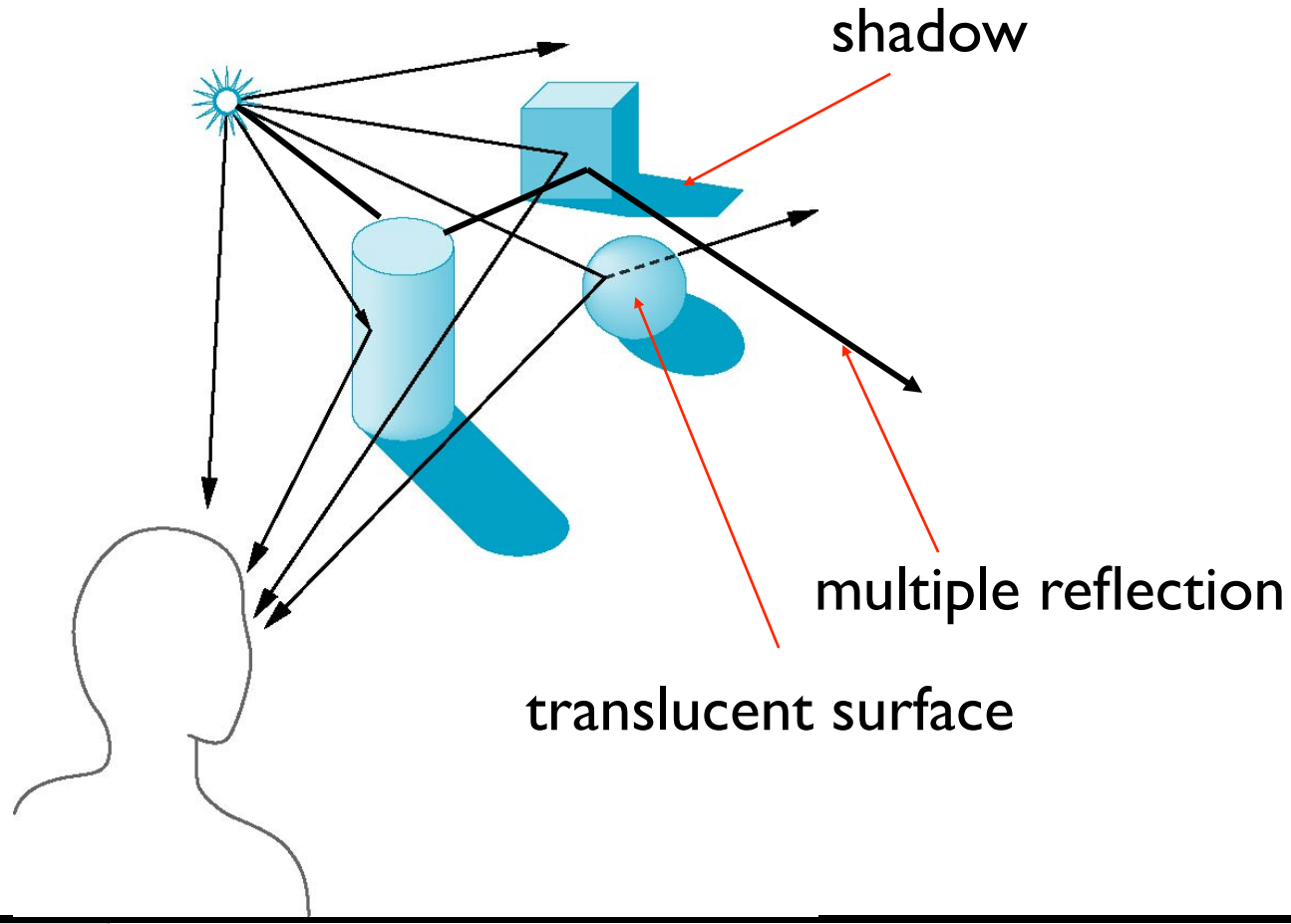
- Why does the shape ?



- Light-material interaction at points -> different color or shade

- Factor
  - Light sources
  - Material properties
  - Location of viewer
  - Surface orientation

Global Effects



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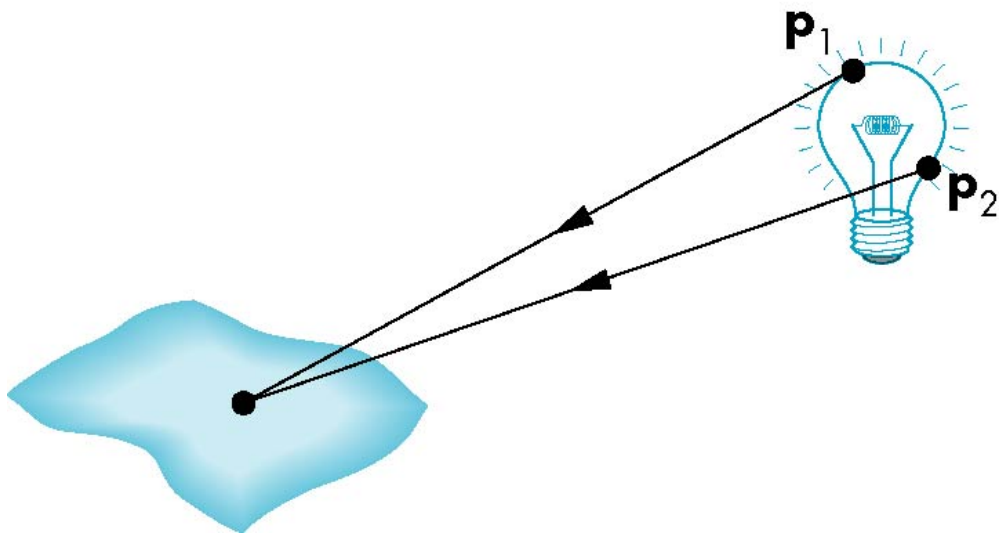
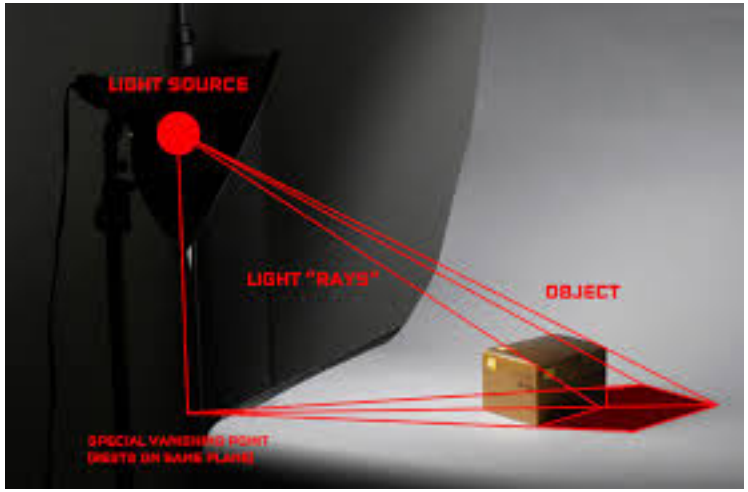
Light Sources

General Difficult !



Simple Light Sources

Point Sources



Point source

- Model with position and color
- Distant source = infinite distance away (parallel)

Spot Light

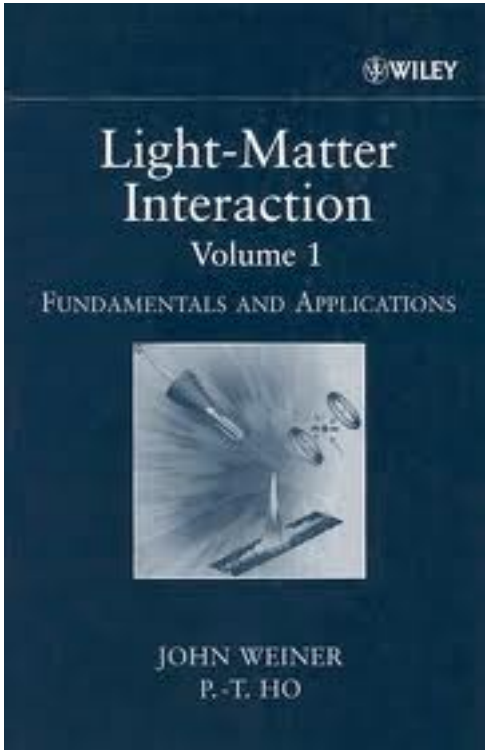
Spotlight

- Restrict light from ideal point source

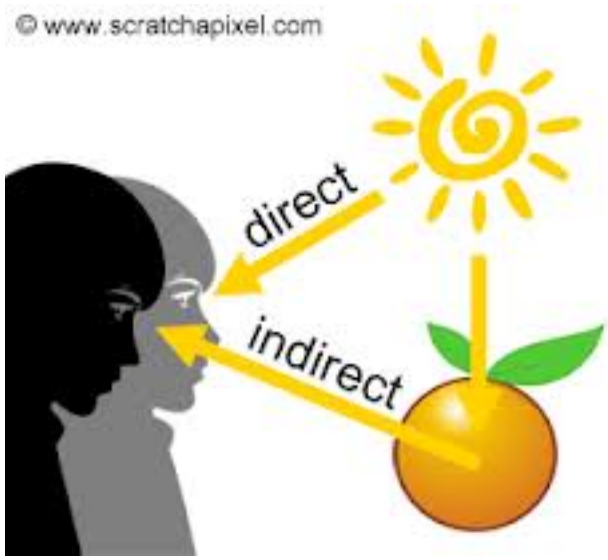
Ambient

Ambient light

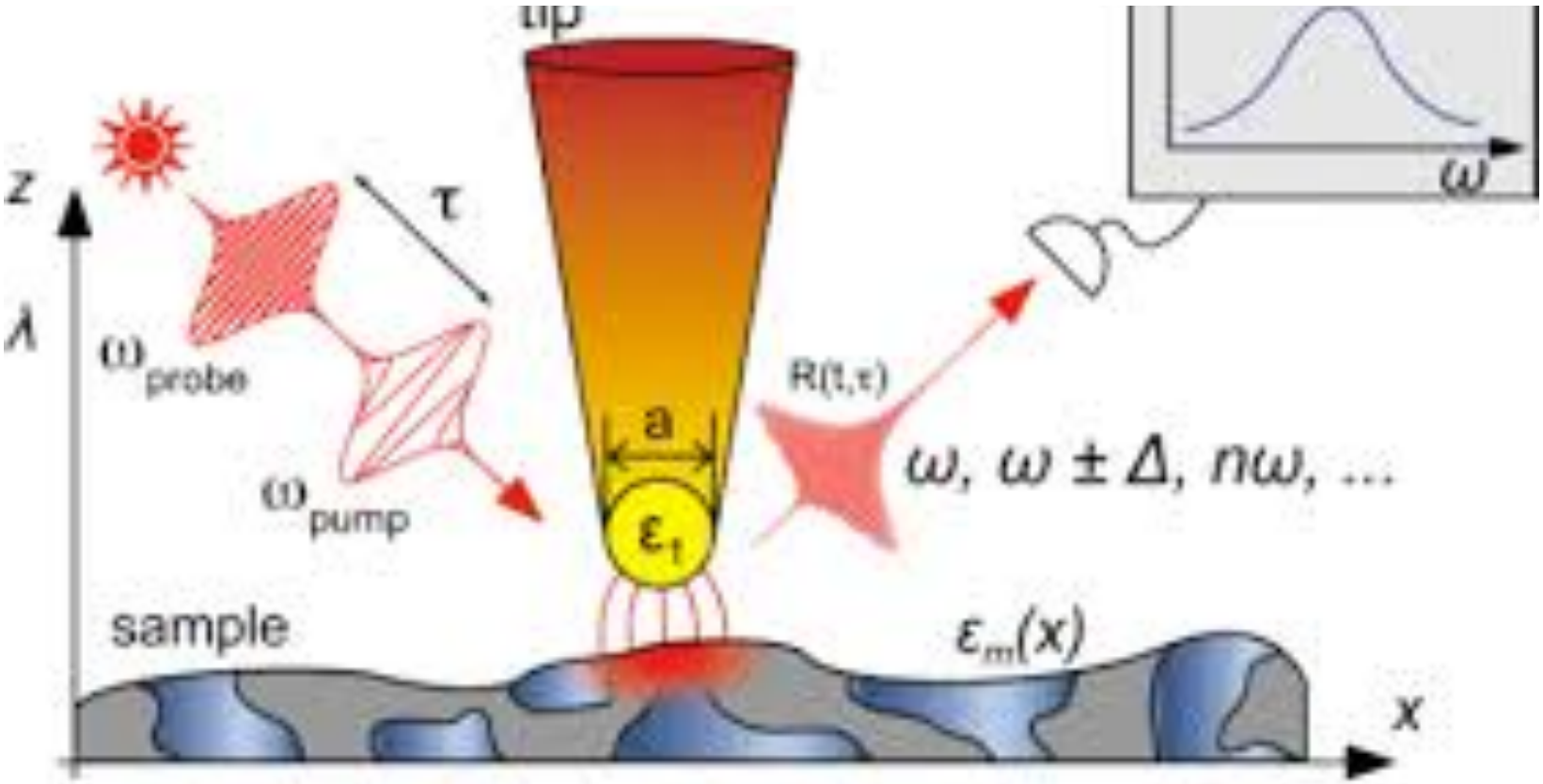
- Same amount of light in scene
- Model contribution of all sources and reflecting surfaces


WILEY
Light-Matter Interaction
Volume 1
FUNDAMENTALS AND APPLICATIONS
JOHN WEINER
P.-T. HO

Indirect/Direct Light

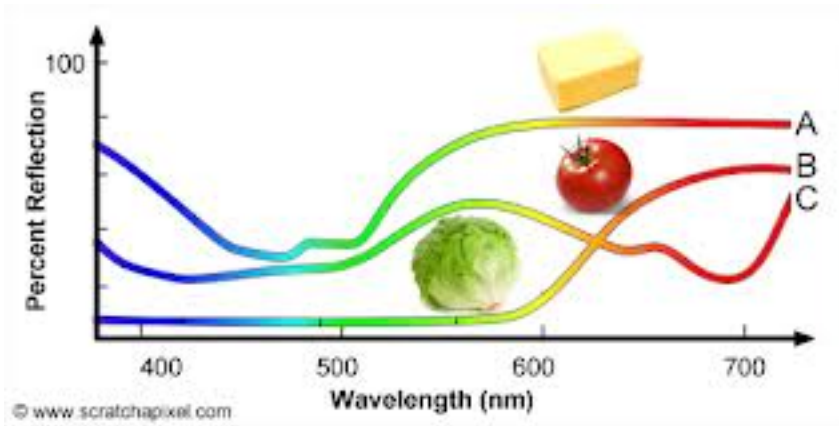


Scatter (reflect) & Absporb



Light strikes object - is partially absorbed & partially scattered (reflected)

Color !

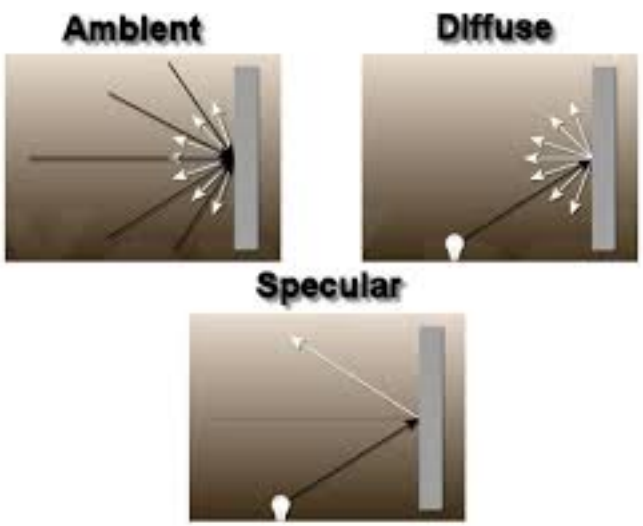


Amount reflected determines the color and brightness of the object

Red surface appears red in white light - red component is reflected and rest is absorbed

The Surface

Reflected light is scattered depending on smoothness and orientation of the surface



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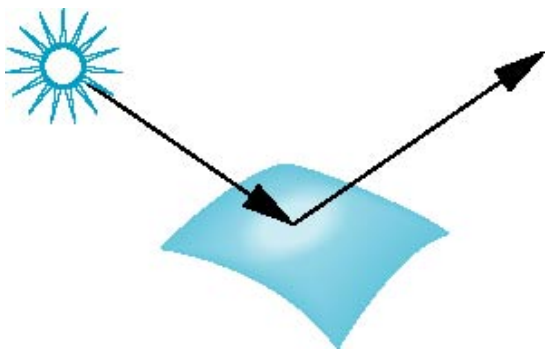
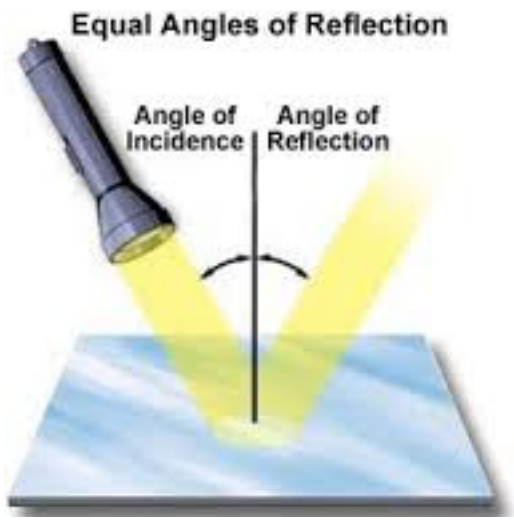
Surface Type - Smooth

- Very Smooth - more reflected light concentrated in one direction – like a perfect mirror





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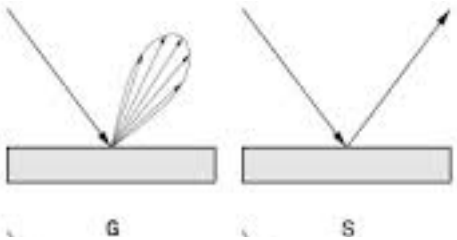
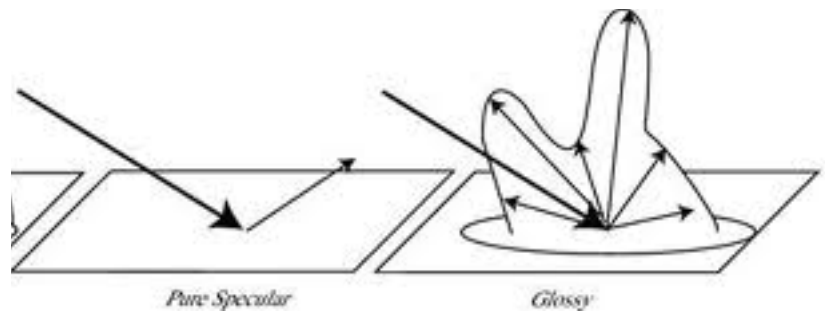
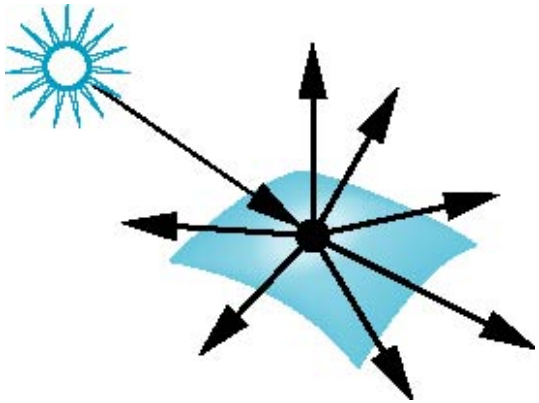
TBT - specular





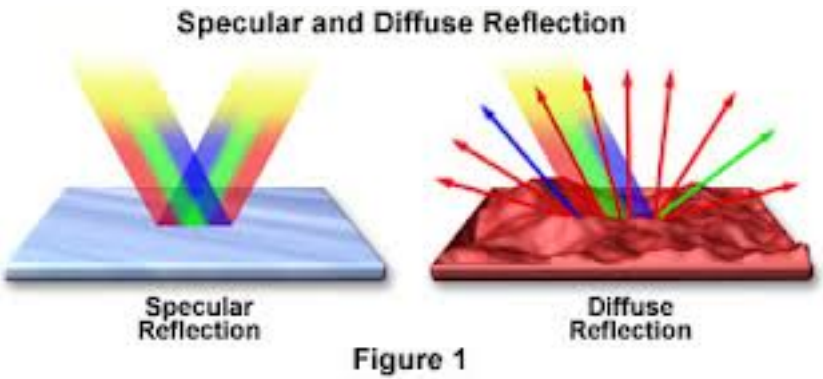
Surface Type - Rough

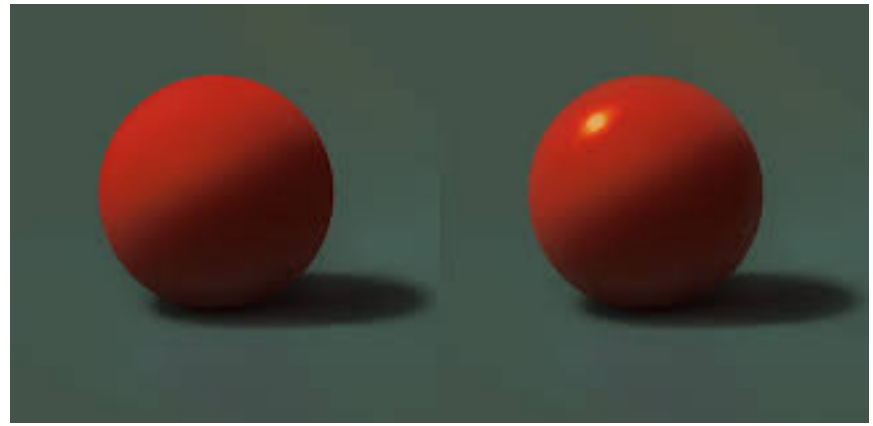
Scatters light in all directions

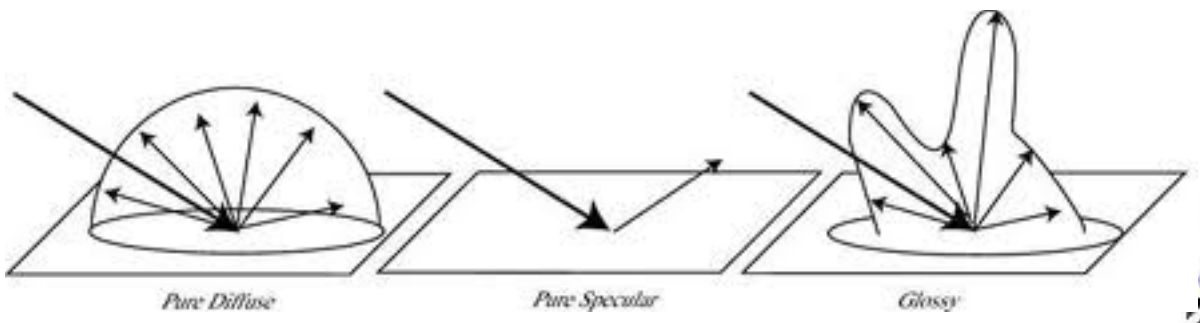


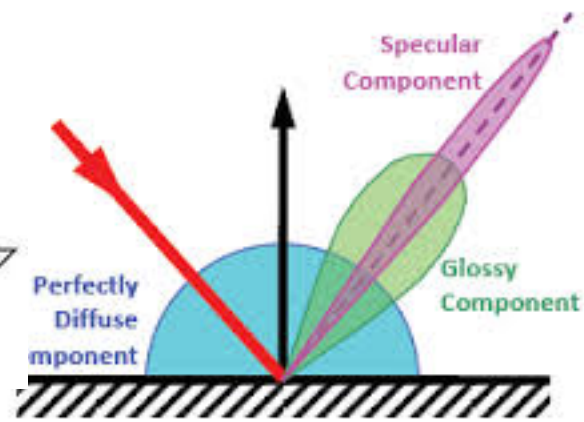
rough surface

Smooth vs. Rough

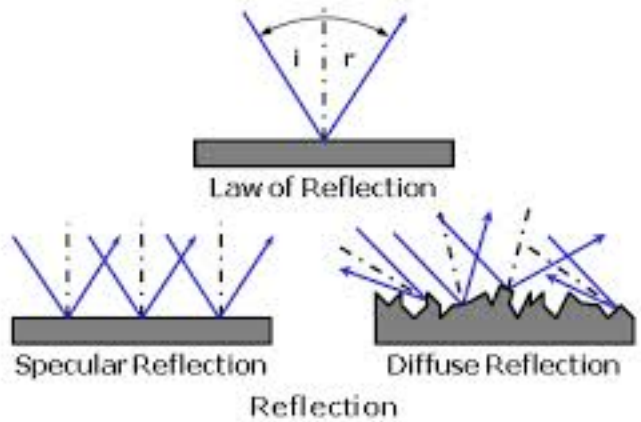








Smooth vs. Rough

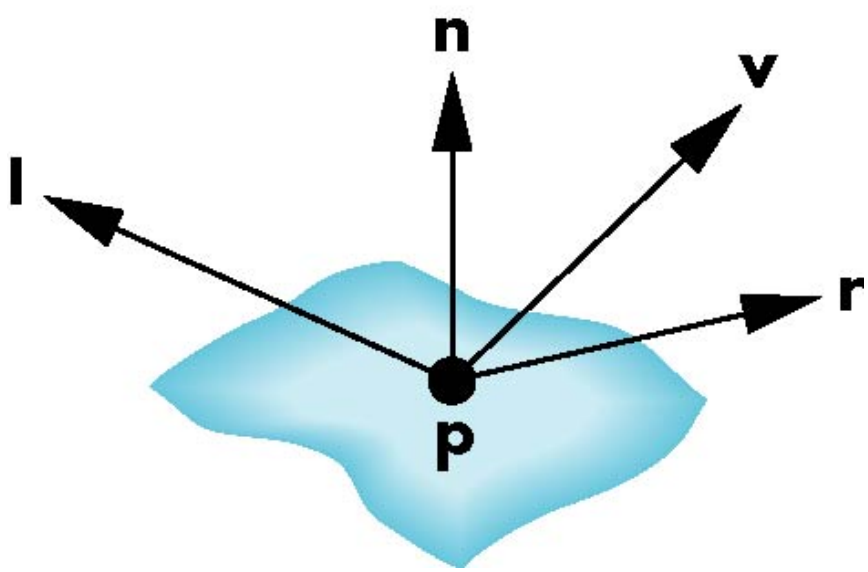


The Phong Illumination Model

Phong Model

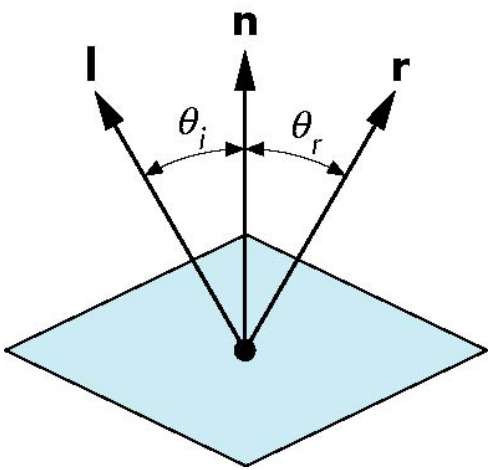
A simple local model that can be computed rapidly

- Has three components
  - Diffuse
  - Specular
  - Ambient
- Uses four vectors
  - To source
  - To viewer
  - Normal
  - Perfect reflector



Ideal Reflector

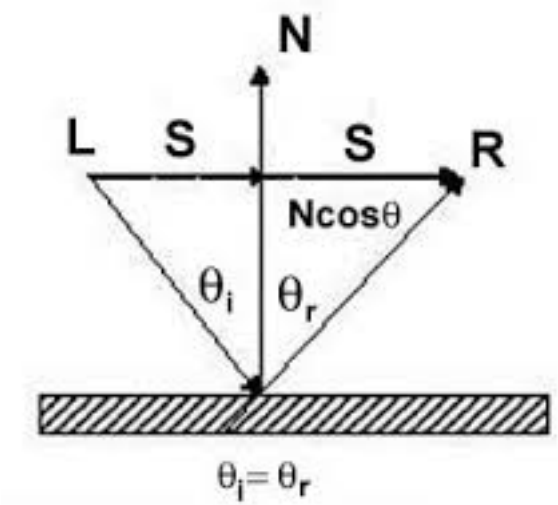
- Normal is determined by local orientation
- Angle of incidence = angle of relection
- The three vectors must be coplanar

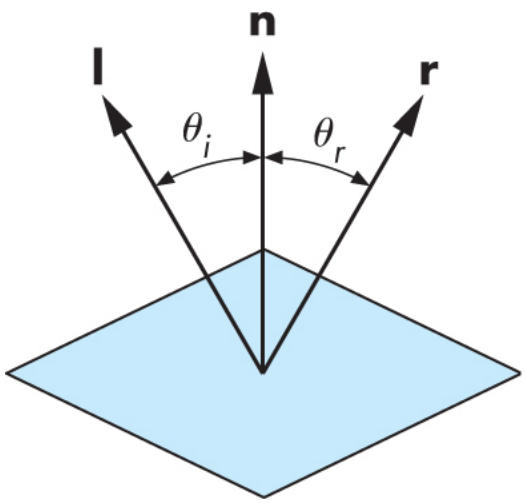


Computing r

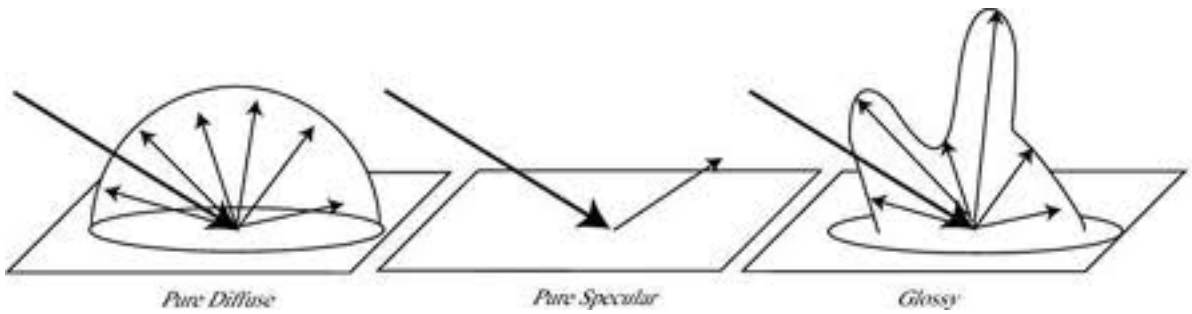
Want all three to be unit length

$$r = 2(l \bullet n)n - l$$



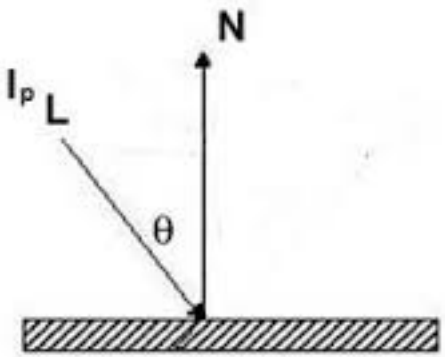


Diffuse

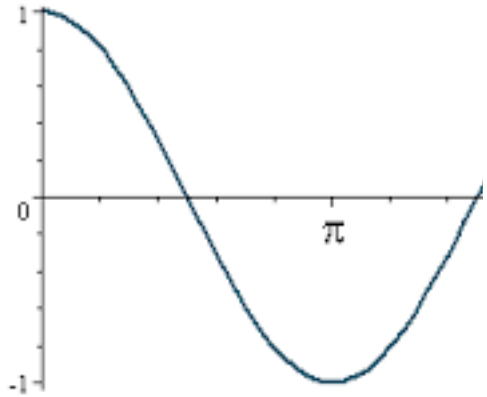


Lambertian Surface

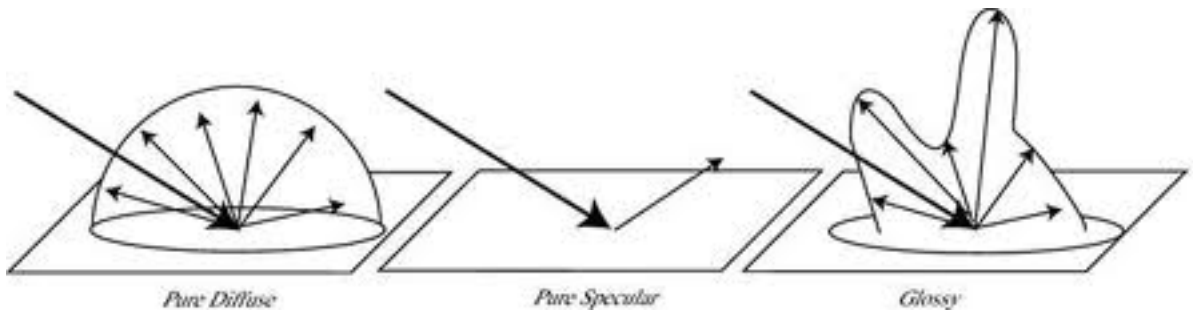
Amount reflected is proportional to vertical component of incoming light



- reflected light ~ cos θ_i
- Or I_d k_d cos θ_i
- cos θ_i = **l** · **n** if vectors normalized
- Three coefficients, k_{dr}, k_{db}, k_{dg} that measure each color component is reflected



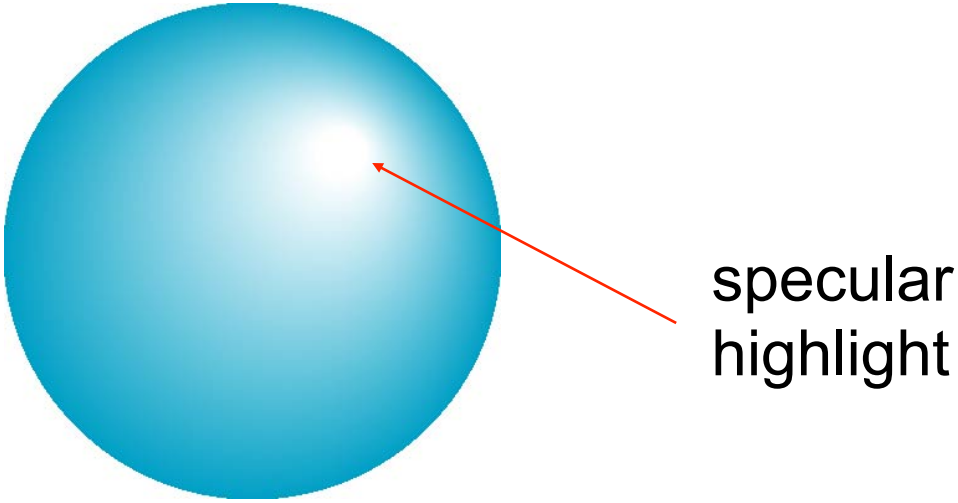
Specular or Glossy Surface



Specular Surfaces

Specular highlights due to incoming light being reflected in directions close to the direction of a perfect reflection

Not Ideal Mirror



Specular Reflections

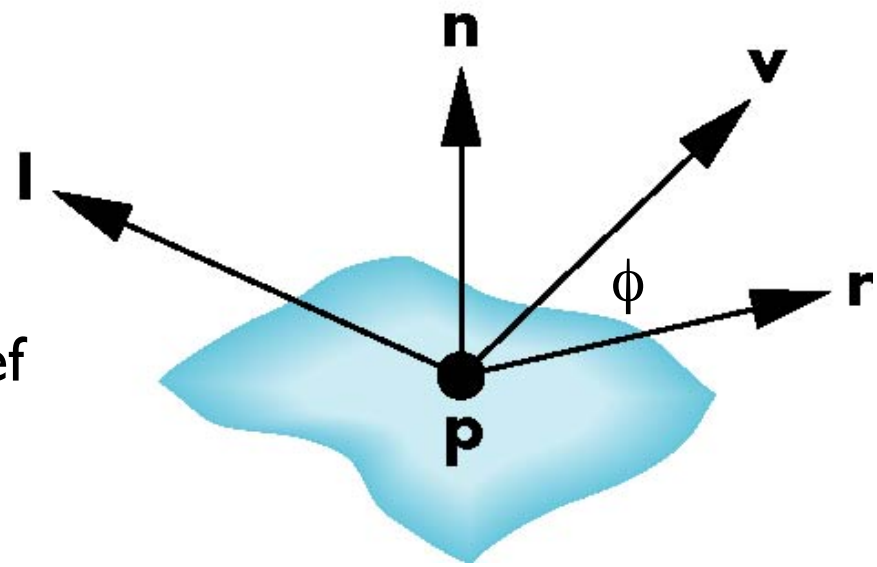
$$\mathbf{I}_r \sim k_s \, \mathbf{I} \cos^{\alpha}\phi$$

reflected intensity

absorption coef

incoming intensity

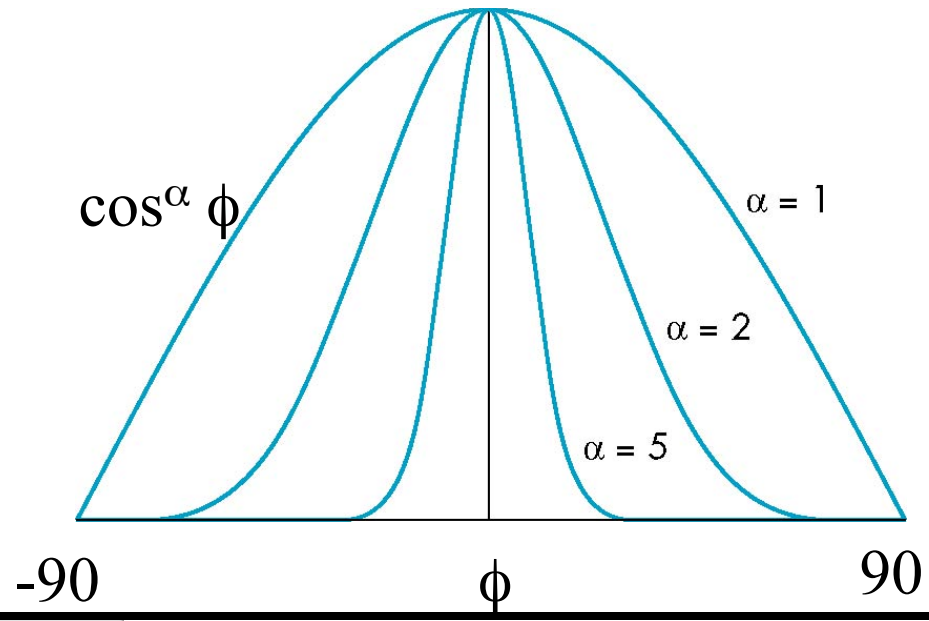
shininess coef



The Shininess Coefficient

α between

- 100 and 200 correspond to metals
- 5 and 10 give surface that look like plastic



Ambient Light

- Result of multiple interactions between (large) light sources and objects in environment
- Amount and color depend on both color of light(s) and material properties of the object
- Add k_a I_a to diffuse and specular terms

reflection coef

intensity of ambient light

Light Source

- We add results from each light source

- Each light source
  - diffuse, specular, and ambient terms
  - Separate red, green and blue components

- Hence, 9 coefficients for each point source
  - I_{dr}, I_{dg}, I_{db}, I_{sr}, I_{sg}, I_{sb}, I_{ar}, I_{ag}, I_{ab}

Material Properties

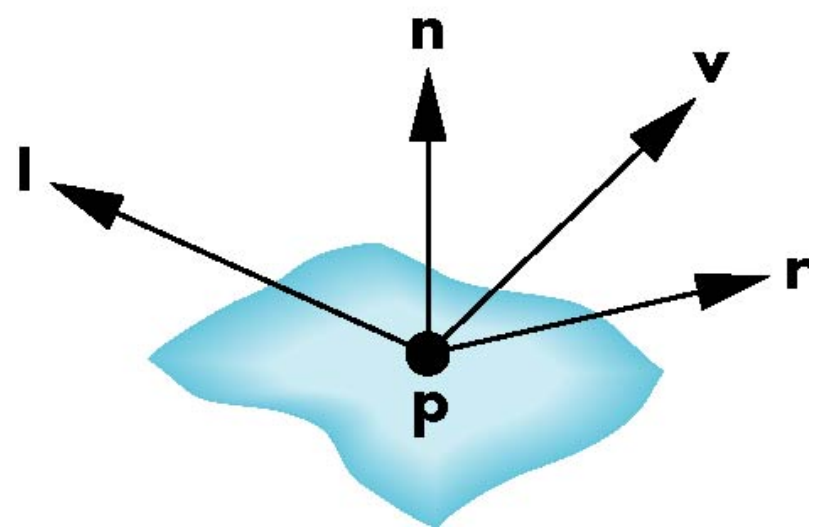
- Material properties match light source properties
  - Nine absorbtion coefficients
    - k_{dr}, k_{dg}, k_{db}, k_{sr}, k_{sg}, k_{sb}, k_{ar}, k_{ag}, k_{ab}
  - Shininess coefficient a

Adding Components

For each light source and each color component, the Phong model can be written (without the distance terms) as

$$I = k_d I_d (\mathbf{l} \cdot \mathbf{n}) + k_s I_s (\mathbf{v} \cdot \mathbf{r})^a + k_a I_a$$

For each color component we add contributions from all sources



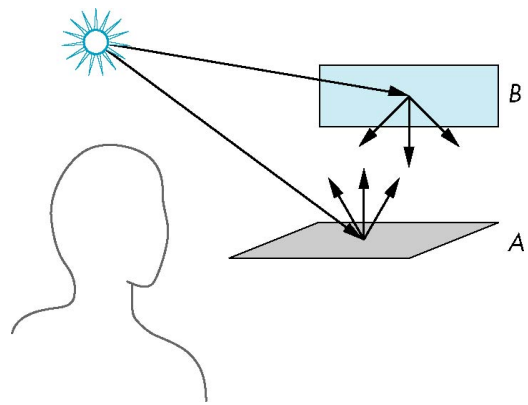
Distance Terms

- Light inversely proportional to square of distance
- A factor added to diffuse and specular components:

$$\frac{1}{(a + bd + cd^2)}$$

$$d = \left\| \bar{p} - \bar{L} \right\|$$

$$a, b, c - \text{constants}$$



- Constant, linear terms soften effect of point source

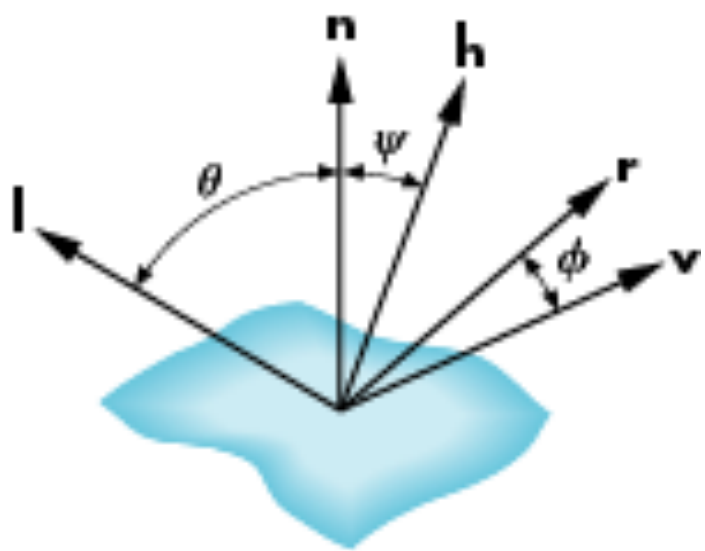
Modified Phong Model

- Specular term requires new reflection and view vector for each vertex
- Blinn approximation – using halfway vector
- And more effiicient

The Halfway Vector

h is normalized vector halfway between **l** and **v**

$$\mathbf{h} = (\mathbf{l} + \mathbf{v}) / |\mathbf{l} + \mathbf{v}|$$

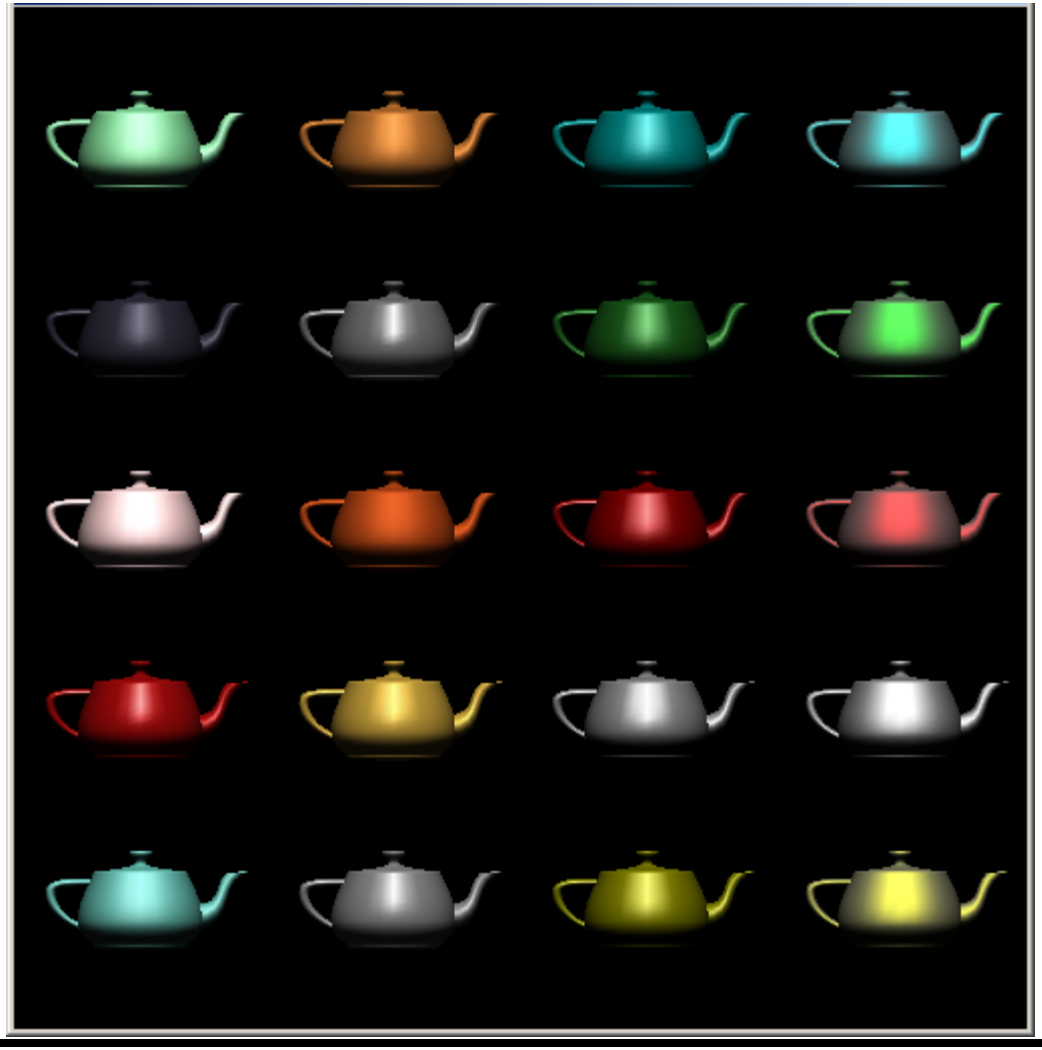


Using the halfway vector

- Replace $(v \cdot r)^a$ by $(n \cdot h)^b$
- b is chosen to match shininess
- Halfway angle half of angle **r** and **v** if coplanar
- Modified Phong, Blinn lighting model
  - OpenGL standard

Teapot Gallery

Only differences in these teapots are the parameters in the modified Phong model



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Computation of Vectors

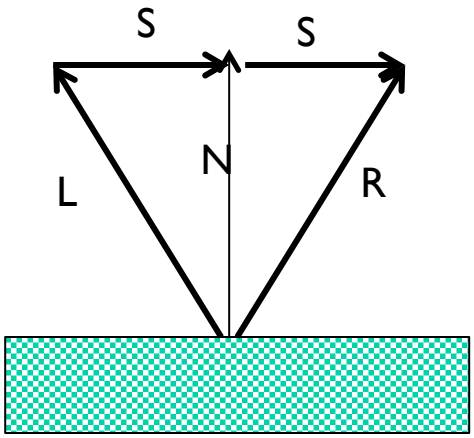
- l and v are specified by the application
- Can compute r from l and n

- Problem is determining n
- Depends on underlying representation of surface

- OpenGL leaves determination of normal to application
  - Exception for GLU quadrics and Bezier surfaces was deprecated

Computing r

Vectors unit length



$$\text{left}\Delta : L + S = N(N \bullet L)$$

$$\text{right}\Delta : N(N \bullet L) + S = R$$

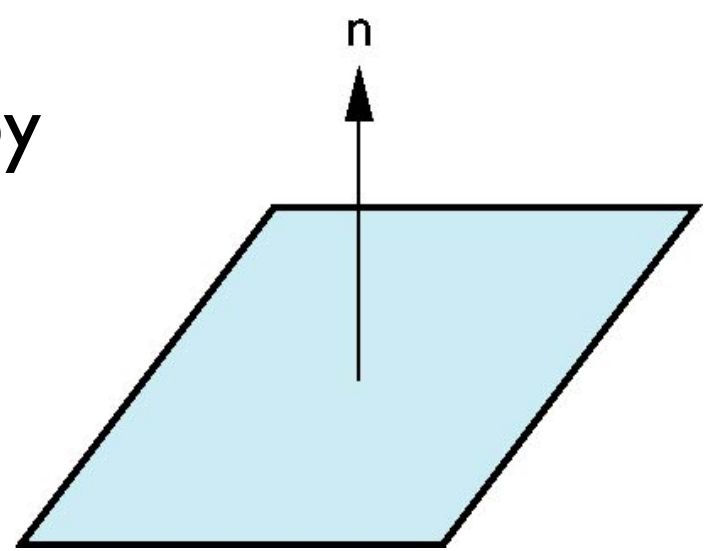
$$R = 2(L \bullet N)N - L$$

Plane Normals

- Equation of plane: $ax+by+cz+d = 0$
- Plane is determined by three points p_0, p_1, p_2 or normal $\mathbf{n}$ and p_0

- Normal can be obtained by

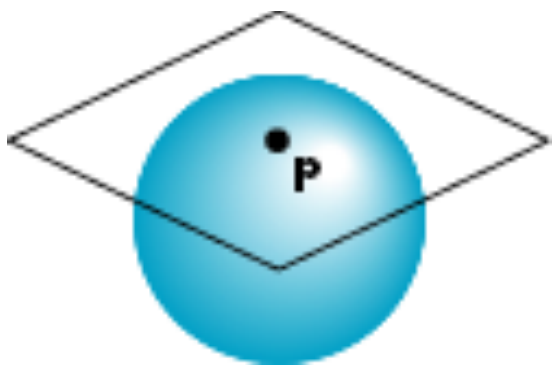
$$\mathbf{n} = (p_2 - p_0) \times (p_1 - p_0)$$



Normal to Sphere

- Implicit function f(x,y.z)=0
- Normal given by gradient
- Sphere $f(p)=p \cdot p^T - 1$

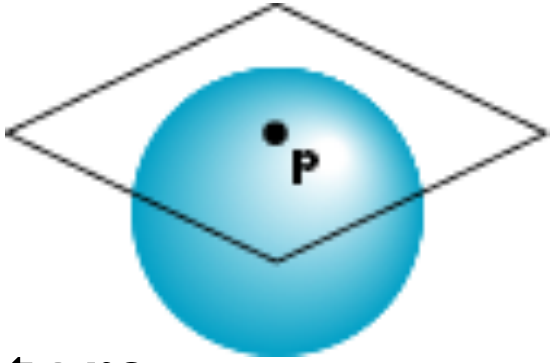
$$n = [\partial f/\partial x, \partial f/\partial y, \partial f/\partial z]^T = p$$



Parametric Form

- For sphere

$$x=x(u,v)=\cos u \sin v$$
$$y=y(u,v)=\cos u \cos v$$
$$z= z(u,v)=\sin u$$



- Tangent plane determined by vectors

$$\partial \mathbf{p}/\partial u = [\partial x/\partial u, \partial y/\partial u, \partial z/\partial u]^T$$
$$\partial \mathbf{p}/\partial v = [\partial x/\partial v, \partial y/\partial v, \partial z/\partial v]^T$$

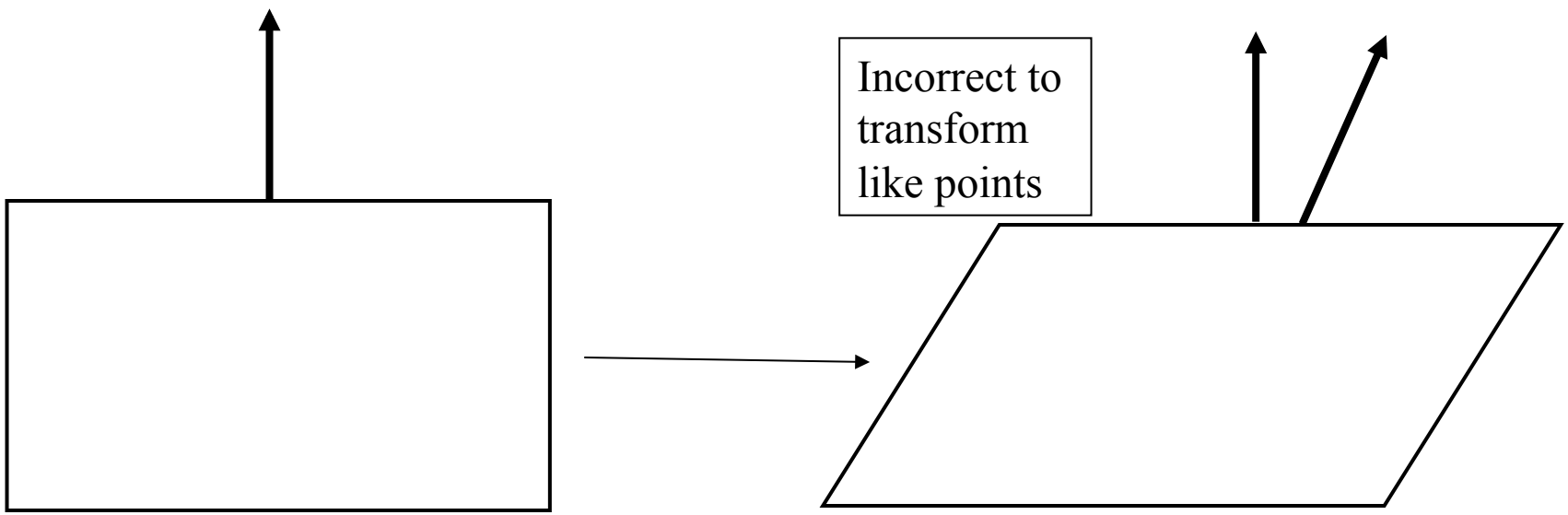
- Normal given by cross product

$$\boldsymbol{n} = \partial \boldsymbol{p}/\partial u \times \partial \boldsymbol{p}/\partial v$$

Transforming Normals

Normals

Algebra tricks to derive correct transform



Transforming Normal

$$p \rightarrow Mp \qquad n \rightarrow Qn \qquad Q = ?$$

$$n^T p = 0$$

$$n^T Q^T M p = 0 \quad \Rightarrow \quad Q^T M = I$$

$$Q = (M^{-1})^T$$

<u>Use Q and not M to transform Normals</u>

Q: What about Rotation ?

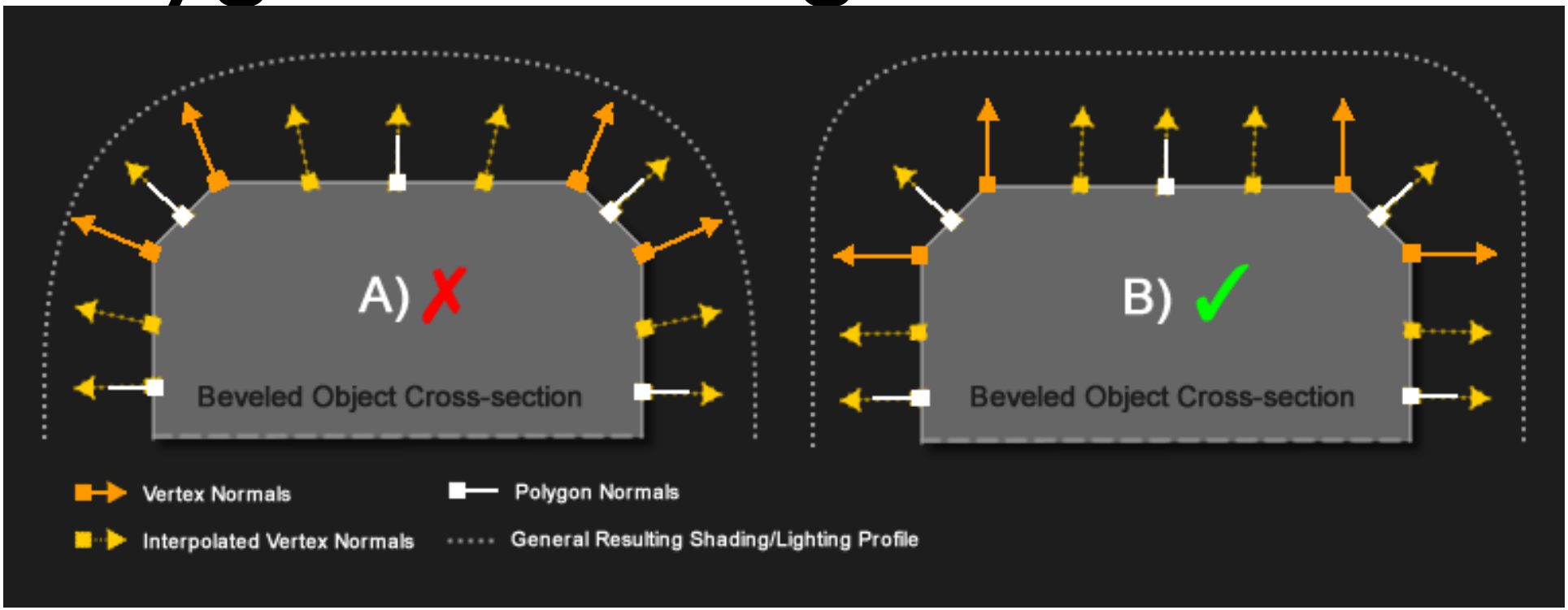
What about Translation ?

General Case

Parametric normals for other simple cases

- Quadrics
- Parameteric polynomial surfaces
  - Bezier surface patches

Polygonal Shading



Polygonal Shading

- Per vertex shading
  - shading calculations are done for each vertex
  - Vertex colors become vertex shades
  - sent to the vertex shader as a vertex attribute
  - Or send parameters to vertex shader & compute shade

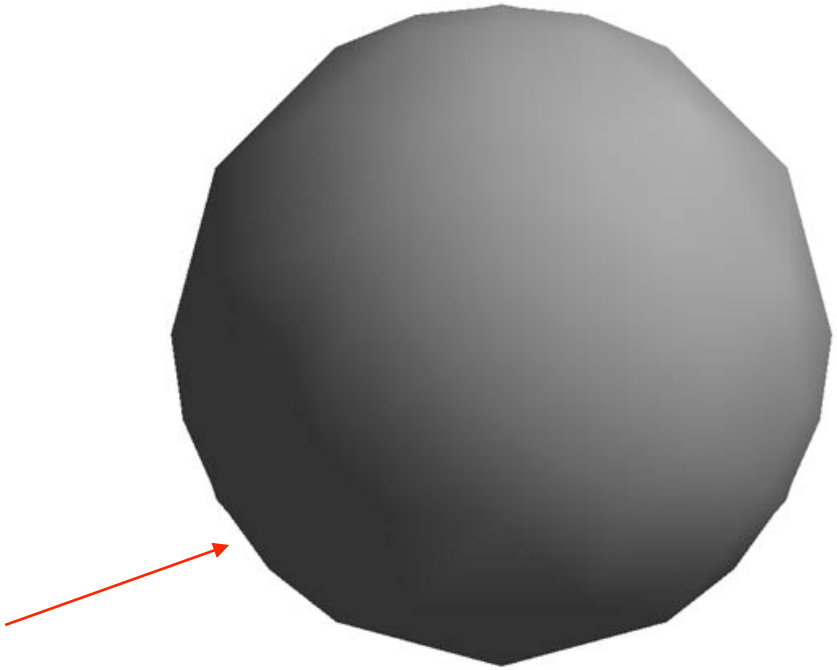
- By default, vertex shades interpolated across object if passed to fragment shader as a varying variable (smooth shading)
- We can also use uniform variables to shade with a single shade (flat shading)

Polygon Normals

- Triangles have a single normal
  - Shades at vertices as computed by the Phong model
  - Identical for a distant viewer (default) or if there is no specular component
- Want different normals at each vertex even though

Smooth Shading

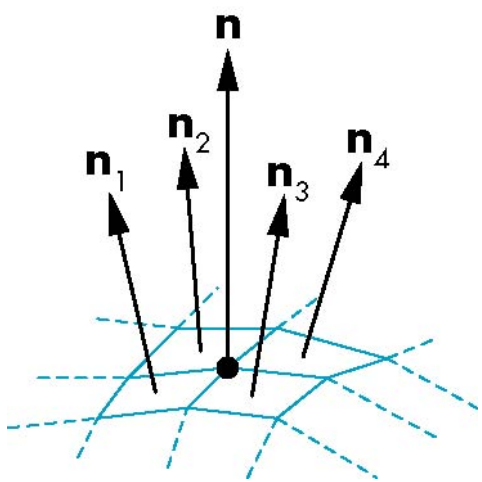
- We can set a new normal at each vertex
- Easy for sphere model
  - If centered at origin **n** = **p**
- Now smooth shading works
- Note *silhouette edge*



Mesh Shading

- Previous example is not general since normal at each vertex is known analytically
- For polygonal models, Gouraud proposed we use the average of normals around a mesh vertex

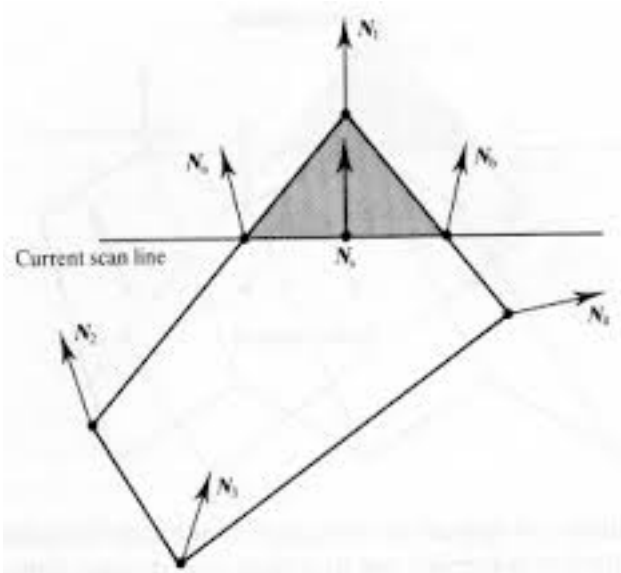
$$n = (n_1+n_2+n_3+n_4)/ |n_1+n_2+n_3+n_4|$$



Interpolation

$$\Delta I_s = \frac{\Delta x}{x_b - x_a}(I_b - I_a)$$

$$I_{s,n} = I_{s,n-1} + \Delta I_s$$



$$\Delta N_{sx} = \frac{\Delta x}{x_b - x_a}(N_{bx} - N_{ax})$$

$$\Delta N_{sy} = \frac{\Delta x}{x_b - x_a}(N_{by} - N_{ay})$$

$$\Delta N_{sz} = \frac{\Delta x}{x_b - x_a}(N_{bz} - N_{az})$$

Gouraud and Phong Shading

- Gouraud Shading
  - Find average normal at each vertex (vertex normals)
  - Apply modified Phong model at each vertex
  - Interpolate vertex shades across each polygon
- Phong shading
  - Find vertex normals
  - Interpolate vertex normals across edges
  - Interpolate edge normals across polygon
  - Apply modified Phong model at each fragment

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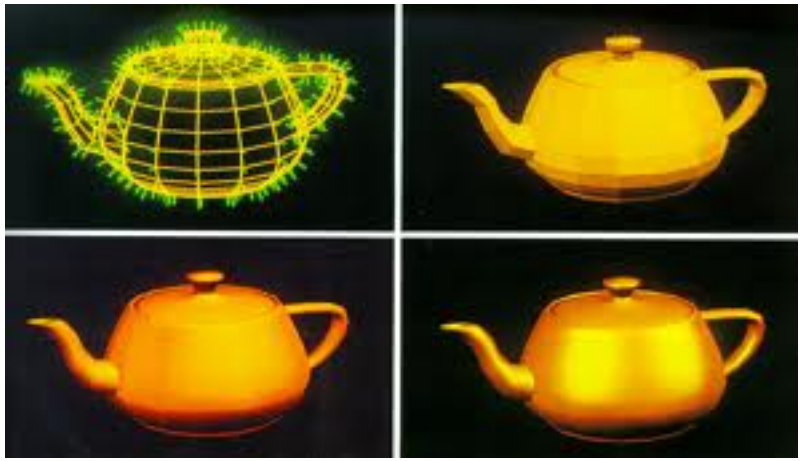
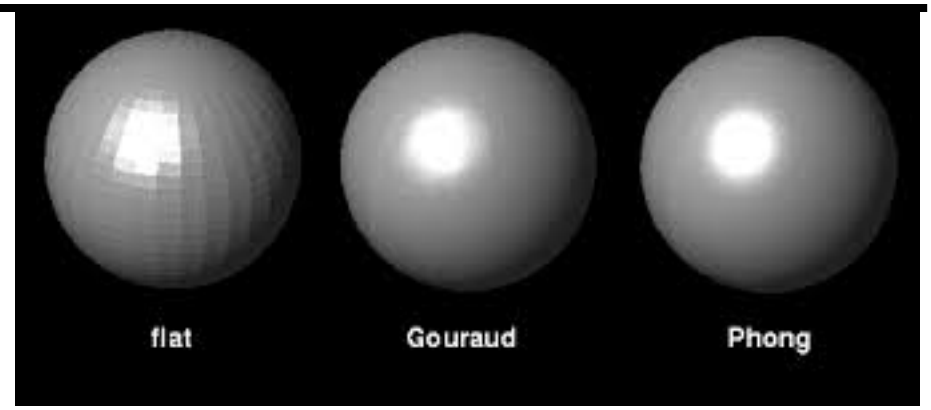
Comparison





Flat Shading

Gouraud Shading

Phong Shading

Gouraud versus Phong

Comparison

- If the polygon mesh approximates surfaces with a high curvatures, Phong shading may look smooth while Gouraud shading may show edges
- Phong shading requires much more work than Gouraud shading
  - Until recently not available in real time systems
  - Now can be done using fragment shaders
- Both need data structures to represent meshes so we can obtain vertex normals

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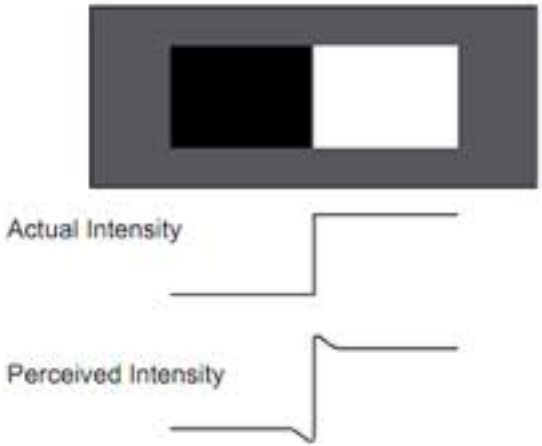
Comparison

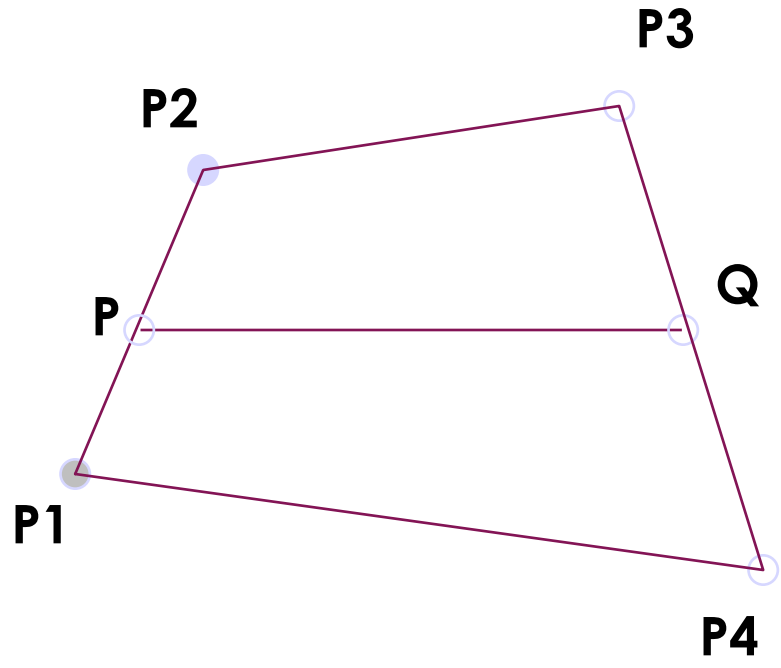
- Specular highlights more accurate in Phong
  - vertex highlight is much sharper
  - a highlight can occur within a polygon
- Mach banding greatly reduced
- Cost is higher because reflection model applied per pixel

Mach Band



Perspective Effects

- Anomalies occur because interpolation is carried out in screen space, after perspective transformation
- Suppose P2 much more distant than P1. P is midway in screen space so gets 50 : 50 intensity (Gouraud) or normal (Phong)
- ... but in world coordinates it is much nearer to P1 than P2



Averaging Normals

- Averaging normals of adjacent faces works
- Beware corrugated surfaces where averaging unduly smooths

GL SL functions

Lay of the Land

- OpenGL shading methods
  - per vertex vs per fragment shading
  - Where to carry out
- Discuss polygonal shading
  - Flat
  - Smooth
  - Gouraud

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OpenGL shading

- Need
  - Normals
  - Material properties
  - Lights
- State-based shading functions have been deprecated
  - glNormal, glMaterial, glLight
- Send attributes to shaders

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Normalization of Vectors

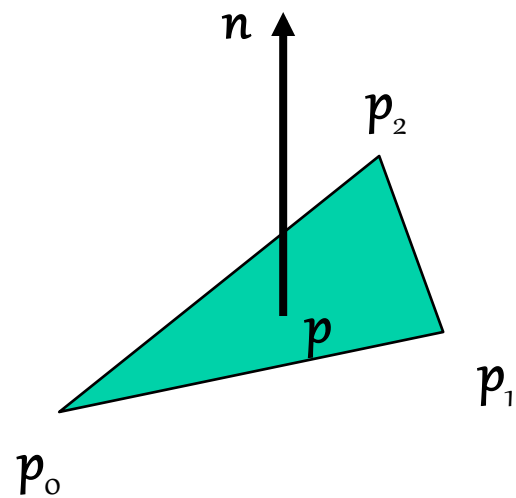
- Cosine terms in lighting computed using dot product
- Unit length vectors simplify calculation
- Set the magnitudes to have unit length but
  - Length can be affected by transformations
  - Note that scaling does not preserved length
- GLSL has a normalization function

Normal for Triangle

plane $\quad n \cdot (p - p_0) = 0$

$n = (p_2 - p_0) \times (p_1 - p_0)$

normalize n $\leftarrow$ $n / |n|$



Note that right-hand rule determines outward face

Point Light Source

For each light source, we can set an RGBA for the diffuse, specular, and ambient components, and for the position

```
vec4 diffuse0 =vec4(1.0, 0.0, 0.0, 1.0);
vec4 ambient0 = vec4(1.0, 0.0, 0.0, 1.0);
vec4 specular0 = vec4(1.0, 0.0, 0.0, 1.0);
vec4 light0_pos =vec4(1.0, 2.0, 3,0, 1.0);
```

Distance and Direction

- Source colors are specified in RGBA
- Position is given in homogeneous coordinates
  - If w =1.0, point source
  - If w =0.0 parallel source given direction vector
- Coefficients in distance terms
  - Quadratic 1/(a+b*d+c*d*d)
  - d is distance from point rendered to light source

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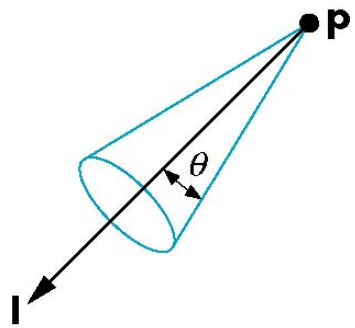
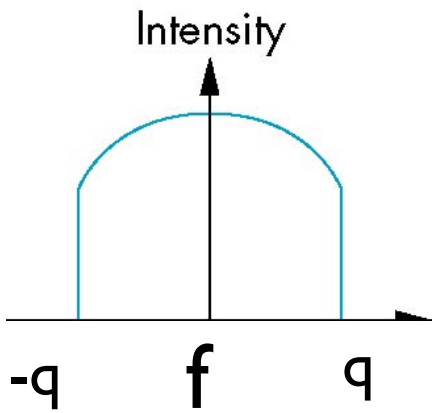
Spotlights

- Derive from point source
  - Direction
  - Cutoff
  - Attenuation Proportional to $\cos^a\phi$





Global Ambient Light

- Ambient light depends on color of light sources
  - A red light in a white room will cause a red ambient term that disappears when the light is turned off
- A global ambient term helpful for testing

Moving Light Sources

- Geometric objects whose positions, directions are affected by model-view matrix
- Depending on where we place the position (direction)
  - Move the light source(s) with the object(s)
  - Fix the object(s) and move the light source(s)
  - Fix the light source(s) and move the object(s)
  - Move the light source(s) and object(s) independently

Material Properties

- Material properties should match light model
- Reflectivities
- w component gives opacity

```
vec4 ambient = vec4(0.2, 0.2, 0.2, 1.0);
vec4 diffuse = vec4(1.0, 0.8, 0.0, 1.0);
vec4 specular = vec4(1.0, 1.0, 1.0, 1.0);
GLfloat shine = 100.0
```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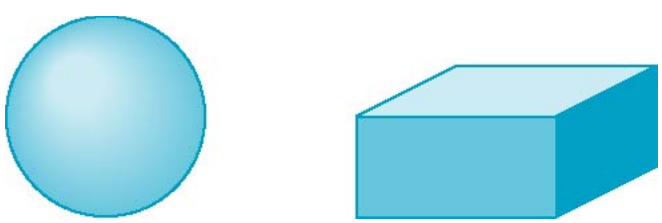
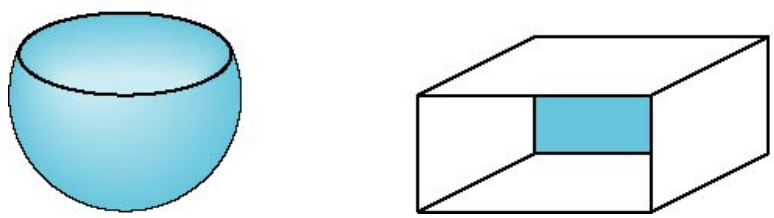
Front and Back Faces

- Every face has a front and back
- For many objects, we never see the back face so ignore
- we can handle in shader



back faces not visible



back faces visible

Emissive Term

- Simulate a light source in OpenGL by giving a material an emissive component

- This component is unaffected by any sources or transformations

Transparency

- Material properties as RGBA values
- A value can be used to make surface translucent
- Default is all surfaces are opaque regardless of A
- Enable blending

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Vertex Lighting Shaders I

```
// vertex shader
in vec4 vPosition;
in vec3 vNormal;
out vec4 color;  //vertex shade

// light and material properties
uniform vec4 AmbientProduct, DiffuseProduct, SpecularProduct;
uniform mat4 ModelView;
uniform mat4 Projection;
uniform vec4 LightPosition;
uniform float Shininess;
```

Vertex Lighting Shaders II

```
void main()
{
    // Transform vertex  position into eye coordinates
    vec3 pos = (ModelView * vPosition).xyz;

    vec3 L = normalize( LightPosition.xyz - pos );
    vec3 E = normalize( -pos );
    vec3 H = normalize( L + E );

    // Transform vertex normal into eye coordinates
    vec3 N = normalize( ModelView*vec4(vNormal, 0.0) ).xyz;
```


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Vertex Lighting Shaders III

```
// Compute terms in the illumination equation
    vec4 ambient = AmbientProduct;

    float Kd = max( dot(L, N), 0.0 );
    vec4  diffuse = Kd*DiffuseProduct;
    float Ks = pow( max(dot(N, H), 0.0), Shininess );
    vec4  specular = Ks * SpecularProduct;
    if( dot(L, N) < 0.0 )  specular = vec4(0.0, 0.0, 0.0, 1.0);
    gl_Position = Projection * ModelView * vPosition;

    color = ambient + diffuse + specular;
    color.a = 1.0;
}
```

Vertex Lighting Shaders IV

```
// fragment shader

in vec4 color;

void main()
{
    gl_FragColor = color;
}
```


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Fragment Lighting Shaders I

```
// vertex shader
in vec4 vPosition;
in vec3 vNormal;

// output values that will be interpolatated per-fragment
out vec3 fN;
out vec3 fE;
out vec3 fL;

uniform mat4 ModelView;
uniform vec4 LightPosition;
uniform mat4 Projection;
```

Fragment Lighting Shaders II

```
void main()
{
    fN = vNormal;
    fE = vPosition.xyz;
    fL = LightPosition.xyz;

    if( LightPosition.w != 0.0 ) {
        fL = LightPosition.xyz - vPosition.xyz;
    }

    gl_Position = Projection*ModelView*vPosition;
}
```

Fragment Lighting Shaders III

```
// fragment shader

// per-fragment interpolated values from the vertex shader
in vec3 fN;
in vec3 fL;
in vec3 fE;

uniform vec4 AmbientProduct, DiffuseProduct, SpecularProduct;
uniform mat4 ModelView;
uniform vec4 LightPosition;
uniform float Shininess;
```

Fragment Lighting Shaders IV

```
void main()
{
    // Normalize the input lighting vectors

   vec3 N = normalize(fN);
    vec3 E = normalize(fE);
    vec3 L = normalize(fL);

    vec3 H = normalize( L + E );
    vec4 ambient = AmbientProduct;
```

Fragment Lighting Shaders V

```
float Kd = max(dot(L, N), 0.0);
    vec4 diffuse = Kd*DiffuseProduct;

    float Ks = pow(max(dot(N, H), 0.0), Shininess);
    vec4 specular = Ks*SpecularProduct;

    // discard the specular highlight if the light's behind the vertex
    if( dot(L, N) < 0.0 )
          specular = vec4(0.0, 0.0, 0.0, 1.0);

    gl_FragColor = ambient + diffuse + specular;
    gl_FragColor.a = 1.0;
}
```

Next

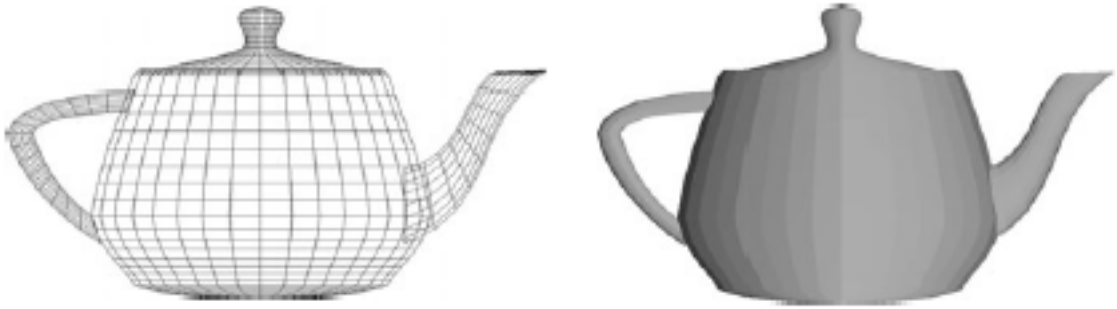

FIGURE 10.41 Rendered teapots.