CSE 6331 Homework 3

Due: Thursday, January 25 by class time

1. A function $T(n)$ (defined for positive integers n) satisfies the following recurrence:

$$T(n) = \begin{cases} c, & \text{if } n \leq 1 \\ 3T(\lfloor n/4 \rfloor) + n, & \text{if } n > 1 \end{cases}$$

   where c ia a positive constant. Prove that $T(n)$ is asymptotically nondecreasing.

2. Determine the tight asymptotic complexity of the following function. Give your answer in Θ notation with a proof.

$$T(n) = \begin{cases} b, & \text{if } n \leq 3 \\ T(\lfloor n/2 \rfloor) + T(\lfloor n/4 \rfloor) + cn, & \text{if } n > 3 \end{cases}$$

3. Determine the tight asymptotic complexity of the following function. Give your answer in Θ notation with a proof.

$$T(n) = \begin{cases} b, & \text{if } n \leq 3 \\ T(\lfloor n/2 \rfloor) + 2T(\lfloor n/4 \rfloor) + cn, & \text{if } n > 3 \end{cases}$$

4. Use the master method to solve the following recurrences.

   (a) $T(n) = 4T(n/2) + n^2$.

   (b) $T(n) = 4T(n/2) + n^2 \log^2 n$.

   (c) $T(n) = 4T(n/2) + n^3$.

5. The running time of an algorithm A is described by the recurrence $T(n) = 7T(n/2)+n^2$. A competing algorithm A' has a running time of $T'(n) = aT'(n/4)+n^2$. What is the largest integer value for a such that A' is asymptotically faster than A.