Designing Localized Algorithms for Barrier Coverage

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ABSTRACT

Global barrier coverage that requires much fewer sensors than full coverage, is known to be an appropriate model of coverage for movement detection applications such as intrusion detection. However, it has been proved that given a sensor deployment, sensors can *not* locally determine whether the deployment provides global barrier coverage, making it impossible to develop localized algorithms, thus limiting its use in practice.

In this paper, we introduce the concept of *local barrier coverage* to address this limitation. Motivated by the observation that movements are likely to follow a shorter path in crossing a belt region, local barrier coverage guarantees the detection of all movements whose trajectory is confined to a slice of the belt region of deployment. We prove that it is possible for individual sensors to locally determine the existence of local barrier coverage, even when the region of deployment is arbitrarily curved. Although local barrier coverage does not always guarantee global barrier coverage, we show that for thin belt regions, local barrier coverage almost always provides global barrier coverage. To demonstrate that local barrier coverage can be used to design localized algorithms, we develop a novel sleep-wakeup algorithm for maximizing the network lifetime, called *Localized Barrier Coverage Protocol (LBCP)*. We show that LBCP provides close to optimal enhancement in network lifetime, while providing global barrier coverage most of the time. It outperforms an existing algorithm called Randomized Independent Sleeping (RIS) by up to 6 times.

Categories and Subject Descriptors

C.2.2 [**Computer-Communication Networks**]: Network Protocols; C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*network topology*

General Terms

Algorithms, Theory

Keywords

Wireless sensor networks, local barrier coverage, coverage, network topology, localized algorithms.

1. INTRODUCTION

Several important applications of wireless sensors involve movement detection, such as when deploying sensors along international borders to detect illegal intrusion, around forests to detect the spread of forest fire, around a chemical factory to detect the spread of lethal chemicals, on both sides of a gas pipeline to detect potential sabotage, etc. *Barrier coverage*, which guarantees that every movement crossing a barrier of sensors will be detected, is known to be an appropriate model of coverage for such applications [6].

Barrier coverage has several advantages over the *full coverage* model, a popular model that requires every point in the deployment region to be covered. First, barrier coverage requires much fewer sensors than full coverage. If the width of the deployment region is three times the sensing range, full coverage requires more than twice the density of barrier coverage. Saving in sensors grows linearly with width [6]. Second, the *sleep-wakeup* problem, that determines a sleeping schedule for sensors to maximize the network lifetime, is polynomial-time solvable for barrier coverage even when sensor lifetimes are not equal [8]. For the full coverage model, on the other hand, the sleep-wakeup problem is NP-Hard even if sensor lifetimes are assumed to be identical [12].

A major limitation of the barrier coverage model, however, is that unlike full coverage, individual sensors can *not* locally determine whether a network *does not* provide barrier coverage [6], making it impossible to develop localized algorithms. Consequently, almost all algorithms developed so far for barrier coverage, including the optimal sleep-wakeup algorithm, are centralized [8]. (The only exception is the Randomized Independent Sleeping (RIS) scheme [6], which does not require any message exchange.) Given the large scale and unattended nature of wireless sensor networks, localized algorithms are essential for scalability. A localized algorithm is also more adaptive to changes in the network, which is expected to be quite frequent in wireless sensor network due to unattended outdoor deployments. Therefore, in order to realize the benefits of the barrier coverage model in movement detection applications, there is a strong need to develop a new model that enables the development of localized algorithms, while retaining the benefits of barrier coverage.

We also observe that the notion of barrier coverage [6], which we will refer to as *global barrier coverage*, requires *every* crossing path to be covered, no matter how long it is. Thus, the sensor deployment in a 50m × 500km border (belt region) as shown in Figure 1, is regarded as *not* providing global barrier coverage due to the existence of an uncovered crossing path (which is more than 499km long). In real life, intruders are highly unlikely to follow such paths; it is more likely that a short path across the belt region is taken.

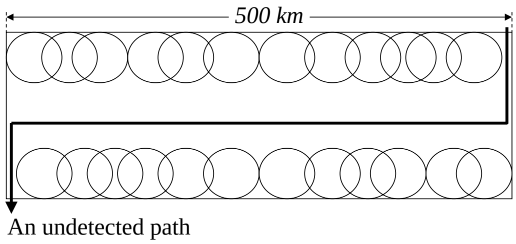

Figure 1: A belt is not global 1-barrier covered because of the existence of a long uncovered crossing path.

Motivated by these observations, we introduce in this paper the concept of *L-local barrier coverage*. It will be formally defined in Section 4, but informally, L-local barrier coverage guarantees the detection of all crossing paths whose trajectory is confined to a slice (of length L) of the belt region of deployment. In other words, if the bounding box that contains the entire trajectory of a crossing path, has a length at most L, then this crossing path is guaranteed to be detected by at least one (or k) sensor(s). For example, the crossing path in Figure 2 is guaranteed to be detected since its bounding box is of length less than L, if the sensor network deployed over this belt region provides L-local barrier coverage. The concept of L-local barrier coverage not only enables the development of localized algorithms, it also generalizes the (global) barrier coverage model; when L is equal to the length of the entire deployment region, L-local barrier coverage is equivalent to global barrier coverage.

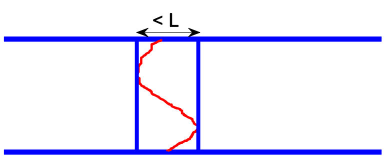

Figure 2: If the network provides L-local barrier coverage, then the crossing path shown is guaranteed to be detected since its bounding box has a length smaller than L.

A key question regarding L-local barrier coverage is how to determine whether a sensor network provides L-local barrier coverage. This question is nontrivial since there are infinitely many bounding boxes (each of length L). In this paper, we prove a theorem that allows a convenient discretization so that instead of checking each of the infinite bounding boxes to establish that a sensor network provides L-local barrier coverage, one only needs to check if the neighborhood of each sensor is barrier covered.

Although local barrier coverage does not always guarantee global barrier coverage (when L is less than the length of the deployment region), we show (by simulation) that for thin belt regions, local barrier coverage almost always provides global barrier coverage. This means that for thin belts, checking locally for the existence of local barrier coverage is sufficient to ensure global barrier coverage in practice. Intuitively, this holds because as the width of the deployment region approaches zero, local barrier coverage and global barrier coverage become equivalent.

To demonstrate that local barrier coverage can be used to design localized algorithms, we develop a sleep-wakeup algorithm for extending the network lifetime, called *Localized Barrier Coverage Protocol (LBCP)*. We show that LBCP provides close to optimal enhancement in network lifetime, while providing global barrier coverage most of the time. It outperforms an existing algorithm called Randomized Independent Sleeping (RIS) by up to 6 times.

Organization: Section 2 describes the network model, and Section 3 mentions some related work. Section 4 constitutes the theoretical foundation of L-local barrier coverage. A critical design issue — how to determine local barrier coverage? — is addressed in Section 5. Section 6 describes our localized sleep-wakeup protocol. Simulation results appear in Section 7. Sections 8 and 9 discuss the future work and conclude the paper, respectively. The proof of a theorem is presented in the Appendix.

2. THE NETWORK MODEL

The network model adopted in this paper is similar to that in [6]. We review here some essential definitions (with necessary modifications to suit the purpose of this paper). We also state a result from [6], which will be used later in this paper.

A sensor network, N, is a collection of sensors with their locations known. We use u to denote both a sensor node as well as the point of its location. We assume that a sensor network is deployed over a belt region. An example belt region is illustrated in Figure 3. To formally define a belt region, let $d(x, y)$ denote the Euclidean distance between two points x and y; and for a point x and a curve l, let $d(x, l)$ be the distance between x and l, i.e., $d(x, l) = \min\{d(x, y) : y \in l\}$. Two curves l_1 and l_2 are said to be *parallel with separation* w if $d(x, l_2) = d(y, l_1) = w$ for all $x \in l_1$ and $y \in l_2$.

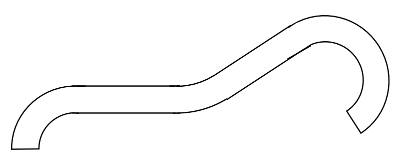

Figure 3: A general belt with two parallel boundaries

Definition 2.1. **[Belt of Width** W**]** *If l_1 and l_2 are two parallel curves with separation W, the region between l_1 and l_2 is referred to as a* belt (region) of width W*. The two curves l_1 and l_2 are the belt's* parallel boundaries.

For ease of presentation, we envision a belt region as roughly going from left to right. With such a convention, the belt's two parallel boundaries may be referred to as the top and the bottom boundary; and the other two boundaries, the left and the right.

Intrusion movement is assumed to occur from top to bottom. Thus, as in [6], a path is said to be a *crossing path* if it crosses from one parallel boundary to the other. A crossing path is *orthogonal* if its length is equal to w, the belt's width. Orthogonal crossing paths are straight lines and, therefore, often referred to as orthogonal crossing lines. For rectangular belts, orthogonal crossing lines are parallel to the belt's left and right sides.

A point p is covered (monitored) by a node u if their Euclidian distance is less than or equal to the sensing range, denoted by r. The sensing region of a node u is the set of all points covered by u. A crossing path is *k-covered* if it intersects the sensing region of at least k distinct sensors. Finally, a sensor network N provides *k-barrier coverage* over a deployment belt region $\mathcal{D}$ if all crossing paths through region $\mathcal{D}$ are k-covered by sensors in N.

DEFINITION 2.2. **[Coverage Graph, $\mathcal{G}(N)$]** *A coverage graph of a sensor network N is constructed as follows. Let $\mathcal{G}(N) = (V, E)$. The set V consists of a vertex corresponding to each sensor. In addition, it has two virtual nodes, s and t to correspond to the left and right boundaries. An edge exists between two nodes if their sensing regions overlap in the deployment region $\mathcal{D}$. An edge exists between u and s (or t) if the sensing region of u overlaps with the left boundary (or right boundary) of the region.*

THEOREM 2.1. *[6] A network N provides k-barrier coverage iff there exist k node-disjoint paths between the two virtual nodes s and t in $\mathcal{G}(N)$.*

Theorem 2.1 enables us to determine whether a belt region is k-barrier covered [6]. First, a coverage graph is constructed using the knowledge of which pairs of sensors have intersecting sensing regions. An algorithm to determine whether there exist at least k node-disjoint paths [11] in the coverage graph is then executed. Existence of k node-disjoint paths in the coverage graph implies the existence of k-barrier coverage.

Remark: A node can be active or sleeping. An active node can monitor the environment and communicate with other nodes; a sleeping node can do neither. When constructing the coverage graph, only active nodes are used.

Remark: Although we use a disk model here for the sensing region, our results hold for all other models for which a coverage graph can be constructed. We address this in Section 4.3.

Remark: We also note here that sensors do not continuously sample the environment and every time a sensor begins to sample the environment there is some startup latency. So, a sensor may not be able to detect an intruder if the intruder just touches the sensor's sensing region. However, if we assume the intruder's maximum movement speed is known, then for a given sampling frequency and a given startup latency, a conservative, smaller-than-actual sensing range can be calculated and used such that if an intruder ever touches this conservative sensing region, then he will stay in the actual, larger sensing region for sufficient time and the sensor will detect the intruder with very high probability.

3. RELATED WORK

We discuss some related work here. The concept of *barrier coverage* (which we call *global barrier coverage* in this paper) is introduced in [6]. A centralized algorithm to determine whether a network provides global barrier coverage is provided there. It is also proved there that it is not possible to locally determine whether a network provides global barrier coverage. The problem of deriving a reliable estimate for ensuring global barrier coverage in a random deployment, which had been an open problem [6], is comprehensively solved in [2]. The estimate derived here can be used in the RIS sleep wakeup algorithm to increase network lifetime while providing global barrier coverage. Although RIS is a purely local sleep wakeup algorithm, using the LBCP algorithm, which is also sufficiently local, can result in upto six-fold increase in the network lifetime.

An optimal sleep-wakeup algorithm for achieving global barrier coverage is proposed in [8]. This is a centralized algorithm. Our LBCP algorithm, on the other hand, is a localized algorithm that provides near-optimal performance, while ensuring global barrier coverage most of the time.

The model of full coverage has been extensively studied. A localized algorithm for determining whether a network does not provide full coverage is presented in [5]. Several heuristic algorithms for sleep-wakeup exist that attempt to maximize the network lifetime while maintaining full coverage [3, 4, 7, 12, 13]. As the sleep-wakeup problem is NP-Hard, no optimal algorithm (centralized or local) exist for this model.

Since local determination of global barrier coverage is not possible [6], our localized algorithm of course cannot guarantee global barrier coverage. However, it can ensure local barrier coverage for appropriately selected values of L, and thereby ensure that all crossing paths that are confined to a box of length at most L will surely be detected. In the unlikely event that a crossing path stretches to more than a length of L across the belt's length, it may still be detected, but is not guaranteed. In summary, since most crossing paths are likely to follow the shortest or close to the shortest path, our algorithm is practically sufficient for ensuring barrier coverage, while extending the network lifetime to close to optimal via local computation.

4. L-LOCAL BARRIER COVERAGE

The concept of L-local barrier coverage is introduced in Section 1. In this section, we formalize this new concept and address a key question: *Given a sensor deployment over a belt region, how does one determine if the deployment provides L-local barrier coverage?*

L-local barrier coverage and its properties are easier to describe and understand in a rectangular belt than in a general belt. So we begin with rectangular belts and then generalize the results for general belts. We first employ the sensing disk model, and then remark on the modifications necessary when other sensing models are used.

4.1 Rectangular Belts

We begin with some definitions. Consider a rectangular region with sensors deployed over it. Recall the definitions of *parallel boundaries* and *orthogonal crossing lines* made in Section 2. Figure 4 illustrates the following definitions.

DEFINITION 4.1. **[L-zone]** *For a positive number L, an L-zone is a slice of the belt region of length L. Two of its edges coincide with the belt's two parallel boundaries, and the other two edges are orthogonal crossing lines separated by a distance of L.*

An L-zone has four boundaries: two parallel boundaries and two orthogonal boundaries. The two orthogonal boundaries happen to be parallel here, but as will be seen later, they are not necessarily parallel in a general belt.

Definition 4.2. [$2d$-**zone**(u)] *For a positive value d, the $2d$-zone of a sensor node u, denoted by $2d$-zone(u), is an L-zone with $L = 2d$, in which the orthogonal crossing line passing through u divides the L-zone into two sections of equal length (each of length d).*

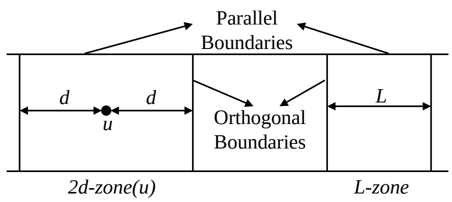

Figure 4: L-zone and $2d$-zone(u)

Recall the definition of k-barrier coverage in Sec. 2.

Definition 4.3. [**L-Local k-Barrier Coverage**] *For a positive number L and a positive integer k, a belt region is said to be L-local k-barrier covered if every L-zone in the region is k-barrier covered.*

Note that if a network provides L-local k-barrier coverage, then it provides M-local k-barrier coverage as well, for all $0 \leq M \leq L$. When $k = 1$, L-local k-barrier coverage is simply referred to as L-local barrier coverage.

We now address the above mentioned question of how to determine if a belt region is L-local k-barrier covered. We begin with a couple of lemmas. The first lemma indicates a condition under which any wide enough L-zone must contain at least one active sensor. Recall that r indicates the sensing range of each sensor node.

Lemma 4.1. *In a rectangular belt, if $d > r$ and $2d$-zone(u) for every active node u is k-barrier covered, then every L-zone with $L \geq 2r$, must contain at least one active node.*

Proof: Assume there is no node in an L-zone with $L \geq 2r$. Consider the node a closest to this L-zone. Without loss of generality, assume a is to the left of the L-zone. Then, the nodes on the right side of a must be to the right side of the L-zone. Therefore, there is no overlap between the coverage area of the nodes to the right of a and that of the nodes to the left of a (including a and those on the same orthogonal crossing line as a) because $L \geq 2r$. Then the nodes on the left side of a (including a and those on the same orthogonal crossing line) should provide k-barrier coverage for $2d$-zone(a), which is impossible because $d > r$. Therefore, there must be at least one active node in the L-zone. □

Even if two zones with overlap are individually k-barrier covered, their union as a single zone is not necessarily k-barrier covered. We prove in the following lemma a condition under which the union of two zones is k-barrier covered. The condition is that one of the two zones is k-barrier covered in a special way, and the other zone is relatively narrow.

Lemma 4.2. *Let A and B be two zones with intersection, with A of length $L_A \geq r$ and B of length $L_B \leq r$. Suppose A is k-barrier covered, but no node in $A - B$ covers A's orthogonal boundary that is contained in B. Then, $A \cup B$ is k-barrier covered.*

Proof: Because A is k-barrier covered, there must be at least k nodes covering A's orthogonal boundary in B. Since these nodes are not in $A - B$ and $L_B \leq r \leq L_A$, these nodes' sensing disks must also cover B's orthogonal boundary that is not in A (this boundary is also an orthogonal boundary of $A \cup B$). Then, the nodes making A k-barrier covered also make $A \cup B$ k-barrier covered. □

The following theorem indicates when, and for what value of L, we can conclude that a rectangular belt is L-local k-barrier covered.

Theorem 4.1. *Consider a rectangular belt with at least one active sensor node. If $2d$-zone(u) for every active node u is k-barrier covered for some $d > r$, then the entire belt is L-local k-barrier covered, with $L = \max\{2d - 2r, d + r\}$.*

Proof: Consider two possible cases: $d \geq 3r$ or $r < d < 3r$.

Case 1: $d \geq 3r$. In this case, $\max\{2d-2r, d+r\} = 2d-2r$. Let $L_1 = 2d - 2r$. We need to show that every L_1-zone is k-barrier covered. Given any L_1-zone as illustrated in Figure 5, there must be at least one active node in its center $2r$-zone according to Lemma 4.1. Let's say node b is in the $2r$-zone. Then, L_1-zone $\subseteq 2d$-zone(b) and the k-barrier coverage of $2d$-zone(b) implies the k-barrier coverage of the L_1-zone.

Case 2: $r < d < 3r$. In this case, $2d - 2r < d + r$. We will show every L_2-zone is k-barrier covered, where $L_2 = d + r > 2r$. Given any L_2-zone as shown in Figure 5, there must be an active node in the L_2-*zone* according to Lemma 4.1. If there is a node n in the center $(d - r)$-zone, then L_2-zone $\subseteq 2d$-zone(n) and therefore the k-barrier coverage of $2d$-zone(n) implies the k-barrier coverage of the L_2-zone.

If there is no node in the center $(d - r)$-zone, then there are nodes in the left or in the right r-zone. Without loss of generality, assume there are nodes in the left r-zone, and let m be the one closest to the center $(d - r)$-zone. By the assumption, $2d$-zone(m) is k-barrier covered. For ease of presentation, let A be $2d$-zone(m) and B be the right r-zone. Since there is no node in the center $(d - r)$-zone and m is the node closest to the center $(d - r)$-zone and $d > r$, no node in $A - B$ ever covers A's orthogonal boundary in B. The length of A is $2d > r$ and the length of B is r. According to Lemma 4.2, $A \cup B$ is k-barrier covered. Since L_2-zone $\subseteq A \cup B$, the L_2-zone is also k-barrier covered. This completes the proof. □

We require $d > r$ in Theorem 4.1. If $d \leq r$, then the k-barrier coverage of every $2d$-*zone*(u) does not imply the k-barrier coverage of every L-zone. We prove this in the next Theorem.

Theorem 4.2. *If $d \leq r$, then for any given value of $L > 0$, there exists a sensor deployment such that even if $2d$-zone(u) for every node u is k-barrier covered, the belt region is not L-local k-barrier covered.*

Proof: Consider a rectangular belt of length $2(L + r)$ or more. Place k sensors in the belt along a same orthogonal crossing line, but otherwise no other nodes in the belt. Now, $2d$-zone(u) for every node u is k-barrier covered, since $d < r$;

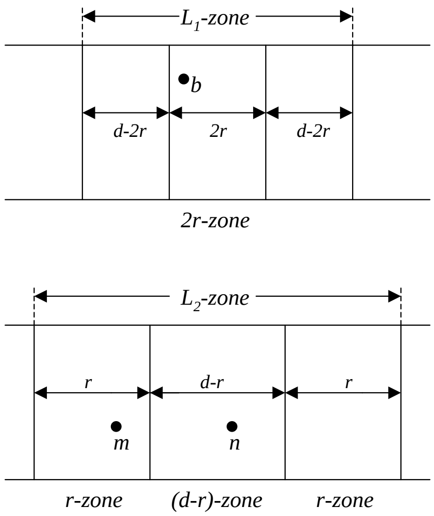

Figure 5: Visualizing the proofs of Theorem 4.1

but there apparently exists an L-zone which is not even 1-barrier covered. □

The L established in Theorem 4.1 is the largest such value possible. In other words, if $L > \max\{2(d-r), d+r\}$, then even if $2d$-zone(u) for every active node u is k-barrier covered, the sensor network does not necessarily provide L-local k-barrier coverage to the entire belt. We prove this in the next theorem.

THEOREM 4.3. *If $L > \max\{2d-2r, d+r\}$, then even if $d > r$, there exists a sensor deployment such that $2d$-zone(u) for every active node is k-barrier covered but the region is not L-local k-barrier covered.*

Proof: If $d \geq 3r$, then $2d - 2r \geq d + r$ and $L > 2d - 2r$. First consider $k = 1$. The example in Figure 6 shows that $2d$-zone(u) for all nodes u is 1-barrier covered but there exists an L-zone that is not barrier covered. For $k > 1$, replicate each node in Figure 6 k times. Now, the $2d$-zone of each active node is k-barrier covered but there exists an L-zone that is not even 1-barrier covered.

If $r < d < 3r$, then $2d - 2r < d + r$ and $L > d + r$. The argument for this case is similar to the previous case except that we now use the deployment shown in Figure 7. □

4.2 General Belts

Now, consider a belt of any shape as defined in Definition 2.1. For mathematical reasons, assume that curvature[1] exists everywhere on the belt's parallel boundaries, and that the largest curvature value over the entire belt boundaries is $1/R$ for some $R > 0$. Assume $R \gg r$, which informally means that the belt is not steeply curved as compared to the sensing range r.

We need to define L-zone for general belts, where, as in the case of rectangular belt, L is supposed to indicate the zone's

[1]Informally, for a curve in the plane, if curvature exists at a point, then there is a circle that best fits with the curve at that point. The reciprocal of that circle's radius is the curvature value at that point. Thus, the curvature value of any point on a circle with radius R is $1/R$, and the curvature value of any point on a straight line is 0.

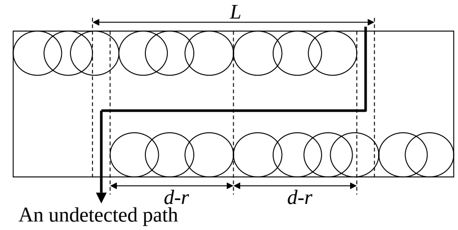

Figure 6: Example for Theorem 4.3 when $d \geq 3r$.

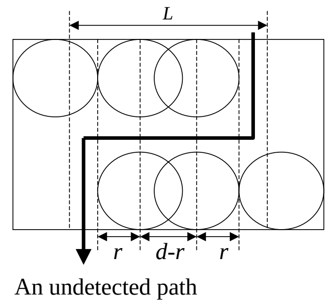

Figure 7: Example for Theorem 4.3 when $r < d < 3r$.

length. Now there is a question — a zone's two parallel boundaries in a general belt are mostly of different lengths; which length should be regarded as *the* length of the zone? We resolve this issue by measuring the zone's middle line, as defined below.

DEFINITION 4.4. [**Middle Line**] *The middle line of a belt is the curve parallel to, and at the middle between, the belt's two parallel boundaries. (So, the width between the middle line and either parallel boundary is one half of the belt's width.)*

DEFINITION 4.5. [**L-zone**] *For a positive value L, an L-zone on a belt is a slice of the belt bounded by the belt's two parallel boundaries and two orthogonal crossing lines, with the length of the zone's middle line being L.*

DEFINITION 4.6. [**$2d$-zone(u)**] *For a positive value d, the $2d$-zone of a sensor node u, denoted by $2d$-zone(u), is an L-zone with $L = 2d$, in which the orthogonal crossing line passing through u divides the L-zone's middle line into two sections of equal length (each of length d).*

These concepts are illustrated in Figure 8 using a circular belt.

With L-zone and $2d$-zone defined as above, let "L-local k-barrier covered" be defined as before (Definition 4.3).

The following theorem generalizes Theorem 4.1, and can be proved along a similar line of reasoning, even though the details are considerably more involved. In order not to interrupt the flow of presenting our main ideas, we defer the proof to the Appendix. In the theorem, W refers to the belt's width, and as before, r is the sensing range. We write $\arcsin(r/2R)$ to mean $\arcsin(r/(2R))$ — the parentheses are omitted for simplicity. The quantity $2(R+W/2)\arcsin(r/2R)$ and the formula for L can be more easily explained using a circular belt. In a circular belt, the inner circle's radius is R. If a zone (i.e., its middle line) is of length

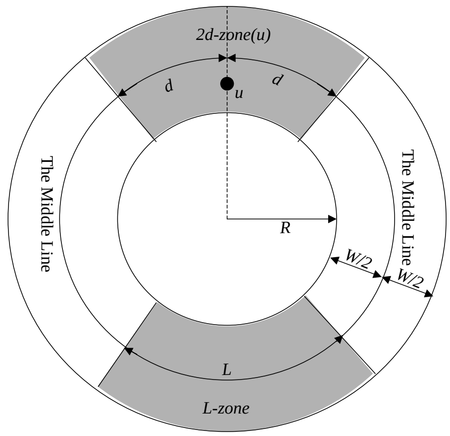

Figure 8: Middle line, L-zone, $2d$-zone

$2(R+W/2)\arcsin(r/2R)$, then the zone's inner boundary's chord is of length r. Similarly, a zone of length $2(R+W/2)\arcsin(2r/2R)$ has its inner boundary's chord being of length $2r$. If a zone is of length $(R+W/2)r/(R+W)$, then the zone's outer boundary is of length r. These formulas are explained in more detail in the Appendix.

THEOREM 4.4. *Consider a belt region with at least one active node deployed in it. Let $1/R$ be the largest curvature value on the belt's two parallel boundaries. If $2d$-zone(u) for every active node u in this belt is k-barrier covered for some $d > 2(R+W/2)\arcsin(r/2R)$, then the entire belt is L-local k-barrier covered, where L equals*

$$\max\left\{2d - 2\left(R+\frac{W}{2}\right)\arcsin\left(\frac{2r}{2R}\right), d + \left(\frac{(R+W/2)r}{R+W}\right)\right\}$$

Note that Theorem 4.4 is indeed a generalization of Theorem 4.1. As R approaches infinity, the belt becomes rectangular and the L in Theorem 4.4 approaches the L in Theorem 4.1.

4.3 Other Sensing Models

In Sections 4.1 and 4.2, we assume that the sensing region is a disk for simplicity. However, all of our results can be easily extended to other sensing models. For example, suppose that the sensing ranges are different in different directions, but there exist a maximum and a minimum value for the sensing ranges. Let $r_{\max}$ be the maximum value of the sensing ranges and $r_{\min}$ be the minimum. The following theorems are corresponding to Theorems 4.1 and 4.4, and can be proved in a similar fashion. (We omit the proof due to space limit.)

THEOREM 4.5. *Consider a rectangular belt with at least one active node deployed in it. If the $2d$-zone(u) for every active node u in this belt is k-barrier covered for some $d > r_{\max}$, then the entire belt is L-local k-barrier covered, with $L = \max\{2d - 2r_{\max}, d + r_{\min}\}$.*

THEOREM 4.6. *Consider a belt region with at least one active node deployed in it. Let $1/R$ be the largest curvature value on the two boundaries of the belt. If $2d$-zone(u) for every active node u in this belt is k-barrier covered for some $d > 2(R+W/2)\arcsin(r_{\max}/2R)$, then the entire belt is L-local k-barrier covered, where L equals*

$$\max\left\{2d - 2\left(R+\frac{W}{2}\right)\arcsin\left(\frac{2r_{\max}}{2R}\right), d + \frac{(R+W/2)r_{\min}}{R+W}\right\}$$

Note that we always get $L \geq d$ in Theorems 4.5 and 4.6 regardless of which sensing model is used as long as $r_{\min} \geq 0$. Therefore, we always can achieve any desired value of L if we make the value of d large enough.

5. IDENTIFYING A 2D-ZONE

Theorems 4.1, 4.4, and 4.6 ensure that in order to determine whether a network provides local barrier coverage, it is sufficient to check whether for some appropriate value d, the $2d$-zone of each node is barrier covered. *If* a sensor is able to identify the boundaries of its $2d$-zone, it can construct a coverage graph for this zone by obtaining sensing neighborhood data from all sensors in this zone. It can then determine whether its $2d$-zone is barrier covered using Theorem 2.1 as discussed in Section 2.

The main issue, therefore, is to develop a mechanism using which a sensor can locally determine the boundaries of its $2d$-zone. This job is trivial if the belt is rectangular or circular so that its parallel boundaries can be described using just a few parameters. For a general belt, however, especially when it is extremely long such as one along an international border, it is unrealistic to assume that each sensor has information about the belt's curvatures in its neighborhood. It is, therefore, nontrivial to recognize a node's $2d$-zone in a general belt region. We develop a heuristic for this nontrivial problem in this section.

Consider a node u which needs to identify its $2d$-zone. It is difficult, if not impossible, to recognize $2d$-zone(u) without knowing the belt's boundaries. Fortunately, since our purpose of recognizing a $2d$-zone is to ensure that it is k-barrier covered, it suffices to identify a region that encloses the $2d$-zone and then ensure that it is k-barrier covered. This will imply that the original $2d$-zone in question is k-barrier covered.

Conceptually, a region enclosing $2d$-zone(u) can be found as follows. Choose a sufficiently large value r_1 such that the entire $2d$-zone(u) is enclosed by the circle C_1 of radius r_1, centered at u. Let l_1 and l_2 be two lines that are tangent to C_1 on the opposite sides of the orthogonal crossing line passing through u, and each intersect the two long boundaries of the belt. The section S of the belt region between l_1 and l_2 evidently contains $2d$-zone(u), as illustrated in Figure 9.

To carry out the above scheme, there are two essential tasks: 1) estimating the value of r_1, and 2) identifying the two lines l_1 and l_2. (We do not need to identify the top and bottom boundaries of region S, because they play no role in constructing the coverage graph.)

For the value of r_1, we want it to be as small as possible. Thus, even though $r_1 = W + d$ is a valid estimate, $r_1 = \sqrt{\left(\frac{d(R+W)}{R+W/2}\right)^2 + W^2}$ can be easily verified to be a tighter bound, where $1/R$ is the biggest curvature value on the long boundaries of the belt.

To address the second issue, which is to select l_1 and l_2, the main idea of our heuristic is for u to choose two far away nodes p and q that are on the opposite sides of the orthogonal crossing line passing through u and satisfy the

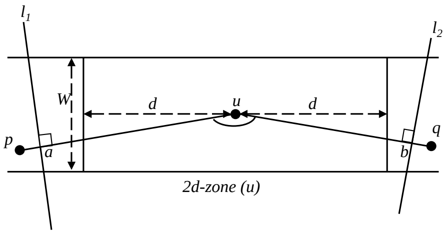

Figure 9: Identified $2d$-zone(u) is the slice of the belt region between lines l_1 and l_2, which contains the real $2d$-zone(u).

two conditions — $d(p,u) \geq r_1$ and $d(q,u) \geq r_1$. We will shortly discuss how to identify two such nodes. Line l_1 then is a line that is perpendicular to $\overline{pu}$ and at a distance of r_1 from u. Similarly, line l_2 is perpendicular to $\overline{qu}$ and r_1 away from u. (See Figure 9 for illustration.). Then, we claim that the slice S of the belt region between l_1 and l_2 contains $2d$-zone(u).

We now describe how to find the two nodes p and q. Meeting the requirement $d(p,u) \geq r_1$ and $d(q,u) \geq r_1$ is easy. Not so easy is to ensure that p and q are on the opposite sides of the orthogonal crossing line passing through u. (Let $l(u)$ denote this crossing line.) Intuitively, if the curvature of the belt is not too large, then two far away nodes on the opposite sides of $l(u)$ should form a large angle at u. Indeed, if we assume $R \gg W$, then there exist two values r_2 and r_2' such that for any two nodes p and q in the belt region with $r_2 \leq d(p,u) \leq r_2'$ and $r_2 \leq d(q,u) \leq r_2'$, it holds that $\angle puq \geq \pi/2$ if and only if p and q are on the opposite sides of $l(u)$. Using some elementary geometry, it can be shown that if $R \geq 3W$, then we can set $r_2 = \sqrt{2}\left(W + \frac{2W^2}{R}\right)$ and $r_2' = \sqrt{2}\left(R - \frac{2W^2}{R}\right)$. Now, u can select two nodes for p and q such that $r' \leq d(p,u) \leq r_2'$, $r' \leq d(q,u) \leq r_2'$, and $\angle puq \geq \pi/2$, where $r' = \max\{r_1, r_2\}$.

We now discuss an optimization to the process of searching for p and q. Since R may be considerably larger than W, so may r_2' than r'. In that case, letting u search all nodes in the range between r' and r_2' will be inefficient. To cut down the search domain, we will use a smaller value r_3 in place of r_2'. Consider two circles C' and C_3 centered at u with radii r', r_3, respectively, where $r' < r_3$. As the value of r_3 increases, the two slices S_1 and S_2 of the belt region that are between C' and C_3 grow, as well. According to Lemma A.2 in the Appendix, if all nodes' $2d$-zones are k-barrier covered, then there must be a value $r_3 > r'$ such that p and q exist in S_1 and S_2, respectively. Otherwise, at least one node's $2d$-zone is not k-barrier covered in the network. Again to keep r_3 as small as possible, we set $r_3 = \sqrt{(2r + r')^2 + 2W^2}$. It can be checked that if $R \gg r$, $R \gg d$, and $R \gg W$, then $r_2' > r_3$, and therefore a node u can use r_3 in place of r_2' in its search for p and q.

We now summarize our method for local identification of $2d$-zone(u).

1. Node u computes the values of r_1, r_2, r_3 using the values of W, d, r, and R as described above. Let $r' = \max\{r_1, r_2\}$.

2. Node u then finds two nodes p and q such that $r' \leq d(p,u) \leq r_3$, $r' \leq d(q,u) \leq r_3$, and $\angle puq \geq \pi/2$. If it can not find two such nodes, then it stops and reports that at least one node's $2d$-zone is not k-barrier covered by the network.

3. It draws a line l_1 perpendicular to $\overline{pu}$ and r_1 away from u. Similarly, it draws a line l_2 perpendicular to $\overline{qu}$ and r_1 away from u.

4. Node u uses the slice S of the belt region between l_1 and l_2 as an estimate for $2d$-zone(u).

We will use the above protocol to identify $2d$-zones in our localized sleep-wakeup protocol (to be presented in the next section). Note that the identification of $2d$-zone needs to be performed only once in the lifetime of a sensor network.

6. A LOCALIZED SLEEP-WAKEUP PROTOCOL

In this section, we use the local barrier coverage concept to design a localized sleep-wakeup algorithm for barrier coverage, called *Localized Barrier Coverage Protocol (LBCP)*, for maximizing the network lifetime. In Section 7, we will show that the LBCP protocol has a close-to-optimal performance and provides global barrier coverage most of the time for thin belt regions.

We first state few assumptions. We assume that each node has a unique ID as is common in newer platforms such as TelosB [10]. We also assume that the network has been localized so that each node knows its own location. In the event of localization inaccuracies, the identified $2d$-zone of a node u may not contain the real $2d$-zone(u). However, the error of the location, denoted by ϵ, only slightly affects the performance of LBCP. For example, in a rectangular belt we only need to increase the value of d to $d' = d + \epsilon$ to insure that the identified $2d'$-zone of a node u contains the real $2d$-zone(u). Further, we assume that a node is able to communicate with all nodes in its (identified) $2d$-zone. With the communication range increasing to $1000ft = 304.8m$ (see Mica2 data sheet [1]), this should be possible in thin belts. We also assume that nodes are able to estimate their remaining lifetimes by observing their battery drainage. Battery drainage rate can be observed in recent mote platforms [9]. Finally, we assume that the MAC protocol does not introduce too much latency; all LBCP packets are sent or received almost immediately.

We now describe the LBCP protocol. First, LBCP calculates the value of d according to the desired value of L. At any time, each node is in one of three states: *active, sleeping*, or *waking-up*. Assume every node is active in the beginning and executes the heuristic method described in Section 5 to identify its $2d$-zone. (Throughout the rest of this section, the term "$2d$-zones" refers to the $2d$-zones so computed.) If any node reports that at least one node's $2d$-zone is not k-barrier covered in the network, the algorithm terminates and reports failure, in which case the value of L needs to be reduced or more sensors need to be deployed. Otherwise, every node u calculates $\gamma_u = \max\{d(u,v) : v \in 2d\text{-zone}(u) \text{ or } u \in 2d\text{-zone}(v)\}$. Then, all nodes go back to sleep and wake up after a short random delay.

Upon waking up, each node executes the following procedure, called WAKEUP, to decide whether to become active or go back to sleep. In the protocol, T is pre-specified.

1. A waking-up node u broadcasts a *Query_W* packet in the range of γ_u.

2. When an active node v receives a *Query_W* packet from a node u, if u is in $2d$-zone(v) and the latter is currently not k-barrier covered, then v replies with a *Required_W* message containing its ID, position and lifetime. Otherwise, v replies with a *Not_Required_W* message containing its ID, position and lifetime.

3. If u receives any *Required_W* packet, u becomes active. If u does not receive any *Required_W* or *Not_Required_W* packet, which means there are no active nodes in $2d$-zone(u), u also becomes active. Otherwise, u goes back to sleep. Whenever u receives a *Required_W* packet or *Not_Required_W* packet, u records the ID, and corresponding positions and lifetimes contained in the packet.

4. If u decides to go back to sleep, u sleeps until T time units later or until the first active node in the range of γ_u is expected to die, whichever occurs earlier. (Waking up after T time is to protect against unanticipated sensor failures, or if the estimation of remaining lifetime is inaccurate.)

5. If u decides to become active, u broadcasts in the range of γ_u a *Decision_Active* packet containing u's ID, position and lifetime.

6. When an active node v receives u's *Decision_Active* packet, v records u's ID, position and lifetime.

Each active node periodically executes the following procedure, called ACTIVE, to decide whether to go back to sleep. Informally, A node u can go back to sleep if for every active node v such that $u \in 2d$-zone(v), $2d$-zone(v) will be k-barrier covered without u. However, two nodes each eligible for going to sleep may sometimes cause damage in barrier coverage if they *both* go to sleep. We let each node u maintain a set $A(u)$ to take care of this subtle problem. Initially, $\forall u, A(u) = \phi$.

1. An active node u broadcasts a *Query_A* packet in the range of γ_u after having been active for T time units, if $2d$-zone(u) will be k-barrier covered without u and the nodes in $A(u)$.

2. Whenever an active node v receives a node u's *Query_A* packet, if $2d$-zone(v) will be k-barrier covered without $A(v) \cup \{u\}$, then v adds u to $A(v)$ and replies with a *Not_Required_A* message. Otherwise, v replies with a *Required_A* message.

3. After issuing a *Query_A*, if u receives a *Not_Required_A* packet from every active node in the range of γ_u and does not receive any *Required_A* packet, then it decides to go to sleep. In that case, u broadcasts a *Decision_Sleep* packet in the range of γ_u and goes to sleep until T time later or until the first active node in the range of γ_u is expected to die, whichever occurs earlier. Otherwise, u stays active and broadcasts in the range of γ_u a *Decision_Continue* packet containing u's ID, position and lifetime.

4. Whenever an active node v receives a node u's *Decision_Sleep* packet, v removes u from its set of active nodes and removes u from $A(v)$. Whenever an active node v receives a node u's *Decision_Continue* packet, v records u's ID, position and lifetime if u is not in its set of active nodes; in addition, v removes u from $A(v)$.

5. If node u stays active, then every T time units it checks whether there have been new active nodes added in $2d$-zone(u) in the past time T (or since its last broadcast of *Query_A*). If so, and if $2d$-zone(u) will be k-barrier covered without the nodes in $A(u) \cup \{u\}$, then u broadcasts a *Query_A* packet again.

In LBCP, a node u communicates only with other nodes in the range of γ_u. When the length of the belt increases, while keeping the density constant, the computing and communication cost of a node remains invariant for a given value of d. In this sense, LBCP is a localized algorithm.

Assume that every node's $2d$-zone is k-barrier covered if all nodes are active. The LBCP protocol's goal is to ensure that every active node's $2d$-zone is k-barrier covered and therefore, by Theorem 4.4, the entire belt region is L-local k-barrier covered. The protocol also attempts to maximize the network life time. The performance of the LBCP protocol varies as d or T is varied.

Remark: The LBCP protocol can be slightly modified so that each active node u knows the remaining lifetime of every active node in the range of $2\gamma_u$. Furthermore, when u goes back to sleep, let it sleep until T time later or until the first active node in the range of $2\gamma_u$ (instead of γ_u) is expected to die. This version of LBCP may reduce the worst-case recovery time when some active node dies, but with a significantly higher message complexity.

Message Complexity: The number of messages transmitted by a node in LBCP protocol is insignificant. Let T be $f(\leq 1)$ times the lifetime of a node, and for any node u the number of nodes in the range of γ_u be at most D. Then, a node executes ACTIVE procedure at most $1/f$ times. There are at most $(2 + D)$ packets transmitted when a node executes the ACTIVE protocol. So, a node sends at most $(2 + D)/f$ packets in executing the ACTIVE procedure in its entire lifetime.

We assume there are at most m disjoint sets of nodes in a node's $2d$-zone such that the nodes in each set provide local barrier coverage for this $2d$-zone. Then, a node sleeps at most mF $(= mT/f)$ time units in its entire lifetime, where F is the life time of a node. There are two possible reasons making a sleeping node wake up. One reason is that it has slept for T time units; another reason is that some active node in its $2d$-node is going to die. a node wakes up at most m/f times for the first reason and at most D times for the second reason. There are at most $(2 + D)$ packets transmitted when a node executes the WAKEUP procedure. So, a node sends at most $(2 + D)(m/f + D)$ packets in executing the WAKEUP procedure in its entire lifetime.

Therefore, the total number of messages sent by a node in its entire lifetime is at most $(2 + D)((1 + m)/f + D)$. If $D = 100$, $f = 0.1$ and m=10, each node will transmit a maximum of $21,420$ packets. Given that transmitting a 60-byte packet consumes 0.01μAh on a Telos mote [10], transmissions of LBCP messages consume about 0.22 mAh, which is insignificant compared to more than $2,000$ mAh of energy reserve in a pair of AA batteries. Note that this analysis gives an upper bound and the real energy consumption may be much smaller than this upper bound.

7. PERFORMANCE EVALUATION

We have implemented LBCP protocol in MATLAB. We have three main results: 1) local barrier coverage almost always implies global barrier coverage when belts are thin, 2) the LBCP protocol provides close to optimal network lifetime while providing global barrier coverage most of the time, and 3) changing the belt from a rectangle to a general belt does not adversely affect the aforementioned performance. We define the network lifetime as the total time when the network is local barrier covered or the total time when the network is global barrier covered.

We use a belt region of dimension $2,000m \times 100m$, unless stated otherwise. Sensors are deployed randomly with uniform distribution. The default sensing range (r) is $30m$, and $k = 1$. For the LBCP protocol, lifetime of each node is 10 weeks, $d = 100m$, and $T = 0$. For every simulation case, 5 random scenarios have been simulated unless stated otherwise. We assume no packet loss, which can be ensured with a suitable reliable data transfer layer.

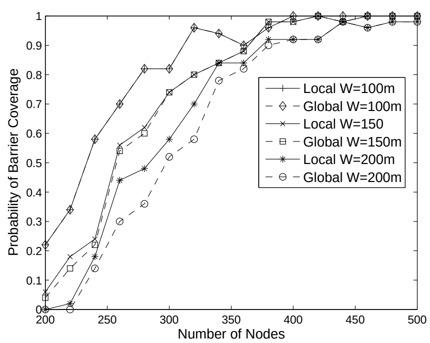

Figure 10: How often is the network local barrier covered vs. global barrier covered when $d = 31m$ and $W = 100m$, $150m$, or $200m$?

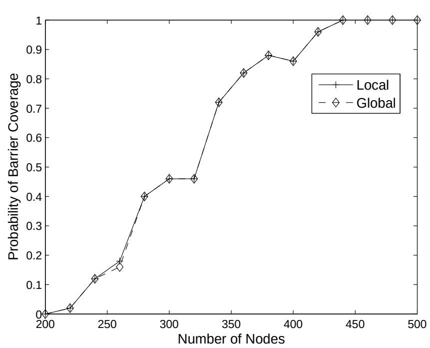

Figure 11: How often is the network local barrier covered vs. global barrier covered when $d = 100m$ and $W = 200m$?

7.1 Local Barrier Coverage vs. Global Barrier Coverage

We vary the density of nodes in random deployments to study the density at which the network begins to provide local barrier coverage and compare it with that for global barrier coverage. For every simulation case, 100 random scenarios have been simulated. To determine if the network provides local barrier coverage, we use Theorem 4.1, which ensures that if $d > r$, then barrier coverage of $2d$-zones of all nodes is sufficient to ensure L-local barrier coverage with $L = \max\{2(d-r), d+r\}$. Hence, we only need to check that the $2d$-zones of all nodes are barrier covered, rather than checking each of the L-zones, of which there are infinitely many.

The results of simulation appear in Figure 10. As can be seen from this figure, when the width (W) is $100m$, the network always provides global barrier coverage whenever it provides local barrier coverage, even if we use a value of d that is close to r. As the width of the region is increased, local barrier coverage does not always ensure global barrier coverage for small d. But, if a larger value of d (e.g., $100m$) is used (which implies a larger value of L in L-local barrier coverage), then local barrier coverage implies global barrier coverage even when the width is large as shown in Figure 11. In summary, for thin belts, local barrier coverage is sufficient for ensuring global barrier coverage, in practice.

7.2 Lifetime Maximization With LBCP

We investigate three main issues here.

1) What level of lifetime improvement is achieved using LBCP and how often does it provide global barrier coverage?

To determine the improvement in lifetime, we compare the performance of LBCP with the optimal (centralized) algorithm of [8] and with Randomized Independent Sleeping (RIS) of [6], which is a localized algorithm. We vary the number of nodes from 500 to 2,000. The simulation results are shown in Figure 12. We make three key observations from this figure. First, although LBCP only strives to provide local barrier coverage, it always provides global barrier coverage as well in our simulations. Second, it outperforms the RIS algorithm by up to 6 times (e.g., providing a lifetime of 246.7 weeks as opposed to 40.3 weeks for RIS when the number of nodes is 2,000). Third, it provides very close to the optimal network lifetime.

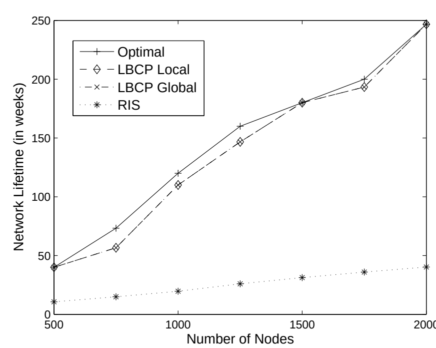

Figure 12: "LBCP Local" denotes that the network is local barrier covered with LBCP and "LBCP Global" denotes that the network is global barrier covered with LBCP. Optimal algorithm and RIS algorithm are both for global barrier coverage.

2) How does the performance of LBCP vary as d is varied?

In Figure 12, we use $d = 100$. For smaller values of d, LBCP does not always provide global barrier coverage; it only ensures local barrier coverage as can be seen in Figure 13. Although local barrier coverage may be sufficient in practice since most movements are expected to follow shortest or close to shortest paths, increasing the value of d ensures global barrier coverage, as well.

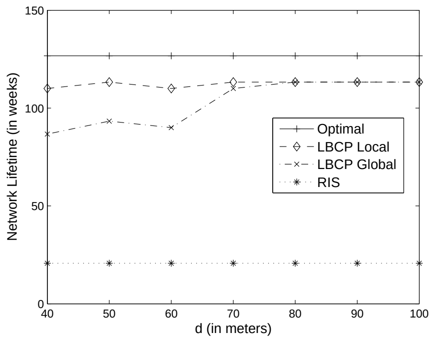

Figure 13: Network lifetime achieved with LBCP as the value of d is varied when 1000 nodes is randomly deployed in the network.

3) How does the performance of LBCP vary as T (the time period for checking the existence of local barrier coverage) is varied?

As can be seen in Figure 14, the performance of LBCP reduces with an increase in T. If T is equal to a node's lifetime, an active node continues to be active until dead, which may reduce the network lifetime. On the other hand, if $T = 0$, an active node checks immediately after a new node becomes active in its $2d$-zone if it can go back to sleep. Using a value close to 0 for T maximizes the network lifetime but involves significant overhead since an active node has to spend significant energy in periodic checking. Notice, however, that when $T = 0$, a sleeping node a wakes up only when the first active node in $2d$-zone(a) is expected to die. We suggest using $[0, 0.1]$ of a node's lifetime for T since even when $T = 0.1$ of a node's lifetime, an active node checks only 9 times in its entire lifetime, while the network life time can still reach 69% of the optimal solution.

7.3 The Performance for General Belts

In this section, we investigate the performance of LBCP combined with the heuristic method developed in Section 5 for determining the barrier coverage in a $2d$-zone for general belts. We consider a semicircular belt whose middle line is $\pi * 1,050m$ long. All other parameters (r, d, W, node lifetime) are the same as described in the beginning of Section 7. Since [6] did not indicate how to set the value of p in the RIS algorithm for a non-rectangular belt, we only compare the performance of LBCP with the optimal algorithm. We vary the number of nodes from 500 to $1,000$. The simulation results are shown in Figure 15. We make two key observations: 1) LBCP provides close to the optimal network lifetime, and 2) LBCP always provides global barrier coverage although it only strives to provide local barrier coverage, indicating that our heuristic (of Section 5) works well in practice.

8. DISCUSSIONS AND FUTURE WORK

Connectivity: If all active nodes' $2d$-zones are k-barrier covered, the network is connected under the assumption that a node is able to communicate (directly or indirectly) with all nodes in its $2d$-zone. This claim can be proved as follows. If an active node u's $2d$-zone is k-barrier covered, there must exist in $2d$-zone(u) two nodes p and q on the opposite sides of the orthogonal line passing u, since u and any nodes on the orthogonal line passing u can not cover the orthogonal boundaries of $2d$-zone(u). (See Lemma A.1 for details.) Since u can communicate with p and q, and p and q can communicate with all active nodes in $2d$-zone(p) and $2d$-zone(q), respectively, it follows that u can communicate with all active nodes in $S = 2d\text{-zone}(u) \cup 2d\text{-zone}(p) \cup 2d\text{-zone}(q)$. Repeating this argument for p and q, we can eventually conclude that u can communicate with all active nodes in the entire belt. Although the assumption that a node is able to communicate with all nodes in its $2d$-zone is reasonable, we will study, in our future work, under what conditions local barrier coverage implies connectivity without this assumption.

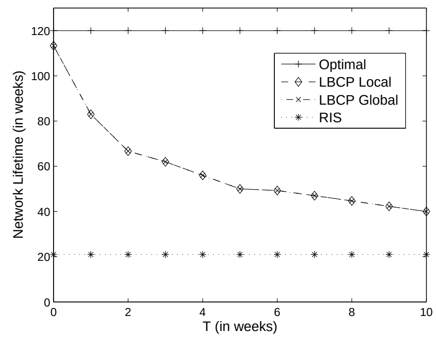

Figure 14: Network lifetime achieved with LBCP as the value of T is varied when 1000 nodes is randomly deployed in the network.

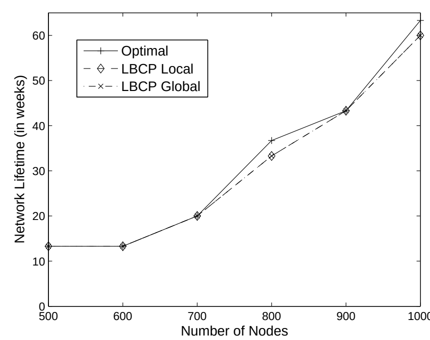

Figure 15: Network lifetime achieved with LBCP as the number of nodes is varied in a general belt.

Quality of Coverage: Global barrier coverage is a binary concept — either a sensor network provides global barrier coverage or it does not. The concept of L-local barrier coverage, on the other hand, can be used to measure the quality of barrier coverage provided by a sensor network. We can determine the maximum value for L such that the sensor network provides L-local barrier coverage, and use L as a measure of the network's quality of barrier coverage. If such measured quality does not meet the desired level, we can then place additional sensors to reach the desired level of quality. We have obtained some interesting results on this problem and will report them in a separate article.

Relationship between local and global barrier coverage: We observe in our simulations that local barrier coverage implies global barrier coverage with a high probability p for thin belts. But, as the belt's width increases, p decreases. It would be interesting to investigate a quantitative relationship between p and network parameters such as L (as in L-local barrier coverage), W (the belt's width), n (the number of nodes), and r (the sensing range).

9. CONCLUSIONS

We proposed a new model of coverage called *local barrier coverage* that made it possible to check locally whether a sensor network ensures that no movement can cross the network without being detected. We then provided a localized algorithm for sensors to determine whether the sensor network provides local barrier coverage. In simulations, we observed that for thin belt regions, the network provided global barrier coverage whenever it provided local barrier coverage. We leveraged the concept of local barrier coverage to develop the first localized sleep-wakeup algorithm for movement detection applications that provided close to optimal enhancement in the network lifetime. We showed that in addition to ensuring global coverage most of the time, local barrier coverage also ensured connectivity under some mild assumptions. By enabling the development of localized algorithms for barrier coverage, our work may have opened up many interesting research problems. For instance, localized algorithms for other tasks such as barrier-coverage network repair may now be explored.

10. REFERENCES

[1] *Mica2 Datasheet*. http://www.xbow.com/Products/Product_pdf_files/Wireless_pdf/MICA2_Datasheet.pdf.

[2] P. Balister, B. Bollobás, A. Sarkar, and S. Kumar. Reliable density estimates for coverage and connectivity in thin strips of finite length. In *Proc. ACM MobiCom*, 2007.

[3] M. Cardei, M. Thai, and W. Wu. Energy-Efficient Target Coverage in Wireless Sensor Networks. In *Proc. IEEE INFOCOM*, 2005.

[4] T. He and et al. Energy-Efficient Surveillance System Using Wireless Sensor Networks. In *Proc. ACM Mobisys*, 2004.

[5] C. Huang and Y. Tseng. The Coverage Problem in a Wireless Sensor Network. In *ACM International Workshop on Wireless Sensor Networks and Applications (WSNA)*, pages 115–121, San Diego, CA, 2003.

[6] S. Kumar, T. H. Lai, and A. Arora. Barrier Coverage with Wireless Sensors. In *Proc. ACM MobiCom*, 2005.

[7] S. Kumar, T. H. Lai, and J. Balogh. On k-Coverage in a Mostly Sleeping Sensor Network. In *Proc. ACM MobiCom*, 2004.

[8] S. Kumar, T. H. Lai, M. E. Posner, and P. Sinha. Optimal Sleep Wakeup Algorithms for Barriers of Wireless Sensors. In *Fourth International Conference on Broadband Communications, Networks, and Systems (IEEE BROADNETS)*, Raleigh, NC, 2007.

[9] B. Kuris and T. Dishongh. SHIMMER Mote:Hardware Guide. *Online at http://www.eecs.harvard.edu/~konrad/projects/shimmer/references/SHIMMER_HWGuide_REV1P3.pdf*, 2006.

[10] J. Polastre, R. Szewczyk, and D. Culler. Telos: Enabling Ultra-Low Power Wireless Research. In *Fourth international symposium on Information processing in sensor networks (IPSN)*, Los Angeles, CA, 2005.

[11] A. Schrijver. *Combinatorial Optimization*. Springer, 2003.

[12] S. Slijepcevic and M. Potkonjak. Power Efficient Organization of Wireless Sensor Networks. In *ICC*, Helsinki, Finland, 2001.

[13] H. Zhang and J. Hou. Maintaining Sensing Coverage and Connectivity in Large Sensor Networks. In *NSF International Workshop on Theoretical and Algorithmic Aspects of Sensor, Ad Hoc Wirelsss, and Peer-to-Peer Networks*, 2004.

APPENDIX

A. PROOF OF THEOREM 4.4

This section provides a proof sketch for Theorem 4.4. Due to space limit, we leave out many details. For simplicity, we write $\arcsin(r/(2R))$ as $\arcsin(r/2R)$.

Lemma A.1. *In a general belt, if $d > 2(R + \frac{W}{2}) \arcsin(\frac{r}{2R})$, then no node u can cover any orthogonal boundary of its $2d$-zone(u). Furthermore, any node on one side of the orthogonal line passing through u can not cover $2d$-zone(u)'s farther orthogonal boundary.*

Proof: First, consider a circular belt of width W, its inner circle being of radius R. That is, the curvature at any point of the belt's inner boundary is of magnitude $1/R$. Consider an L_2-zone, with a line of length r, an arc of length L_1, $\angle A$, $\angle B$ as shown in Figure 16. We know that $\angle B = 2\angle A = 2\arcsin(r/2R)$. Therefore, $L_1 = 2R\arcsin(r/2R)$, and $L_2 = 2(R + W/2)\arcsin(r/2R)$. The distance between the two orthogonal boundaries of L_2-zone is r. If a node u is at the intersection of the L_2-zone's inner circular boundary with one of its orthogonal boundaries, then u's sensing range can barely cover the other orthogonal boundary of the L_2-zone. If $L > L_2$, then the sensing range of any node on one orthogonal boundary of an L-zone can not cover the other orthogonal boundary since the distance between the two orthogonal boundaries is larger than r, and any node outside of an L-zone can not cover the farther orthogonal boundary of the L-zone. Therefore, if $d > L_2$, since $2d$-zone(u) can be divided into two d-zones at the orthogonal line passing through u, no node u can cover any orthogonal boundary of its $2d$-zone(u), and any node on one side of the orthogonal line passing through u can not cover $2d$-zone(u)'s farther orthogonal boundary.

Next, we show that the lemma holds for a circular belt of width W, whose inner circle is of radius $R' > R$. It is clear that $2(R' + \frac{W}{2})\arcsin(\frac{r}{2R'}) < 2(R + \frac{W}{2})\arcsin(\frac{r}{2R})$. So, in this belt, if $d > 2(R + \frac{W}{2})\arcsin(\frac{r}{2R})$, we also get $d > 2(R' + \frac{W}{2})\arcsin(\frac{r}{2R'})$, which implies that no node u can cover any orthogonal boundary of its $2d$-zone(u), and any node on one side of the orthogonal line passing through u can not cover $2d$-zone(u)'s farther orthogonal boundary.

Now, we consider a general belt. By assumption, the curvature at any point on the belt's parallel boundaries is of magnitude at most $1/R$. That is, at any point on the parallel boundaries, the curvature is of magnitude $1/R'$, with $R' \geq R$. Therefore, if $d > 2(R+W/2)\arcsin(r/2R)$, no node u can cover any orthogonal boundary of its $2d$-zone(u), and any node on one side of the orthogonal line passing through u can not cover $2d$-zone(u)'s farther orthogonal boundary in a general belt. □

LEMMA A.2. *In a general belt, if* $d > 2(R+\frac{W}{2})\arcsin(\frac{r}{2R})$, *and the* $2d$*-zone(*u*) of every active node* u *is* k*-barrier covered, then each* M*-zone with* $M \geq 2(R+W/2)\arcsin(2r/2R)$ *must contain at least one active node.*

Proof: For a circular belt, if the inner circle's radius is R and $M_2 = 2(R+W/2)\arcsin(2r/2R)$, then the length of the chord of the M_2-zone on the inner circle is $2r$, as shown in Figure 16. In this case, the sensing disks of any two nodes outside of the M_2-zone and on the opposite sides of the M_2-zone have no overlap in the M_2-zone. It is clear that if $R' > R$, $2(R'+W/2)\arcsin(2r/2R') < 2(R+W/2)\arcsin(2r/2R)$. As in the proof of Lemma A.1, in a general belt whose biggest curvature value on the two parallel boundaries is $1/R$, if $L \geq 2(R+W/2)\arcsin(r/R)$, then the sensing disks of any two nodes outside of the M_2-zone and on the opposite sides of the M_2-zone have no overlap in the M_2-zone. Clearly, the conclusion is true for any $M \geq M_2$ if it is true for M_2.

Now, assume there are no nodes in an M-zone with $M \geq M_2$. Consider the node a whose orthogonal crossing line is closest to the M-zone. Then, there is no node in the region between a's orthogonal crossing line and the M-zone (as well as the M-zone itself). Since there is no overlap between the sensing disks of any two nodes on the opposite sides of the M-zone, the nodes on the same side of the M-zone as a (including a) should provide k-barrier coverage for $2d$-zone(a) (including its farther orthogonal boundary), which is impossible according to Lemma A.1. Therefore, there must be at least one node in each M-zone. □

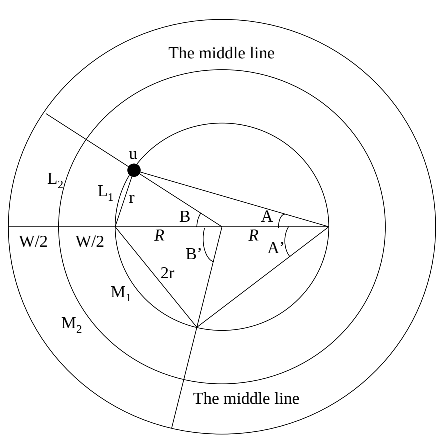

Figure 16: Visualizing the proofs of Lemmas A.1 and A.2

LEMMA A.3. *In a general belt, let* A *be an* L_1*-zone and* B *be an* L_2*-zone. Assume that the intersection of* A *and* B *is non-empty, and* $L_2 \leq \frac{R+W/2}{R+W}r$. *Suppose* A *is* k*-barrier covered, and no node in* $A - B$ *ever covers* A*'s orthogonal boundary that is contained in* B*. Then,* $A \cup B$ *is* k*-barrier covered.*

Proof: Let a_1 be the A's orthogonal boundary that is in B, and a_2 be the A's orthogonal boundaries that is not in B. Let b_1 be the B's orthogonal boundary that is in A, and b_2 the B's orthogonal boundaries that is not in A. So, a_2 and b_2 are also the orthogonal boundaries of $A \cup B$.

First, we prove that the sensing disk of any node in B covers both of the orthogonal boundaries of B, b_1 and b_2. Consider see a circular blet with the inner circle's radius being R. If $L_2 = \frac{R+W/2}{R+W}r$, then the length of the longer parallel boundary (on the outer circle) of B is r and the length of the corresponding chord $\leq r$. Let p be an arbitrary node in B, and consider the passing orthogonal line b_p of p and b_1, we get a sub-zone of B. Clearly, the lengths of the parallel boundaries of this sub-zone $\leq r$, and the distance between p and $b_1 \leq r$. Therefore, the sensing disk of p covers b_1. Similarly, we can prove that the sensing disk of p covers b_2. If $R' > R$, $\frac{R'+W/2}{R'+W}r = (1 - \frac{W}{2(R'+W)})r > \left(1 - \frac{W}{2(R+W)}\right) r = \frac{R+W/2}{R+W}r$. Following the idea used in the proof of Lemma A.1, we can prove that for a general belt whose biggest curvature value is $1/R$ on the parallel boundaries, if $L_2 \leq \frac{R+W/2}{R+W}r$, the lengths of the parallel boundaries of $B \leq r$, and the sensing disk of any node in B covers both of the orthogonal boundaries of B.

Because A is k-barrier covered, there must be at least k nodes covering a_1. Since these nodes are not in $A - B$, they must be in B, or outside of B but on the other side of b_2 as compared to a_1. If they are outside of B, their sensing disks must cover b_2 if they cover a_1. If they are in B, their sensing disks also must cover b_2. Therefore, the nodes making A k-barrier covered also make $A \cup B$ k-barrier covered. □

Proof of Theorem 4.4: The proof is similar to that of Theorem 4.1, but now we will use Lemmas A.2 and A.3 in place of Lemmas 4.1 and 4.2.

Assume that $2d$-zone(u) is k-barrier covered for every active node u. Let $L_1 = 2d - 2(R + W/2)\arcsin\left(\frac{2r}{2R}\right)$ and $L_2 = d + \frac{R+W/2}{R+W}r$.

Case 1: $L_1 \geq L_2$. Let $M = 2(R + W/2)\arcsin\left(\frac{2r}{2R}\right)$. By Lemma A.2, given any L_1*-zone*, there is at least one node b in its center M-zone. Then, L_1-zone $\subseteq 2d$-zone(b) and hence L_1-zone is k-barrier covered.

Case 2: $L_1 < L_2$. Given any L_2-zone, it can be divided into three parts: the center L_c-zone, and the two L_s-zones on the opposite sides of the center zone, where $L_c = \left(d - \frac{R+W/2}{R+W}r\right)$ and $L_s = \frac{R+W/2}{R+W}r$. Then, it can be proved in a similar manner as in the proof of Theorem 4.1 (case 2) using Lemma A.3 in place of Lemma 4.2 that L_2-zone is k-barrier covered.

Therefore, if the $2d$-zone of every active node is k-barrier covered, then every L-zone is k-barrier covered, where $L = \max\{L_1, L_2\}$. □